Table 1: **Properties of the Continuous-Time Fourier Transform**

$$x(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega$$

$$X(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

Property	**Aperiodic Signal**	**Fourier transform**
	$x(t)$	$X(\omega)$
	$y(t)$	$Y(\omega)$
Linearity	$ax(t) + by(t)$	$aX(\omega) + bY(\omega)$
Time-shifting	$x(t - t_0)$	$e^{-j\omega t_0} X(\omega)$
Frequency-shifting	$e^{j\omega_0 t} x(t)$	$X(\omega - \omega_0)$
Conjugation	$x^*(t)$	$X^*(-\omega)$
Time-Reversal	$x(-t)$	$X(-\omega)$
Time- and Frequency-Scaling	$x(at)$	$\frac{1}{\lvert a\rvert} X\left(\frac{\omega}{a}\right)$
Convolution	$x(t) * y(t)$	$\sqrt{2\pi} X(\omega) Y(\omega)$
Multiplication	$x(t)y(t)$	$\frac{1}{\sqrt{2\pi}} X(\omega) * Y(\omega)$
Differentiation in Time	$\frac{d^n}{dt^n} x(t)$	$(j\omega)^n X(\omega)$
Integration	$\int_{-\infty}^{t} x(t) dt$	$\frac{1}{j\omega} X(\omega) + \pi X(0) \delta(\omega)$
Differentiation in Frequency	$t^n x(t)$	$j^n \frac{d^n}{d\omega^n} X(\omega)$

Table 2: **Basic Continuous-Time Fourier Transform Pairs**

Signal	**Fourier transform**
1	$\sqrt{2\pi}\,\delta(\omega)$
e^{jat}	$\sqrt{2\pi}\,\delta(\omega-a)$
$\cos at$	$\frac{\sqrt{2\pi}}{2}[\delta(\omega-a)+\delta(\omega+a)]$
$\sin at$	$\frac{\sqrt{2\pi}}{2j}[\delta(\omega-a)-\delta(\omega+a)]$
$\cos at^2$	$\frac{1}{\sqrt{2a}}\cos\left(\frac{\omega^2}{4a}-\frac{\pi}{4}\right)$
$\sin at^2$	$\frac{-1}{\sqrt{2a}}\sin\left(\frac{\omega^2}{4a}-\frac{\pi}{4}\right)$
$\sum_{n=-\infty}^{+\infty}\delta(t-nT)$	$\frac{\sqrt{2\pi}}{T}\sum_{k=-\infty}^{+\infty}\delta\left(\omega-\frac{2\pi k}{T}\right)$
$\Pi(at)$	$\frac{1}{\sqrt{2\pi a^2}}\ \mathrm{sinc}\left(\frac{\omega}{2\pi a}\right)$
$\mathrm{sinc}(at)$	$\frac{1}{\sqrt{2\pi a^2}}\ \Pi\left(\frac{\omega}{2\pi a}\right)$
$\mathrm{sinc}^2(at)$	$\frac{1}{\sqrt{2\pi a^2}}\ \Lambda\left(\frac{\omega}{2\pi a}\right)$
$t^n,\ n\in\mathbb{N}$	$j^n\sqrt{2\pi}\,\delta^{(n)}(\omega)$
$t^{-n},\ n\in\mathbb{N}$	$-j\sqrt{\frac{\pi}{2}}\,\frac{(-j\omega)^{n-1}}{(n-1)!}\mathrm{sgn}(\omega)$
$\lvert t\rvert^{\alpha},\ 0>\alpha>-1$	$\frac{-2}{\sqrt{2\pi}}\,\frac{\sin(\pi\alpha/2)\Gamma(\alpha+1)}{\lvert\omega\rvert^{\alpha+1}}$
$\delta(t)$	$1/\sqrt{2\pi}$
$\delta(t-t_0)$	$e^{-j\omega t_0}/\sqrt{2\pi}$
$\Lambda(at)$	$\frac{1}{\sqrt{2\pi a^2}}\ \mathrm{sinc}^2\left(\frac{\omega}{2\pi a}\right)$
$\mathrm{Heaviside}(t)$	$\sqrt{\frac{\pi}{2}}\left(\frac{1}{j\pi\omega}+\delta(\omega)\right)$
$\mathrm{sgn}(t)$	$\sqrt{\frac{2}{\pi}}\,\frac{1}{j\omega}$
$e^{-\alpha t^2}\alpha>0$	$\frac{1}{\sqrt{2\alpha}}\cdot e^{-\frac{\omega^2}{4\alpha}}$
$\mathrm{e}^{-a\lvert x\rvert}\ a>0$	$\sqrt{\frac{2}{\pi}}\,\frac{a}{a^2+\omega^2}$
$e^{-at}\mathrm{Heaviside}(t),\ a>0$	$\frac{1}{\sqrt{2\pi}(a+j\omega)}$
$\log\lvert t\rvert$ $\gamma=\lim_{n\to\infty}\left(\sum_{k=1}^{n}\frac{1}{k}-\ln(n)\right)=\int_1^\infty\left(\frac{1}{\lfloor x\rfloor}-\frac{1}{x}\right)dx.$	$-\frac{\sqrt{\pi/2}}{\lvert\omega\rvert}-\sqrt{2\pi}\gamma\delta(\omega),$
$(\mp jt)^{-\alpha},\ 0<\alpha<1$	$\frac{\sqrt{2\pi}}{\Gamma(\alpha)}u(\pm\omega)(\pm\omega)^{\alpha-1}$
$J_0(t)$	$\sqrt{\frac{2}{\pi}}\,\frac{\Pi\left(\frac{\omega}{2}\right)}{\sqrt{1-\omega^2}}$
$J_n(t)$	$\sqrt{\frac{2}{\pi}}\,\frac{(-j)^nT_n(\omega)\Pi\left(\frac{\omega}{2}\right)}{\sqrt{1-\omega^2}}$

Table 3: **Properties of the Laplace Transform** $X(s) = \int_0^\infty x(t)e^{-st}\,dt$

Property	**Signal**	**Transform**
Linearity	$ax_1(t) + bx_2(t)$	$aX_1(s) + bX_2(s)$
Time shifting	$x(t - t_0)$	$e^{-st_0}X(s)$
Shifting in the s-Domain	$e^{s_0 t}x(t)$	$X(s - s_0)$
Time scaling	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{s}{a}\right)$
Conjugation	$x^*(t)$	$X^*(s^*)$
Convolution	$x_1(t) * x_2(t)$	$X_1(s)X_2(s)$
Differentiation in the Time Domain	$\frac{d^n}{dt^n}x(t)$	$s^n X(s) - \sum_{k=1}^{n} s^{k-1}x^{(n-k)}(0)$
Differentiation in the s-Domain	$t^n x(t)$	$(-1)^n \frac{d^n}{ds^n}X(s)$
Integration in the Time Domain	$\int_{-\infty}^{t} x(\tau)d(\tau)$	$\frac{1}{s}X(s)$

Table 4: **Laplace Transforms of Elementary Functions**

Signal	**Transform**	**Region of convergence**
$\delta(t)$	1	All s
$\delta(t-\tau)$	$\mathrm{e}^{-\tau s}$	All s
$\text{Heaviside}(t)$	$\frac{1}{s}$	$\Re e\{s\} > 0$
$-\text{Heaviside}(-t)$	$\frac{1}{s}$	$\Re e\{s\} < 0$
$\text{Heaviside}(t-\tau)$	$\frac{\mathrm{e}^{-\tau s}}{s}$	$\Re e\{s\} > 0$
$\frac{t^{n-1}}{(n-1)!}\text{Heaviside}(t)$	$\frac{1}{s^n}$	$\Re e\{s\} > 0$
$-\frac{t^{n-1}}{(n-1)!}\text{Heaviside}(-t)$	$\frac{1}{s^n}$	$\Re e\{s\} < 0$
$-t^{\frac{1}{n}}\,\text{Heaviside}(t)$	$\frac{\Gamma(\frac{1}{n}+1)}{s^{\frac{1}{n}+1}}$	$\Re e\{s\} > 0$
$e^{-\alpha t}\text{Heaviside}(t)$	$\frac{1}{s+\alpha}$	$\Re e\{s\} > -\Re e\{\alpha\}$
$-e^{-\alpha t}\text{Heaviside}(-t)$	$\frac{1}{s+\alpha}$	$\Re e\{s\} < -\Re e\{\alpha\}$
$\frac{t^{n-1}}{(n-1)!}e^{-\alpha t}\text{Heaviside}(t)$	$\frac{1}{(s+\alpha)^n}$	$\Re e\{s\} > -\Re e\{\alpha\}$
$-\frac{t^{n-1}}{(n-1)!}e^{-\alpha t}\text{Heaviside}(-t)$	$\frac{1}{(s+\alpha)^n}$	$\Re e\{s\} < -\Re e\{\alpha\}$
$e^{-\alpha\|t\|}$	$\frac{2\alpha}{\alpha^2-s^2}$	$-\Re e\{\alpha\} < \Re e\{s\} < \Re e\{\alpha\}$
$\delta(t-T)$	e^{-sT}	All s
$[\cos\omega_0 t]\text{Heaviside}(t)$	$\frac{s}{s^2+\omega_0^2}$	$\Re e\{s\} > 0$
$[\sin\omega_0 t]\text{Heaviside}(t)$	$\frac{\omega_0}{s^2+\omega_0^2}$	$\Re e\{s\} > 0$
$[e^{-\alpha t}\cos\omega_0 t]\text{Heaviside}(t)$	$\frac{s+\alpha}{(s+\alpha)^2+\omega_0^2}$	$\Re e\{s\} > -\Re e\{\alpha\}$
$[e^{-\alpha t}\sin\omega_0 t]\text{Heaviside}(t)$	$\frac{\omega_0}{(s+\alpha)^2+\omega_0^2}$	$\Re e\{s\} > -\Re e\{\alpha\}$
$u_n(t) = \frac{d^n\delta(t)}{dt^n}$	s^n	All s
$u_{-n}(t) = \underbrace{\text{Heaviside}(t) * \cdots * \text{Heaviside}(t)}_{n \text{ times}}$	$\frac{1}{s^n}$	$\Re e\{s\} > 0$