Igor Fabrici, František Kardoš eds.

Workshop
Cycles and Colourings 2012

9th – 14th September 2012
Nový Smokovec

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Dear Participant,


The scientific programme of the workshop consists of 50 minute lectures of invited speakers and of 20 minute contributed talks. This booklet contains abstracts as were sent to us by the authors.

Invited speakers:

Stephan Brandt, Southern Denmark University, Odense, Denmark
Michael Henning, University of Johannesburg, South Africa
Alexandr V. Kostochka, Sobolev Institute of Mathematics, Novosibirsk, Russia
University of Illinois at Urbana-Champaign, United States
Mariusz Meszka, AGH University of Science and Technology, Kraków, Poland
János Pach, École Polytechnique Fédérale de Lausanne, Switzerland
Hungarian Academy of Sciences, Budapest, Hungary
Jean-Sébastien Sereni, LIAFA (Université Denis Diderot), Paris, France
Douglas West, Zhejiang Normal University, Jinhua, China
University of Illinois at Urbana-Champaign, United States

Have a pleasant and successfull stay in Nový Smokovec.

Organising Committee:

Igor Fabrici
František Kardoš
Tomáš Madaras
Roman Soták
Contents

Preface iii
Contents iv

Abstracts 1

van Aardt S. An infinite family of 2-hypohamiltonian digraphs . . . . 1
Adamus L. Sufficient conditions for cycles in bipartite digraphs . . . . 1
Adiwijaya A classification of some corona product graphs based on its
f-chromatic index ............... 1
Anholcer M. Group irregularity strength of graphs .................. 2
Bezegová L. Numbers of edges and vertices in magic-type graphs . . . 2
Bode J.-P. Grundy critical graphs ................................ 3
Bonamy M. Edge-choosability of planar graphs with ∆ ≥ 7 .......... 3
Brandt S. The circumference of the square of a connected graph .... 4
Buytás Cs. Generalized line graphs ................................ 5
Burger A. P. An infinite family of planar hypohamiltonian oriented
graphs ........................................ 6
Candráková B. k-regular graphs and multigraphs with the circular chro-
matic index close to k .. .......................... 6
Chiba S. Hamilton cycles and Tutte cycles in claw-free graphs ....... 7
Cichacz S. Distance magic labeling of product of two graphs ....... 7
Coroničová Hurajová J. Extremal graphs with respect to vertex be-
tweenness ..................................... 8
Crevals S. Upper bounds for the independent and total domination
number of grid graphs .......... .......................... 8
Drgas-Burchardt E. Unique factorization some classes of graphs with
respect to prime graphs ................. 9
Dvořák Z. Crossing number of periodic graphs ...................... 10
Fiedorowicz A. Star colouring of graphs and related topics .......... 10
Frick M. The Traceability Conjecture and generalized hypotraceability 11
Furmańczyk H. Equitable coloring of corona multiproducts of graphs 11
Furuya M. Forbidden subgraphs and the existence of a 2-walk ....... 12
Galčík F. Optimal semi-matchings in bipartite graphs ............... 12
Görlüch A. Cordial labeling of hypertrees ................................... 13
Habuszczak M. On Ramsey minimal graphs .......................... 14
Henning M. A. Cycles in graphs and colorings in hypergraphs ....... 15
Holub P. Degree/diameter problem in honeycomb networks ........ 16
Hudák D. Necessary condition for 1-planarity .......................... 16
Hudák P. Light edges and paths in 4-critical planar graphs ........... 17
Jakovac M. The k-path vertex cover of the strong and the lexicographic
product ........................................... 17
Kardoš F. Almost all fullerene graphs are Hamiltonian ............... 18
Kemnitz A. d-strong total colorings of graphs ...................... 18
Klešč M. The crossing numbers in special products of graphs ....... 18
<table>
<thead>
<tr>
<th>Author</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kostochka A. V.</td>
<td>On critical graphs with few edges</td>
<td>19</td>
</tr>
<tr>
<td>Král D.</td>
<td>Quasirandom permutations</td>
<td>19</td>
</tr>
<tr>
<td>Kupavskii A.</td>
<td>Distance graphs with large chromatic number and arbitrary girth</td>
<td>19</td>
</tr>
<tr>
<td>Kyppard J.</td>
<td>Induced cycles of Pascal’s polytopes</td>
<td>21</td>
</tr>
<tr>
<td>Li H.</td>
<td>Implicit degrees and Hamiltonian graph theory</td>
<td>21</td>
</tr>
<tr>
<td>Lichiardopol N.</td>
<td>Proof of Caccetta-Häggkvist conjecture for oriented graphs with positive minimum out-degree and of independence number 2</td>
<td>21</td>
</tr>
<tr>
<td>Lindner C.</td>
<td>How to embed a partial odd-cycle system</td>
<td>22</td>
</tr>
<tr>
<td>Lőwenstein C.</td>
<td>Transversals in hypergraphs</td>
<td>22</td>
</tr>
<tr>
<td>Lukot'ka R.</td>
<td>Maximal 4-degenerated induced subgraph of a planar graph</td>
<td>23</td>
</tr>
<tr>
<td>Máčajová E.</td>
<td>Eulerian trails in regular graphs of odd order</td>
<td>23</td>
</tr>
<tr>
<td>Mazák J.</td>
<td>Oddness of snarks with small cyclic connectivity</td>
<td>24</td>
</tr>
<tr>
<td>Meszáka M.</td>
<td>Palettes in block colourings of combinatorial structures</td>
<td>24</td>
</tr>
<tr>
<td>Mockovčiaková M.</td>
<td>Star chromatic index of trees and outerplanar graphs</td>
<td>25</td>
</tr>
<tr>
<td>Naroski P.</td>
<td>The eulerian tight cycles in hypergraphs and NP-completeness</td>
<td>25</td>
</tr>
<tr>
<td>Naserasr R.</td>
<td>Mapping planar graphs into projective cubes</td>
<td>26</td>
</tr>
<tr>
<td>Nedela R.</td>
<td>Hamiltonicity of a certain class of cubic graphs</td>
<td>27</td>
</tr>
<tr>
<td>Pach J.</td>
<td>The origins and big unsolved problems of Geometric Graph Theory</td>
<td>27</td>
</tr>
<tr>
<td>Pavlič P.</td>
<td>Roman domination number of products of paths and cycles</td>
<td>28</td>
</tr>
<tr>
<td>Peterin I.</td>
<td>The b-chromatic index of a graph</td>
<td>28</td>
</tr>
<tr>
<td>Petrosyan P. A.</td>
<td>Interval colorings of complete multipartite graphs</td>
<td>29</td>
</tr>
<tr>
<td>Pettersson V. H.</td>
<td>On the maximum length of coil-in-the-box codes in dimension 8</td>
<td>29</td>
</tr>
<tr>
<td>Polláková T.</td>
<td>Supermagic graphs having a saturated vertex</td>
<td>30</td>
</tr>
<tr>
<td>Przybyło J.</td>
<td>On the facial Thue choice index via entropy compression</td>
<td>30</td>
</tr>
<tr>
<td>Rautenbach D.</td>
<td>On graphs with maximal independent sets of few sizes</td>
<td>31</td>
</tr>
<tr>
<td>Rollóvá E.</td>
<td>Circuit covers and nowhere-zero flows in signed graphs</td>
<td>31</td>
</tr>
<tr>
<td>Rosa A.</td>
<td>Block colourings of designs revisited</td>
<td>32</td>
</tr>
<tr>
<td>Ryjáček Z.</td>
<td>Distance-locally disconnected graphs</td>
<td>32</td>
</tr>
<tr>
<td>Saputro S. W.</td>
<td>The locating chromatic number of composition product of graphs</td>
<td>32</td>
</tr>
<tr>
<td>Scheidweiler R.</td>
<td>Neighbour distinguishing gap colorings</td>
<td>33</td>
</tr>
<tr>
<td>Schiermeyer I.</td>
<td>Hunting for rainbow bulls and diamonds</td>
<td>33</td>
</tr>
<tr>
<td>Schrejer J.</td>
<td>On balanced separations of graphs</td>
<td>34</td>
</tr>
<tr>
<td>Semanišin G.</td>
<td>Weighted k-path vertex cover</td>
<td>35</td>
</tr>
<tr>
<td>Sereni J.-S.</td>
<td>Cycles (and balls) in (planar) graphs</td>
<td>36</td>
</tr>
<tr>
<td>Škrabuláková E.</td>
<td>Upper bounds for the facial Thue choice number of graphs</td>
<td>36</td>
</tr>
<tr>
<td>Šparl P.</td>
<td>Maximum induced matching of hexagonal graphs</td>
<td>37</td>
</tr>
<tr>
<td>Steffen E.</td>
<td>1-factor- and cycle covers of cubic graphs</td>
<td>37</td>
</tr>
<tr>
<td>Šugerek P.</td>
<td>Palindromfree facial edge coloring of plane graphs</td>
<td>38</td>
</tr>
<tr>
<td>Taranenko A.</td>
<td>On the structure of Lucas cubes</td>
<td>39</td>
</tr>
<tr>
<td>Timková M.</td>
<td>H-force number in distance graphs</td>
<td>39</td>
</tr>
<tr>
<td>Author</td>
<td>Title</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>----------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>Treglown A.</td>
<td>Monochromatic triangles in three-coloured graphs</td>
<td>39</td>
</tr>
<tr>
<td>Tuza Zs.</td>
<td>Hypercycle systems</td>
<td>40</td>
</tr>
<tr>
<td>Węsek K.</td>
<td>$b$-bounded circular coloring of graphs</td>
<td>40</td>
</tr>
<tr>
<td>West D. B.</td>
<td>Extending choosability results to paintability</td>
<td>40</td>
</tr>
<tr>
<td>Włoch A.</td>
<td>On generalized Pell numbers and their relations with Fibonacci numbers</td>
<td>41</td>
</tr>
<tr>
<td>Włoch I.</td>
<td>On kernels by monochromatic paths in edge-coloured digraphs</td>
<td>42</td>
</tr>
<tr>
<td>Zelenyuk Yu.</td>
<td>Symmetries and colorings of groups</td>
<td>42</td>
</tr>
</tbody>
</table>

**List of Participants**

**Programme of the Conference**
An infinite family of 2-hypohamiltonian digraphs

Susan van Aardt
(joint work with Marietjie Frick, Alewyn Burger, Bernardo Llano, and Rita Zuazua)

A digraph is hamiltonian if it contains a cycle that visits every vertex. If a digraph $D$ is nonhamiltonian but $D - S$ is hamiltonian for every set $S$ of $r$ vertices in $V(D)$, we say $D$ is $r$-hypohamiltonian. A 1-hypohamiltonian digraph is simply called hypohamiltonian. Many different methods for constructing hypohamiltonian digraphs have appeared in the literature. In fact, Skupień [1] observed that there are superexponentially many hypohamiltonian digraphs on $n$ vertices for $n \geq 6$. However, no $r$-hypohamiltonian digraphs have yet been reported in the literature for any $r > 1$. We present an infinite family of 2-hypohamiltonian digraphs. It is still an open question whether 3-hypohamiltonian digraphs exist.

References


Sufficient conditions for cycles in bipartite digraphs

Lech Adamus

We discuss bipartite counterparts of classical sufficient conditions for cycles in digraphs, particularly degree conditions of Woodall- and Meyniel-type. We show also a counterexample to the size condition which exists in literature for 20 years, and formulate a new conjecture and prove it in a case of long cycles.

A classification of some corona product graphs based on its $f$-chromatic index

Adiwijaya
(joint work with A. N. M. Salman, Edy Tri Baskoro, Bayu Erfianto, and Djoko Suprijanto)

Let $G = (V, E)$ be a simple graph and $f : V \to \mathbb{N}$ be a function. An $f$-coloring of graph $G(V, E)$ is a generalized edge-coloring such that every vertex $v$ in $V$ has at most $f(v)$ edges colored with a same color. The minimum number of colors of an $f$-coloring of $G$ is the $f$-chromatic index $\chi'_f(G)$ of $G$. Based on the $f$-chromatic
index, a graph $G$ can be either in class $C_f1$, if $\chi'_f(G) = \Delta_f(G)$, or in class $C_f2$, if $\chi'_f(G) = \Delta_f(G) + 1$, where $\Delta_f(G) = \max_{v \in V}[d(v)/f(v)]$. The corona product of a graph $G$ with a graph $H$, denoted by $G \circ H$, is a graph obtained by taking one copy of an $n$-vertex graph $G$ and $n$ copies of $H$, namely $H_1, H_2, \ldots, H_n$, and then for $i = 1, 2, \ldots, n$, joining by an edge the $i$-th vertex of $G$ with every vertex of $H_i$. In this paper, we provide a classification of the corona product of a complete graph with some graphs based on its $f$-chromatic index. Since the corona product of any two graphs is not commutative, we also give a classification of the corona product of a complete graph with some graphs.

**Group irregularity strength of graphs**

Marcin Anholcer

(joint work with Sylwia Cichacz)

We investigate the group irregularity strength ($s_g(G)$) of graphs, i.e. the smallest value of $s$ such that taking any Abelian group $G$ of order $s$, there exists a function $f : E(G) \to G$ such that the sums of edge labels at every vertex are distinct. We prove that for any connected graph $G$ of order at least 3, $s_g(G) = n$ if $n \neq 4k + 2$ and $s_g(G) \leq n + 1$ otherwise, except the case of some infinite family of stars. We also show that $s_g(G) \in \{n, n + 1\}$ for disconnected graphs with no bipartite component and for some special classes of graphs with bipartite components. Finally we prove that it is possible to distinguish the vertices of $G$ with the elements of any Abelian group of order $t$ for almost all the integers $t \geq s_g(G)$.

**Numbers of edges and vertices in magic-type graphs**

Ludmila Bezegová

(joint work with Jaroslav Ivančo)

A graph is called magic if it admits a labelling of the edges by pairwise different positive integers such that the sum of the labels of the edges incident with a vertex is independent of the particular vertex. A magic graph is supermagic if the set of labels of edges consists of consecutive positive integer. A graph $G$ is called degree-magic if it admits a labelling of the edges by integers $1, 2, \ldots, |E(G)|$ such that the sum of the labels of the edges incident with any vertex $v$ is equal to $\frac{1+|E(G)|}{2}\deg(v)$. A degree-magic labelling of a graph is generalization of the supermagic labelling of regular graphs.

We present known results on numbers of edges and vertices of magic graphs, of supermagic graphs and new results of degree-magic graphs.
Grundy critical graphs

Jens-P. Bode
(joint work with Piotr Borowiecki and Arnfried Kemnitz)

A proper $k$-coloring $c$ of the vertices of a graph $G$ using all $k$ colors $1,\ldots,k$ is called Grundy $k$-coloring if for each vertex $u$ and all colors $i$ with $1 \leq i < c(u)$ there exists an adjacent vertex $v$ with $c(v) = i$. The Grundy number $\Gamma(G)$ is defined to be the maximum $k$ for which a Grundy $k$-coloring of $G$ exists. A graph $G$ is called Grundy $k$-critical if $\Gamma(G) = k$ but $\Gamma(G - v) < k$ for every $v \in V(G)$.

We determine all Grundy $k$-critical graphs for $k \leq 5$ and all Grundy $k$-critical graphs of order $n \leq 11$. Furthermore, Grundy critical graphs being induced subgraphs of grid graphs are considered.

Edge-choosability of planar graphs with $\Delta \geq 7$

Marthe Bonamy
(joint work with Benjamin Léveque and Alexandre Pinlou)

For simple graphs, we consider the problem of edge coloring, that is, the problem of coloring the edges of a graph in such a way that no two edges with the same color are incident. Given a graph $G$, we denote $\chi'(G)$ the minimal number of colors needed to edge color $G$. Given a simple graph $G = (V, E)$, the list extension of edge coloring consists in considering a list assignment $L : E \to \mathcal{P}(\mathbb{N})$ and trying to edge color $G$ in such a way that the color of every edge $e \in E$ must belong to $L(e)$. We denote $\chi'_L(G)$ the smallest integer such that $G$ is edge $L$-colorable for any $L$ with $|L(e)| \geq \chi'_L(G)$ for any $e \in E$.

Obviously, for $\Delta(G)$ the maximum degree of $G$, $\chi'_L(G) \geq \chi'(G) \geq \Delta(G)$. A deep conjecture in graph theory, the List Coloring Conjecture states that $\chi'(G)$ should be equal to $\chi'_L(G)$ on any simple graph $G$.

Vizing [4] proved that $\chi'(G)$ has only two possible values as $\chi'(G) \leq \Delta(G) + 1$. When restricting the problem on planar graphs, it was shown by Sanders and Zhao [3] that $\chi'(G) = \Delta(G)$ for any simple planar graph $G$ with $\Delta \geq 7$.

While the same should hold also for list edge coloring according to the conjecture, it is only known for $\Delta \geq 12$ [1]. We show here that $\chi'_L(G) = \Delta(G)$ for any simple planar graph $G$ with $\Delta(G) \geq 7$ and no triangle sharing an edge with a cycle of length four, which improves previously known results [2, 5]. The corresponding proof works also for total list coloring.

References

The circumference of the square of a connected graph

Stephan Brandt
(joint work with Janina Müttel and Dieter Rautenbach)

Fleischner’s celebrated result [1] states that the square of every two-connected graph is hamiltonian. We investigate how small the circumference of the square of a connected graph of order $n$ can be. We present a lower bound for the smallest circumference which is logarithmic in $n$. The bound is tight for infinitely many values of $n$ and for each such $n$ there is a unique tree of order $n$ attaining this bound.

Harary and Schwenk [2] proved that a tree of order $n \geq 3$ is hamiltonian, if and only if it is a caterpillar, i.e. a tree where the deletion of all its leaves leaves a path. Let $S_{r,r,r}$ denote the graph obtained from a claw $K_{1,3}$ by subdividing each edge exactly $r - 1$ times. It is easy to see that a tree is a caterpillar if and only if it does not contain $S_{2,2,2}$ as a subgraph. We investigate the question how large the circumference of the square of a tree that does not contain $S_{r,r,r}$ as a subgraph can be. For the case $r = 3$ we present a lower bound for the circumference of the square which is of the order of the square root of $n$ and which is again tight for infinitely many values of $n$.

References

Generalized line graphs

Csilla Bujtás
(joint work with S. Aparna Lakshmanan and Zsolt Tuza)

Putting the concept of line graph in a more general setting, the $k$-line graph $L_k(G)$ of a graph $G$ has the $K_k$-subgraphs of $G$ as its vertices, and two vertices of $L_k(G)$ are adjacent if the corresponding copies of $K_k$ in $G$ share $k - 1$ vertices. The two vertices in the symmetric difference of these two $K_k$-subgraphs may be adjacent (type 1) or non-adjacent (type 2) in $G$, yielding the $k$-anti-Gallai graph $\Delta_k(G)$ (type-1 edges) and the $k$-Gallai graph $\Gamma_k(G)$ (type-2 edges) forming an edge-partition of $L_k(G)$. If $k = 1$, then $L_1(G) = K_{|V(G)|}$, $\Delta_1(G) = G$ and $\Gamma_1(G) = \overline{G}$; if $k = 2$, then $L_2(G)$ is the usual line graph; and if $k = 3$, then $L_3(G)$ is the triangle graph of $G$.

In the talk, we give a unified characterization for nontrivial connected graphs $F$ and $G$ such that the Cartesian product $F \square G$ is a $k$-line graph. For $k = 3$ this answers a question of Bagga [2], yielding the necessary and sufficient condition that $F$ is a complete graph and $G$ is the line graph of a triangle-free graph (or vice versa).

The other main topic of the talk is the time complexity of the recognition problem of $k$-line graphs and of $k$-anti-Gallai graphs. Using a recent result of Anand et al. [1], which says that 2-anti-Gallai graphs are hard to recognize, moreover using our characterization theorem, we prove that the recognition problem of $k$-line graphs and that of $k$-anti-Gallai graphs are NP-complete for each $k \geq 3$. This hardness is in sharp contrast to the forbidden subgraph characterization [3] and linear-time recognizability of line graphs. We also put some remarks on the recognition problem of $k$-Gallai graphs.

\section*{References}


An infinite family of planar hypohamiltonian oriented graphs

Alewyn P. Burger
(joint work with Marietjie Frick and Susan van Aardt)

Carsten Thomassen asked in 1976 whether there exists a planar hypohamiltonian oriented graph. We answer his question by presenting an infinite family of planar hypohamiltonian oriented graphs, the smallest of which has order 9. A computer search showed that 9 is the smallest possible order of a hypohamiltonian oriented graph.

References


k-regular graphs and multigraphs with the circular chromatic index close to k

Barbora Candráková
(joint work with Edita Máčajová)

The circular chromatic index of a graph $G$ is the infimum of all rational numbers $p/q$, such that there exists a circular $p/q$-edge-coloring of the graph $G$. It is an interesting problem to determine which possible values of the circular chromatic index are attained for $k$-regular graphs with chromatic index greater than $k$.

In this work we provide a construction of $k$-regular graphs which shows that all the values $k + 2/m$ for an integer $m \geq 2$ are attained as circular chromatic indices. So far, no similar result has been known for $k$-regular graphs with the exception of $k = 4$. Moreover, we construct $k$-regular multigraphs with the circular chromatic index equal to $k + a/m$ where $a \in [2, \ldots, \lfloor k/2 \rfloor]$ and $m \geq 2$. 
Hamilton cycles and Tutte cycles
in claw-free graphs

Shuya Chiba
(joint work with Roman Čada, Kenta Ozeki, Petr Vrana, and Kiyoshi Yoshimoto)

Most of the results in this talk are motivated by Matthews and Sumner’s conjecture [2] (every 4-connected claw-free graph is Hamiltonian) and Thomassen’s conjecture [4] (every 4-connected line graph is Hamiltonian). Since every line graph is claw-free, Thomassen’s conjecture is a special case of Matthews and Sumner’s conjecture. However it is known that a result on closures due to Ryjáček [3] implies that the above conjectures are equivalent. On the other hand, as a possible approach to solve the above conjectures, Jackson [1] conjectured that every 2-connected claw-free graph has a Tutte cycle, where a cycle $C$ of a graph $G$ is called a Tutte cycle of $G$ if (i) $C$ is a Hamilton cycle of $G$, or (ii) $|V(C)| \geq 4$ and every component of $G - V(C)$ has at most three neighbors on $C$. Note that every Tutte cycle $C$ of a 4-connected graph $G$ is a Hamilton cycle, since otherwise the neighbors of a component of $G - V(C)$ form a cut set of order at most three, contradicting 4-connectedness of $G$, and hence Jackson’s conjecture implies Matthews and Sumner’s conjecture. Our main result is to show that the converse also holds. We show that Matthews and Sumner’s conjecture implies Jackson’s conjecture.

References

[1] B. Jackson, Concerning the circumference of certain families of graphs, Memorandum 1076, Univ. of Twente, Enschede (1992), 87–94.
[2] M. M. Matthews, D. P. Sumner, Hamiltonian results in $K_{1,3}$-free graphs, J. Graph Theory 8 (1984), 139–146.

Distance magic labeling of product of two graphs

Sylwia Cichacz
(joint work with Marcin Anholcer, Iztok Peterin, and Aleksandra Tepeh)

Let $G = (V, E)$ be a graph of order $n$. A distance magic labeling of $G$ is a bijection $\ell : V \rightarrow \{1, 2, \ldots, n\}$ for which there exists a positive integer $k$ such that $\sum_{x \in N(v)} \ell(x) = k$ for all $v \in V$, where $N(v)$ is the neighborhood of $v$. 

7
In this talk we deal with strong product of graphs. We introduce a natural subclass of distance magic graphs. For this class we show that it is closed for the direct product with regular graphs and closed as a second factor for lexicographic product with regular graph. In addition we show that a graph $C_m \times C_n$ is distance magic if and only if $n = 4$ or $m = 4$ or $m, n \equiv 0 \pmod{4}$.

Moreover we deal also with $\Gamma$-distance labeling. We will show that if $G$ is an $r$-regular of order $n$, then the strong product $C_4 \times G$ is $\Gamma$-distance magic for any Abelian group of order $4n$. We will also present that a graph $C_m \times C_n$ is $\mathbb{Z}_{mn}$-distance magic if and only if $m \in \{4, 8\}$ or $n \in \{4, 8\}$ or $m, n \equiv 0 \pmod{4}$. It will be shown that if $m, n \not\equiv 0 \pmod{4}$ then $C_m \times C_n$ is not $\Gamma$-distance magic for any Abelian group $\Gamma$ of order $mn$.

**Extremal graphs**

**with respect to vertex betweenness**

**Jana Coroníčová Hurajová**

(joint work with Jelena Govorčin, Tomáš Madaras, and Riste Škrekovski)

The centrality index is a real-valued function which helps us to quantify the role that a given object plays in the network. One very important and quite often used centrality is the betweenness centrality. The betweenness centrality of a vertex is defined in [1] as the relative number of shortest paths between all pairs of vertices passing through given vertex.

Let $B_{\min}$ and $B_{\max}$ denotes the smallest and the largest value of betweenness of vertices of a graph, respectively. We focus are attention on minimization of $B_{\max}$ as well as on maximization of $B_{\min}$ for graphs of a several given classes of graphs.

**REFERENCES**


**Upper bounds for the independent and total domination number of grid graphs**

**Simon Crevals**

(joint work with Patric Östergård)

The domination numbers of grid graphs have gotten a lot of attention recently, and finally Chang’s conjecture [1] has been proved [2]. In this talk we present an upper bound for the independent domination number of grid graphs that
equals the domination number of grid graphs. Since the domination number is a lower bound for the independent domination number, the independent domination number is now known for all grid graphs.

In the second part of the presentation we give an upper bound for the total domination number of grid graphs, which is asymptotically better than the previously best known upper bound [3]. All upper bounds are proved by construction.

References


Unique factorization some classes of graphs with respect to prime graphs

Ewa Drgas-Burchardt

A set W of vertices of a graph G is a module in G if for any two vertices x, y ∈ W, the equality $N_G(x) \setminus W = N_G(y) \setminus W$ is satisfied. We say that an n-vertex graph is prime, if it does not include any k-element modules, where $2 \leq k \leq n-1$. In other words the n-vertex graph is prime if it cannot be gained by the substitution of at least 2-vertex and at most $(n-1)$-vertex graph to another graph as its vertex.

Let $H, G_1, \ldots, G_n$ be graphs and $V(H) = \{v_1, \ldots, v_n\}$. By $H[G_1, \ldots, G_n]$ we mean a graph obtained by simultaneous substitution of $G_i$ to $H$ instead of $v_i$ for all $i \in [n]$. The graph $H[G_1, \ldots, G_n]$ is known as the generalized lexicographic product of the graphs $G_1, \ldots, G_n$ and the base graph $H$ (see [2]).

In 1967 Tibor Gallai [1] proved a theorem on the modular decomposition of a graph. Gallai’s theorem, let us introduce a graph as a unique composition of the generalized lexicographic products, whose all base graphs are prime. In this talk we transfer features of such a composition of a graph onto a composition of a class of graphs, giving unique factorization theorems for classes of graphs.

References

Crossing number of periodic graphs

Zdeněk Dvořák
(joint work with Bojan Mohar)

A *cyclic periodic graph* is obtained by taking a number of copies of a fixed graph (*a tile*) and by joining them in a cyclic fashion by adding edges between prescribed vertices of consecutive tiles. It is known [1] that the average crossing number per tile in cyclic periodic graphs tends to a limit as the number of tiles goes to infinity. We give an estimate on the convergence rate of this limit, thus providing an algorithm to approximate it to arbitrary precision.

References


Star colouring of graphs and related topics

Anna Fiedorowicz
(joint work with Mariusz Hałuszczak)

A proper $k$-colouring of the vertices of a graph is called a *star colouring*, if every two colour classes induce a star forest. The minimum number of colours in a star colouring of a graph $G$ is called the *star chromatic number* of $G$. Star colourings are closely related to acyclic colourings, introduced by Grünbaum in [3].

Recently, star colourings of graphs have received a lot of attention. For instance, Albertson et al. proved that any graph $G$ with maximum degree $\Delta$ has a star colouring with at most $\Delta(\Delta - 1) + 2$ colours [1]. When $\Delta = 3$, they improved it to 7. Another approach was presented by Bu et al. in [2]. Namely, they studied the relationship between the star chromatic number and the maximum average degree of a graph.

In this talk we consider star colourings of graphs with maximum degree at most 4. In particular, we prove that if we additionally assume that the maximum average degree of a graph $G$ is less than 3, then $G$ can be star coloured with at most 7 colours.

References

The Traceability Conjecture and generalized hypotraceability

Marietjie Frick
(joint work with Susan van Aardt, Alewyn Burger, Bernardo Llano, and Rita Zuazua)

A digraph of order $n$ is $k$-traceable ($k \leq n$) if each of its subdigraphs of order $k$ is traceable. The Traceability Conjecture asserts that for $k \geq 2$ every $k$-traceable oriented graph of order at least $2k - 1$ is traceable. A digraph $D$ of order $n$ is $r$-hypotraceable ($r \leq n - 2$) if $D$ is not traceable but the deletion of any $r$ vertices from $D$ results in a traceable digraph. We present a brief overview of results on the Traceability Conjecture and investigate its connection with the existence of $r$-hypotraceable oriented graphs.

Equitable coloring of corona multiproducts of graphs

Hanna Furmańczyk
(joint work with Marek Kubale and Vahan V. Mkrtchyan)

A graph is equitably $k$-colorable if its vertices can be partitioned into $k$ independent sets in such a way that the number of vertices in any two sets differ by at most one. The smallest $k$ for which such a coloring exists is known as the equitable chromatic number of $G$ and denoted $\chi_e(G)$. It is known that this problem is NP-hard in general case. Moreover, the problem remains NP-hard for corona graphs. In this paper we obtain some results regarding the equitable chromatic number for $l$-corona product $G \circ l H$, where $G$ is an equitably 3- or 4-colorable graph and $H$ is an $r$-partite graph, a path, a cycle or a complete graph. Our proofs are constructive in that they lead to polynomial algorithms for equitable coloring of such graph products provided that there is given an equitable coloring of $G$. Moreover, we confirm Equitable Coloring Conjecture for corona products of such graphs. This paper extends our results from [1].

References

Forbidden subgraphs  
and the existence of a 2-walk  

Michitaka Furuya  

A \textit{k-walk} of a graph is a spanning closed walk meeting each vertex at most \(k\) times. Thus a 1-walk of a graph is a Hamiltonian cycle of the graph.

For a real number \(t > 0\), we say that a graph \(G\) is \(t\)-\textit{tough} if for every \(S \subseteq V(G)\) with \(c(G - S) \geq 2\), we have \(c(G - S) \leq |S|/t\), where \(c(G - S)\) denotes the number of components of \(G - S\). Jackson and Wormald [1] showed that if \(G\) has a \(k\)-walk, then \(G\) is \((1/k)\)-tough. On the other hand, infinitely many \((1/k)\)-tough graphs have no \(k\)-walk. Hence there is a gap between two properties “having a \(k\)-walk” and “\((1/k)\)-tough”. Recently, Ota and Sueiro [2] gave a characterization of sets \(\mathcal{H}_k\) such that every connected \(\mathcal{H}_k\)-free graph of sufficiently large order is \((1/k)\)-tough.

In this talk, we consider sets of forbidden subgraphs that imply the existence of a \(k\)-walk in a connected graph of sufficiently large order. For \(k = 2\), we show that such sets correspond with \(\mathcal{H}_2\).

References


Optimal semi-matchings in bipartite graphs  

František Galčík  
(joint work with Ján Katrenič and Gabriel Semanišin)

The problem of finding an optimal semi-matching is a generalization of the problem of finding classical matching in bipartite graphs. This problem is equivalent to a parallel machine scheduling problem with eligibility constraints in which each job has a pre-determined set of machines capable of processing the job and with the optimization objective to balance the load of machines. A \textit{semi-matching} in a bipartite graph \(G = (U, V, E)\) with \(n\) vertices and \(m\) edges is a set of edges \(M \subseteq E\) such that each vertex in \(U\) is incident to at most one edge in \(M\). An \textit{optimal semi-matching} is a semi-matching with \(\text{deg}_M(u) = 1\) for all \(u \in U\) and the minimal value of \(\sum_{v \in V} \frac{\text{deg}_M(v)}{2} \cdot \frac{(\text{deg}_M(v) + 1)}{2}\). We propose a schema that allows a reduction of the studied problem to a variant of the maximum bounded-degree semi-matching problem. Comparing to previous works, we use a different approach based on a result from [1]. The proposed schema [3] yields to two algorithms for computing
an optimal semi-matching. The first one runs in time $O(\sqrt{n} \cdot m \cdot \log n)$ that is the same as the time complexity of the currently best known algorithm [2]. The second one, that utilizes an algorithm from [5], is randomized and computes an optimal semi-matching with high probability in $O(n^{\omega} \cdot \log^{1+o(1)} n)$, where $\omega$ is the exponent of the best known matrix multiplication algorithm. Since $\omega \leq 2.38$, this algorithms breaks through $O(n^{2.5})$ barrier for dense graphs.

References


Cordial labeling of hypertrees

Agnieszka Görlich

(joint work with Sylwia Cichacz and Zsolt Tuza)

Let $H = (V, E)$ be a hypergraph with vertex set $V = \{v_1, \ldots, v_n\}$ and edge set $E = \{e_1, \ldots, e_m\}$. A vertex labeling $c : V \to \mathbb{N}$ induces an edge labeling $c^* : E \to \mathbb{N}$ by the rule $c^*(e_i) = \sum_{v_j \in e_i} c(v_j)$. For integers $k \geq 2$ we study the existence of labelings satisfying the following condition: every residue class modulo $k$ occurs exactly $\lfloor n/k \rfloor$ or $\lceil n/k \rceil$ times in the sequence $c(v_1), \ldots, c(v_n)$ and exactly $\lfloor m/k \rfloor$ or $\lceil m/k \rceil$ times in the sequence $c^*(e_1), \ldots, c^*(e_m)$. Hypergraph $H$ is called $k$-cordial if it admits a labeling with these properties.

This problem was introduced by Hovey [1] for graphs. He raised the conjecture (still open for $k > 5$) that if $H$ is a tree graph, then it is $k$-cordial for every $k$.

We present some new results for hypertrees (connected hypergraphs without cycles) and present various sufficient conditions on $H$ to be $k$-cordial. These results generalize Cahit’s theorem [2] which states that every tree graph is 2-cordial.
On Ramsey minimal graphs

Mariusz Hałuszczak

Let $F$ be a graph and let $\mathcal{G}$, $\mathcal{H}$ denote nonempty families of graphs. We write $F \rightarrow (\mathcal{G}, \mathcal{H})$ if in any 2-coloring of the edges of $F$, with red and blue, there is a red subgraph isomorphic to some graph from $\mathcal{G}$ or a blue subgraph isomorphic to some graph from $\mathcal{H}$. The graph $F$ is said to be a $(\mathcal{G}, \mathcal{H})$-minimal graph if $F \rightarrow (\mathcal{G}, \mathcal{H})$ and $F - e \not\rightarrow (\mathcal{G}, \mathcal{H})$ for $e \in E(F)$. The set of all $(\mathcal{G}, \mathcal{H})$-minimal graphs (up to isomorphism) is called the Ramsey set $\mathcal{R}(\mathcal{G}, \mathcal{H})$.

We present a procedure, which on the basis of the set of some special graphs, generates an infinite family of $(K_1 + mG, \mathcal{H})$-minimal graphs, where $m \geq 2$, $G$ is a connected graph and $\mathcal{H}$ is a family of 2-connected graphs. Moreover, graphs obtained by this procedure can be obtained in linear time with respect to theirs order.

References

Cycles in graphs and colorings in hypergraphs

Michael A. Henning
(joint work with Ingo Schiermeyer and Anders Yeo)

The first part of this talk focuses on results on domination in graphs. Let $G$ be a connected graph of order $n$ with minimum degree at least two. Let $\gamma(G)$ and $\gamma_t(G)$ denote the domination and total domination numbers, respectively, of $G$. We introduce the weighted cycle property of a graph and show how this can be used to obtain bounds in graphs with given girth ([3]). In particular, we show that if $G$ has girth $g \geq 3$, then $\gamma_t(G) \leq \frac{n}{2} + \max\left(1, \frac{n}{2(g+1)}\right)$ and this bound is sharp. We also investigate results on domination in graphs with certain forbidden cycles. We show ([2]) that if $n > 18$ and if $G$ has no induced 6-cycle, then $\gamma_t(G) \leq 6n/11$. We show ([1]) that if $n \geq 14$ and $G$ has no induced 4-cycle or 5-cycle, then $\gamma(G) \leq 3n/8$. Both these results are sharp.

The second part of this talk focuses on colorings in hypergraphs. A hypergraph is 2-colorable if there is a 2-coloring of the vertices with no monochromatic hyperedge. Let $\mathcal{H}_k$ denote the class of all $k$-uniform $k$-regular hypergraphs. We exhibit a surprising connection between disjoint total dominating sets in graphs, 2-coloring of hypergraphs, and even cycles in digraphs. We show ([4]) that the following statements are equivalent for all $k \geq 2$. (1) Every $k$-regular graph contains two disjoint total dominating sets. (2) Every hypergraph in $\mathcal{H}_k$ is 2-colorable. (3) Every $(k - 1)$-regular digraph has an even cycle. As a consequence of this result, we have that every hypergraph in $\mathcal{H}_k$ is 2-colorable, provided $k \geq 4$. We discuss a strengthening of this results. For this purpose, we define a set $X$ of vertices in a hypergraph $H$ to be a free set in $H$ if we can 2-color $V(H) \setminus X$ such that every edge in $H$ receives at least one vertex of each color. We conjecture that for $k \geq 4$, every hypergraph $H \in \mathcal{H}_k$ has a free set of size $k - 3$ in $H$. We show that the bound $k - 3$ cannot be improved for any $k \geq 4$ and we prove our conjecture when $k \in \{5, 6, 7, 8\}$. For every $k \geq 13$, we prove ([5]) that every hypergraph $H \in \mathcal{H}_k$ of order $n$ has a free set of size at least $n/14$. For any $\epsilon$ where $0 < \epsilon < 1$ and for sufficiently large $k$, we prove that every hypergraph $H \in \mathcal{H}_k$ of order $n$ has a free set of size at least $c_k n$, where $c_k = 1 - 6(1 + \epsilon) \ln(k)/k$, and so $c_k \to 1$ as $k \to \infty$.

References

Degree/diameter problem in honeycomb networks

Přemek Holub

(joint work with Mirka Miller, Hebert Perez-Roses, and Joe Ryan)

The degree diameter problem involves finding the largest graph (in terms of the number of vertices) subject to constraints on the degree and the diameter of the graph. Beyond the degree constraint there is no restriction on the number of edges (apart from keeping the graph simple) so the resulting graph may be thought of as being embedded in the complete graph. In a generalisation of this problem, the graph is considered to be embedded in some connected host graph, in this paper the honeycomb network. We consider embedding the graph in the $k$-dimensional honeycomb grid and provide upper and lower bounds for the optimal graph. The particular cases of dimensions 2 and 3 are examined in detail.

Necessary condition for 1-planarity

Dávid Hudák

(joint work with Július Czap)

A graph is called 1-planar if it can be drawn in the plane with at most one crossing per any edge. Although the concept of 1-planarity was introduced in 1965, the problem of characterization is still not solved and seems to remain open. Since there is a 1-planar subdivision of every graph $G$, there is no chance to characterize 1-planar graphs by theorem of Kuratowski type. In the paper [1] it was observed that the class of 1-planar graphs is not closed under the operation of edge contraction. Hence, the family of 1-planar graphs is not minor closed and the Robertson-Seymour theorem could not be used. Moreover, the authors of [2] found some strictly different properties of 1-planar graphs when compared to planar graphs. These results indicate the impossibility of characterizing 1-planar graphs by finite number of forbidden minors.

We concentrate on describing some useful verifications, which will determine non-1-planar graphs. We show that every 1-planar drawing of any 1-planar graph on $n$ vertices has at most $n - 2$ crossings. With essential help of this result we characterize the 1-planarity of some classes of graphs.

References

Light edges and paths in 4-critical planar graphs

Peter Hudák

A graph $G$ is $k$-critical if $\chi(G) = k$ and $\chi(H) < \chi(G)$ for every proper sub-graph $H$ of $G$. It is well known that every planar graph is 4-colorable, therefore class of 4-critical planar graphs is extremal class of planar graphs (in sense of colorability). Koester [2] proved that every 4-critical planar graph contains vertex of degree at most 4 and this bound is best possible. We will prove that every 4-critical planar graph contains edge of low weight.

Fabrici and Jendrol’ [1] showed that if 3-connected planar graph $G$ contains $k$-vertex path $P_k$ then it contains $P_k$ such that $\Delta_G(P_k) \leq 5k$. We will prove similar theorem for 4-critical planar graphs and show that bound is in this case exponential.

References


The $k$-path vertex cover of the strong and the lexicographic product

Marko Jakovac

(joint work with Andrej Taranenko)

A subset $S$ of vertices of a graph $G$ is called a $k$-path vertex cover if every path of order $k$ in $G$ contains at least one vertex from $S$. We denote by $\psi_k(G)$ the minimum cardinality of a $k$-path vertex cover in $G$. In this talk improved lower and upper bounds for $\psi_k$ of the strong product of paths are derived. It is shown that for $\psi_3$ those bounds are tight. For the lexicographic product bounds are presented for $\psi_k$, moreover $\psi_2$ and $\psi_3$ are exactly determined for the lexicographic product of two arbitrary graphs. As a consequence the independence and the dissociation number of the lexicographic product are given.

References

Almost all fullerene graphs are Hamiltonian
František Kardoš

Fullerene graphs, i.e. cubic planar 3-connected graphs with pentagonal and hexagonal faces, are conjectured to be Hamiltonian. This is a special case of a conjecture of Barnette. We show that if a fullerene graph is large enough, it contains a Hamiltonian cycle.

d-strong total colorings of graphs
Arnfried Kemnitz
(joint work with Massimiliano Marangio)

If \( c : V \cup E \rightarrow \{1, 2, \ldots, k\} \) is a proper total coloring of a graph \( G = (V, E) \) then the palette \( S[v] \) of a vertex \( v \in V \) is the set of colors of the incident edges and the color of \( v \): \( S[v] = \{c(e) : e = vw \in E\} \cup \{c(v)\} \). A total coloring \( c \) distinguishes vertices \( u \) and \( v \) if \( S[u] \neq S[v] \). A d-strong total coloring of \( G \) is a proper total coloring that distinguishes all pairs of vertices \( u \) and \( v \) with distance \( 1 \leq d(u, v) \leq d \). The minimum number of colors of a d-strong total coloring is called d-strong total chromatic number \( \chi''_d(G) \) of \( G \).

Let \( n_i \) denote the maximum number of vertices of degree \( i \) that are of pairwise distance at most \( d \) and let \( \mu''_d(G) = \max\{\min\{j : \binom{j}{i+1} \geq n_i\} : \delta(G) \leq i \leq \Delta(G)\} \). Obviously, \( \mu''_d(G) \) is a lower bound for \( \chi''_d(G) \).

We determine \( \chi''_d(G) \) completely for paths, give exact values for cycles, and disprove a general conjecture by Zhang et al. [1] on an upper bound for \( \chi''_d(G) \).

REFERENCES


The crossing numbers in special products of graphs
Marián Klešč

The crossing number of a graph \( G \) is denoted by \( \text{cr}(G) \) and is defined to be the least number of edge crossings in any drawing of \( G \) in the plane. The general problem of determining this invariant is NP-hard even for cubic graphs. The exact values of crossing numbers are known only for some families of graphs.
According to their special structure, Cartesian products of special graphs are one of few graph classes for which exact values of crossing numbers were obtained. Recently, some authors started to study the crossing numbers for join products of graphs. In the talk, we give the exact values of crossing numbers for new infinite families of graphs and we find the upper bound of crossing numbers for some other graphs. All these graphs are obtained as categorical product and as strong product of two graphs.

**On critical graphs with few edges**

Aleksandr V. Kostochka
(joint work with Matthew Yancey)

Dirac in 1957, Gallai in 1963 and Ore in 1967 asked to evaluate $f_k(n)$ – the minimum number of edges in a $k$-critical $n$-vertex graph for $k \geq 4$ and $n > k + 1$. We find $f_4(n)$ exactly for every $n \geq 6$ and asymptotically for every fixed $k \geq 5$. The result has interesting implications, in particular, a very simple proof of Grötzsch Theorem that every planar triangle-free graph is 3-colorable.

**Quasirandom permutations**

Daniel Kráľ
(joint work with Oleg Pikhurko)

For permutations $\sigma$ and $\pi$ of length $|\sigma| \leq |\pi|$, let $t(\sigma, \pi)$ be the probability that the restriction of $\pi$ to $|\sigma|$ random points is isomorphic to $\sigma$. We show that every sequence $\pi_j$ of permutations with increasing lengths such that $t(\sigma, \pi_j)$ tends to $1/4!$ for every 4-point permutation $\sigma$ is quasirandom, i.e., $t(\sigma, \pi_j)$ tends to $1/|\sigma|!$ for every permutation $\sigma$. This answers a question posed by Graham.

**Distance graphs with large chromatic number and arbitrary girth**

Andrey Kupavskii

In this article we consider a problem related to two famous combinatorial topics. One of them concerns the chromatic number of the space. The chromatic number $\chi(\mathbb{R}^n)$ of the space $\mathbb{R}^n$ is the minimum number of colors needed to paint the points of the space $\mathbb{R}^n$ so that no two points at unit distance apart receive the same color. The following asymptotic lower and upper bounds are due to A. Raigorodskii and D. Larman, C. Rogers respectively:

$$(\zeta_{\text{low}} + o(1))^n \leq \chi(\mathbb{R}^n) \leq (3 + o(1))^n, \quad \zeta_{\text{low}} = 1.239\ldots$$
We study graphs that arise naturally in the context of the problem of finding the chromatic number of the space. Graph $G = (V, E)$ is called a distance graph in the space $\mathbb{R}^n$, if $V$ is a subset of $\mathbb{R}^n$ and

$$E \subseteq \{\{x, y\} : x, y \in V, |x - y| = 1\}.$$  

On the one hand, it follows from the definitions that for any such distance graph $G$, $\chi(G) \leq \chi(\mathbb{R}^n)$. On the other hand, N.G. de Bruijn and P. Erdős proved that there exists a distance graph $G'$ in $\mathbb{R}^n$ with finite number of vertices such that $\chi(G') = \chi(\mathbb{R}^n)$.

The second question that lies at the basis of this article is the following. Can we construct graphs with arbitrarily large chromatic number and arbitrary girth (the length of the shortest cycle)? The positive answer to this question was given by P. Erdős in 1959.

It is natural to ask how big can the chromatic number of a distance graph be if we additionally require that the graph has no cliques (complete subgraphs) or cycles of fixed size. The question, whether there is a distance graph in the plane with chromatic number 4 and without triangles (which are both cliques of size 3 and cycles of length 3) was asked by P. Erdős in 1975. It was answered positively. Moreover, P. O’Donnell in 2000 proved that for any $l \in \mathbb{N}$ there exists a distance graph in the plane with chromatic number 4 and girth greater than $l$.

We consider the following three families of distance graphs in $\mathbb{R}^n$: $C(n, k)$, $G_{\text{odd}}(n, k)$, $G(n, k)$ are the families of all distance graphs that do not contain complete subgraphs of size $k$, odd cycles of length $\leq k$ and cycles of length $\leq k$ respectively. Next we define the following quantities:

$$\zeta_k = \liminf_{n \to \infty} \max_{G \in C(n, k)} (\chi(G))^{1/n}, \quad \zeta_{k, \text{odd}} = \liminf_{n \to \infty} \max_{G \in G_{\text{odd}}(n, k)} (\chi(G))^{1/n},$$

$$\xi_k = \liminf_{n \to \infty} \max_{G \in G(n, k)} (\chi(G))^{1/n}.$$  

It follows from the asymptotical upper bound on the chromatic number of the space that these values do not exceed three. The values $\zeta_k$ and $\zeta_{k, \text{odd}}$ were considered in several papers. It is known that $\zeta_k \geq c_k$, where $c_k > 1$ and $\lim_{k \to \infty} c_k = c_{\text{low}}$ and that for any fixed $k$ we have $\xi_{k, \text{odd}} > 1$.

However, previous approaches failed to provide any estimate for $\xi_k$, $k \geq 4$. The main result of this article is the following

**Theorem 1** For any fixed $k \geq 3$ we have $\xi_k > 1$.

The proof of this theorem is based on the analysis of a random subgraph of the distance graph $G_{4n} = (V_{4n}, E_{4n})$, where

$$V_{4n} = \{x = (x_1, \ldots, x_{4n}) : x_i \in \{0, 1\}, x_1 + \ldots + x_{4n} = 2n\},$$

$$E_{4n} = \{\{x, y\} : (x, y) = n\}.$$  

Here $(,)$ denotes the Euclidean scalar product. The main ingredients of the proof are the Lovász local lemma and theorem from the paper by Frankl and Rödl.
Theorem 2 [Frankl, Rödl, 1987] For any $\epsilon > 0$ there exists $\delta > 0$ such that for any subset $S$ of $V_{4n}$, $|S| \geq (2 - \delta)^{4n}$, the number of edges in $S$ (the cardinality of $E_{4n}|S|$) is greater than $(4 - \epsilon)^{4n}$.

Induced cycles of Pascal’s polytopes

Jorma Kyppö

This presentation is focused on the induced cycles of Pascal’s Polytopes. The extension of Pascal’s Arithmetical Triangle to the dimension $N$ is called the Pascal’s Tetrahedron or Pascal’s Pyramid and the extension to the fourths dimension is called Pascal’s Pentachoron. In this context, the extension to the dimension $N$ is called Pascal’s Polytope. The concept of the induced or chordless cycle means an induced subgraph of the undirected graph $G$. Induced cycles are also called as chordless cycles or graph holes. The problem of monitoring the long induced cycles in hypercubes is known as the coil-in-the-box problem and the problem of searching for the long induced paths in hypercubes is known as the snake-in-the-box problem. This presentation is aimed at the induced cycles on Pascal’s Polytopes.

Implicit degrees and Hamiltonian graph theory

Hao Li

(joint work with Junqing Cai and Wantao Ning)

Dirac showed in 1952 that every graph of order $n$ is hamiltonian if any vertex is of degree at least $n/2$. In order to deal with the case that some vertices may have small degrees, Zhu, Li and Deng defined implicit degrees for a vertex $v$ by using the degrees of vertices that has distance at most 2 from $v$ and obtained sufficient conditions with implicit degrees for Hamiltonian problems in 1989. In this talk, we present new results on implicit degree conditions for Hamiltonicity, cyclable, pancyclic, etc. These results are from several recent papers.

Proof of Caccetta-Häggkvist conjecture for oriented graphs with positive minimum out-degree and of independence number 2

Nicolas Lichiardopol

In his paper [1], A. Razborov says that Chudnovski and Seymour proved that an out-regular oriented graph of out-degree $d \geq 2$, of independence number 2
and of order at most $3d$ contains a directed triangle, which means that Caccetta-Häggkvist conjecture is verified by such an oriented graph. He says also that to the best of his knowledge, the question is still open without the restriction of out-regularity. In this paper, we give a complete answer, by proving that for $d \geq 2$, any oriented graph of minimum out-degree $d \geq 2$, of independence number 2 and of order at most $3d$ contains a directed triangle. Additionally, we prove that any oriented graph of minimum out-degree $d \geq 1$, of independence number 2 and of order at most $4d$ contains a directed cycle of length at most 4. A simple observation on the girth of a non-acyclic oriented graph of independence number 2, allows us to state that the Caccetta-Häggkvist conjecture is true for oriented graphs of minimum out-degree at least 1 and of independence number 2.

References


How to embed a partial odd-cycle system

Curt Lindner

A $k$-cycle system of order $n$ is a pair $(X,C)$ where $C$ is a collection of edge disjoint $k$-cycles which partitions the edge set of $K_n$ with vertex set $X$. The spectrum for $k$-cycle systems for all $k$ is by now well known. A partial $k$-cycle system is a pair $(X,P)$, where $P$ is a collection of edge disjoint $k$-cycles of $K_n$ with vertex set $X$. (The edges of a partial $k$-cycle system do not necessarily partition $K_n$.) The partial $k$-cycle system $(X,P)$ of order $n$ is said to be embedded in the $k$-cycle system $(Y,C)$ of order $m$ provided $n \leq m$, $X \subseteq Y$, and $P \subseteq C$. We would like $m$ to be as small as possible. Fairly recently Darryn Bryant has obtained the best possible embedding for partial 3-cycle systems (= Steiner triple systems) by showing that any partial 3-cycle system of order $n$ can be embedded in a 3-cycle system of order $\approx 2n + 1$. This is the only known best possible embedding for partial odd-cycle systems. This talk shows that any partial $(2k+1)$-cycle system of order $n$ can be embedded in a $(2k+1)$-cycle system of order $(2k+1)(2n+1)$. This is not the best possible and, in fact, the author is not exactly sure what the best possible embedding is. This problem is wide open for $2k + 1 \geq 5$.

Transversals in hypergraphs

Christian Löwenstein

(joint work with Michael A. Henning)

For $k \geq 2$, let $H$ be a $k$-uniform hypergraph on $n$ vertices and $m$ edges. The transversal number $\tau(H)$ of $H$ is the minimum number of vertices that intersect
every edge. Chvátal and McDiarmid [1] proved that \( \tau(H) \leq (n + \lfloor \frac{k}{2} \rfloor m)/(\lfloor \frac{3k}{2} \rfloor) \).
When \( k = 3 \), the connected hypergraphs that achieve equality in the Chvátal-McDiarmid Theorem were characterized by Henning and Yeo [2]. In this talk, we characterize the connected hypergraphs that achieve equality in the Chvátal-McDiarmid Theorem for \( k = 2 \) and for all \( k \geq 4 \).

References


Maximal 4-degenerated induced subgraph of a planar graph

Robert Lukoťka

(joint work with Ján Mazák and Xuding Zhu)

Every planar graph \( G \) has a vertex of degree five or less. Therefore we can collect vertices of degree five or less from \( G \) until we collect the whole graph \( G \) (this shows every planar graph is 5-degenerated). If we can collect only vertices of maximum degree four, to be able to collect vertices we may need to delete some vertex first.

We show that in any planar graph of minimum degree five, we can delete a vertex such that we can subsequently collect at least 6 vertices of degree four or less. Moreover in any planar graph \( G \) we can always delete no more that 1/9 of the vertices of \( G \), so that after removing these vertices we can subsequently collect all the remaining vertices in \( G \).

We conjecture that in any planar graph \( G \) we can always delete no more that 1/12 of the vertices of \( G \), so that after removing these vertices we can collect the rest of the graph. Even stronger we conjecture that we can always delete a vertex in \( G \), so that we can collect at least 11 vertices.

We discuss several other similar problems for some classes of planar graphs.

Eulerian trails in regular graphs of odd order

Edita Máčajová

(joint work with Martin Škoviera)

Every vertex \( v \) in a connected \( 2d \)-regular graph \( G \) divides any eulerian trail \( T \) into \( d \) closed subtrails \( T_1, T_2, \ldots, T_d \) based at \( v \). If \( G \) has odd order, the number
of edges of $G$ is $d$ times an odd number, so one may ask whether $T$ and $v$ can be chosen in such a way that all the subtrails $T_i$ have odd length. The answer is positive (but trivial) for $d = 1$, and is positive also for $d = 2$. The purpose of this talk is to give a positive answer for $d = 3$ and sketch the proof. The motivation for this research comes from signed graphs: the result is crucial for determining the flow number of an arbitrary signed eulerian graph.

**Oddness of snarks with small cyclic connectivity**

Ján Mazák  
(joint work with Robert Lukoťka, Edita Máčajová and Martin Škoviera)

The oddness of a given bridgeless cubic graph $G$ is the smallest possible number of odd cycles in a 2-factor of $G$. It transpired that oddness is a useful parameter allowing us to express how close a cubic graph is to being 3-edge-colourable. Every 3-edge-colourable cubic graph has a 2-factor containing only even cycles, hence it has oddness 0. For each positive integer $k$, there are many bridgeless cubic graphs with oddness $2k$. All these graph have chromatic index 4 and are called snarks. Various deep conjectures have been proved for 3-edge-colourable cubic graphs and some of them have been proved for oddness 2, so it is natural to try small snarks with oddness 4 as possible counterexamples to those conjectures. We have managed to construct a non-trivial snark with oddness 4 on 44 vertices, improving the previously known best bound of 52.

We also describe constructions of infinite families of snarks with given oddness and rather small order. An additional parameter in those constructions is cyclic edge-connectivity $\lambda$; we look at $\lambda \in \{2, 3, 4, 5, 6\}$ (no snarks with higher cyclic edge-connectivity are known). Our results improve some of the previously known bounds.

In addition, we employ perfect matching polytope (a linear programming approach to matchings in graphs) to establish some lower bounds on the order of snarks with given oddness. In particular, a snark with oddness $o$ has at least $(5 + 1/4)o$ vertices.

**Palettes in block colourings of combinatorial structures**

Mariusz Meszka

Let $c : E(G) \mapsto C$ be a proper edge colouring of a graph $G$, i.e., incident edges of $G$ get distinct colours. A palette of a vertex $v \in V(G)$ with respect to $c$ is the set $S_c(v)$ of colours of edges incident to $v$. Two vertices of $G$ are distinguished by a colouring $c$ if their palettes are distinct. Given a colouring $c$ of $G$, the number
of distinct palettes among all vertices of $G$ is denoted by $p_c(G)$. The minimum value of $p_c(G)$ taken over all possible proper edge colourings of $G$ is called the \textit{palette index} and is denoted by $\tilde{s}(G)$.

By analogy, colourings with palettes may be carried over combinatorial designs. A \textit{proper block colouring} of a design $(V, \mathcal{B})$ is a mapping $c : \mathcal{B} \rightarrow C$ such that for any $B, B' \in \mathcal{B}$, $c(B) \neq c(B')$ whenever $B \cap B' \neq \emptyset$. Given a design $(V, \mathcal{B})$ and its proper colouring $c$, a \textit{palette} of an element $v \in V$ with respect to $c$ is the set $S_c(v)$ of colours of blocks containing $v$. Similarly as for graphs, the minimum number $\tilde{s}(V, \mathcal{B})$ of distinct palettes taken over all block colourings of a design $(V, \mathcal{B})$ is called the \textit{palette index} of $(V, \mathcal{B})$.

Colourings with respect to the minimum number of palettes will be discussed. In particular, the palette index of complete graphs will be determined. Moreover, upper bounds on the palette index for Steiner triple systems and related designs will be presented.

**Star chromatic index of trees and outerplanar graphs**

Martina Mockovčiaková  
(joint work with Ludmila Bezegová, Borut Lužar, Roman Soták, and Riste Škrekovski)

Let $\varphi : E \rightarrow \{1, 2, \ldots, k\}$ be a proper edge coloring of a graph $G$. A coloring $\varphi$ is a \textit{star edge coloring} if there is no bi-colored 4–cycle and no bi–colored path of length 4 in $G$. The \textit{star chromatic index} of $G$, denoted by $\chi'_s(G)$, is the minimum number of colors required for such coloring.

Recently, Dvořák, Mohar and Šámal [J. Graph Theory, in press] obtained a near-linear upper bound on $\chi'_s$ in terms of the maximum vertex degree of graph. They also provided some bounds for complete graphs and subcubic graphs.

We present some results on the star edge coloring of outerplanar graphs; we determine the best possible upper bound on the star chromatic index of subcubic outerplanar graphs and trees. We also derive an upper bound for outerplanar graphs, which is best possible up to constant.

**The eulerian tight cycles in hypergraphs and NP-completeness**

Paweł Naroski  
(joint work with Zbigniew Lonc)

A \textit{tight cycle} in a 3-uniform hypergraph $H$ is a cyclic ordering of some of its vertices $(v_0, \ldots, v_{s-1})$ – possibly with repetitions – such that $v_iv_{i+1}v_{i+2}$ is an edge
of $H$ for $i = 0, \ldots, s - 1$ (addition is performed modulo $s$) and these edges are pairwise different i.e. $v_i v_{i+1} v_{i+2} \neq v_j v_{j+1} v_{j+2}$ for $i \neq j$. We say that a tight cycle $(v_0, \ldots, v_{s-1})$ in $H$ is eulerian if it traverses all of its edges i.e. $s = |E(H)|$.

We show that a problem of deciding if a given 3-uniform hypergraph has an eulerian tight cycle is $NP$-complete.

**References**


**Mapping planar graphs into projective cubes**

Reza Naserasr

The projective cube of dimension $2k$, denoted $PC(2k)$, is the graph obtained from the hypercube of dimension $2k+1$ by identifying each pair of antipodal vertices. With the terminology of Cayley graphs, $PC(2k)$ is $(\mathbb{Z}_2^{2k}, (e_1, e_2, \ldots, e_{2k}, J)$ where the $e_i$’s are the vectors of the standard basis and $J$ is the all-1 vector. A homomorphism of a graph $G$ to a graph $H$ is a mapping of $V(G)$ to $V(H)$ which preserves adjacency. We raised the following question:

*Given two integers $r \geq k$, what is the smallest subgraph of $PC(2k)$ to which every planar graph of odd-girth $2r + 1$ admits a homomorphism to?*

In this talk, we show that this question, surprisingly captures many well-known theorems and conjectures such as the four colour theorem, Grötzsch’s theorem, Jeager’s conjecture, Seymour’s conjecture on edge-coloring of planar graphs. We also show a strong relation between this question and the development of theories such as edge-coloring, fractional coloring, circular coloring for the class of planar graphs. In particular, the question is also related to the characterization of binary clutters.

**References**

Hamiltonicity of a certain class of cubic graphs

Roman Nedela

(joint work with Klavdija Kutnar and Dragan Marušič)

The problem of existence of a Hamiltonian cycle was originally investigated on the dodecahedron. Although hundreds of papers deals with the problem of existence of a Hamiltonian cycle in a graph, there is a lack of results on the hamiltonicity of cubic graphs. Among others, it is well-known that to decide whether a cubic graph is Hamiltonian is an NP-complete problem.

A graph is cyclically \( k \)-edge connected if it admits no edge-cut of size \(< k\) separating two cycles. Graphs without cycle-separating edge-cuts were classified in an early work by Lovász, between cubic graphs there are just two of them \( K_4 \) and \( K_{3,3} \). No cubic 8-cyclically connected non-hamiltonian cubic graph is known.

A truncation of a map \( M \) on a closed surface is a cubic map arising by expanding each vertex into a face. Employing the method introduced in [1] we show the following.

**Theorem.** Let \( M \) be a triangulation of a closed surface and with a cyclically 4-connected dual and let \( v \) denote number of vertices of the truncation \( X = T(M) \). Then

- \( X \) is hamiltonian provided \( v \equiv 2 \mod 4 \),
- \( X \) admits a hamiltonian path if \( v \equiv 0 \mod 4 \).

Note that a dual of a polyhedral triangulation is (cyclically) 3-connected and it has a cycle-separating 3-cut if and only if the triangulation has a separating cycle of length 3.

**References**


The origins and big unsolved problems of Geometric Graph Theory

János Pach

We revisit the original problems that led to the birth of Geometric Graph Theory and briefly summarize some recent developments concerning them. The topics discussed include Schur’s conjecture, Conway’s thrackle conjecture, quasiplanar graphs.
Roman domination number of products of paths and cycles

Polona Pavlič
(joint work with Janez Žerovnik)

Roman domination is an historically inspired variety of domination in graphs, in which vertices are assigned a value from the set \{0, 1, 2\} in such a way that every vertex assigned the value 0 is adjacent to a vertex assigned the value 2. The Roman domination number is the minimum possible sum of all values in such an assignment. Using an algebraic approach, which bases on path algebra, we present an $O(C)$-time algorithm for computing the Roman domination numbers of special classes of graphs called polygraphs, which include rotagraphs and fascigraphs. Using this algorithm we determine formulas for the Roman domination numbers of the Cartesian and the direct products of path and cycles, where one of the factors is fixed.

REFERENCES


The $b$-chromatic index of a graph

Iztok Peterin
(joint work with Marko Jakovac)

A $b$-edge coloring of a graph $G$ is a proper edge coloring of $G$ such that each color class contains an edge that has at least one incident edge in every other color class and the $b$-chromatic index of a graph $G$ is the largest integer $\varphi'(G)$ for which $G$ has a $b$-edge-coloring with $\varphi'(G)$ colors.
We present an upper and a lower bound for $\phi'(G)$. The $\phi'(G)$ is studied with respect to diameter and girth of $G$. We study $\phi'(G)$ on trees, complete graphs, and complete multipartite graphs. We also give an exact answer for $\phi'(G)$ for all cubic graphs. The latest result is an analogue of $b$-vertex colorings of cubic graphs in [1].

References


Interval colorings
of complete multipartite graphs

Petros A. Petrosyan

An edge-coloring of a graph $G$ with colors $1, \ldots, t$ is an interval $t$-coloring if all colors are used, and the colors of edges incident to each vertex of $G$ are distinct and form an interval of integers. A graph $G$ is interval colorable if it has an interval $t$-coloring for some positive integer $t$. For an interval colorable graph $G$, the least and the greatest values of $t$ for which $G$ has an interval $t$-coloring are denoted by $w(G)$ and $W(G)$, respectively. In this talk we present some bounds for $w(K_{n_1,\ldots,n_k})$ and $W(K_{n_1,\ldots,n_k})$ of the interval colorable complete $k$-partite graph $K_{n_1,\ldots,n_k}$. We also formulate some open problems on interval colorings of complete multipartite graphs.

On the maximum length of coil-in-the-box codes in dimension 8

Ville H. Pettersson

(joint work with Patric R. J. Östergård)

The coil-in-the-box problem deals with finding the longest simple chordless cycle in an $n$-cube, also known as an $n$-dimensional spread 2 circuit code, or an $n$-coil. This problem has been solved earlier for $n \leq 7$. A computational approach based on canonical augmentation is here used to solve the problem for $n = 8$ and show that the maximum length of a chordless cycle in an 8-cube is 96. Several new 8-coils of length 96 are presented.
Supermagic graphs having a saturated vertex

Tatiana Poláková

A graph is called supermagic if it admits a labeling of the edges by pairwise different consecutive integers such that the sum of the labels of the edges incident with a vertex is independent of the particular vertex. We establish some conditions for graphs with a saturated vertex to be supermagic. Inter alia we show that complete multipartite graphs $K_{1,n,n}$ and $K_{1,2,\ldots,2}$ are supermagic.

On the facial Thue choice index via entropy compression

Jakub Przybyło

A sequence is nonrepetitive if it contains no identical consecutive subsequences. An edge colouring of a path is nonrepetitive if the sequence of colours of its consecutive edges is nonrepetitive. By the celebrated construction of Thue, it is possible to generate nonrepetitive edge colourings for arbitrarily long paths using only three colours. A recent generalization of this concept [3] implies that we may obtain such colourings even if we are forced to choose edge colours from any sequence of lists of size 4 (while sufficiency of lists of size 3 remains an open problem). As an extension of these basic ideas, Havet, Jendroľ, Soták and Škrabuľáková [4] proved that for each plane graph, 8 colours are sufficient to provide an edge colouring so that every facial path is nonrepetitively coloured. We prove that the same is possible from lists, provided that these have size at least 12. We thus improve the previous bound of 291 (proved by means of the Lovász Local Lemma in [6]). Our approach is based on the Moser-Tardos entropy-compression method [5] and its recent extensions by Grytczuk, Kozik and Micek [2], and by Dujmović, Joret, Kozik and Wood [1].

References

On graphs with maximal independent sets of few sizes

Dieter Rautenbach
(joint work with Rommel Barbosa and Márcia Santana)

We prove that for every $r$ and $\Delta$, there are only finitely many graphs of minimum degree at least 2, maximum degree at most $\Delta$, and girth at least 7 that have maximal independent sets of at most $r$ different sizes. Furthermore, we prove several results restricting the degrees of such graphs. Our contributions generalize known results on well-covered graphs.

Circuit covers and nowhere-zero flows in signed graphs

Edita Rollová
(joint work with Edita Máčajová, André Raspaud, and Martin Škoviera)

A circuit cover for a given graph is a set of circuits of the graph such that each edge of the graph belongs to at least one circuit. It is clear that a graph admits a circuit cover if and only if it contains no bridges, or equivalently if it admits a nowhere-zero flow. A signed graph is a graph where each edge is either positive or negative. Since it is reasonable for a signed graph to admit a signed circuit cover and a nowhere-zero flow at the same time, by Bouchet [1] a signed circuit needs to be a balanced circuit or a barbell. A circuit of a signed graph is called balanced, if it contains an even number of negative edges, otherwise it is unbalanced. A barbell is a signed graph consisting of two edge-disjoint unbalanced circuits joined with a path. Thus given a signed graph $G$, a signed circuit cover $C$ of $G$ is a set of signed circuits of $G$ such that each edge of $G$ is contained in at least one of them. The length of $C$ is the sum of the lengths of the signed circuits in $C$. In the talk we will mention several bounds on the length of a shortest circuit cover in a signed graph $G$, depending on the existence of a nowhere-zero $k$-flow in $G$. In particular we show that a signed graph $G$ that admits a nowhere-zero 2-flow has a circuit cover with total length at most $\frac{4}{3} \cdot |E(G)|$. This bound is tight for infinitely many graphs.

References

Block colourings of designs revisited

Alexander Rosa

Block colourings of designs, both classical and its variants, and also the so-called specialized block colourings, offer an extensive collection of interesting problems. We will discuss some old and new concepts, results and open problems in colourings of block designs, mostly for Steiner triple systems and Steiner systems $S(2, 4, v)$.

Distance-locally disconnected graphs

Zdeněk Ryjáček

(joint work with Mirka Miller and Joe Ryan)

For an integer $k \geq 1$, we say that a (finite simple undirected) graph $G$ is $k$-locally disconnected if, for any $x \in V(G)$, the set of vertices at distance at most $k$ from $x$ induces in $G$ a disconnected graph. In the paper we study the asymptotic behavior of the number of edges of a $k$-locally disconnected graph on $n$ vertices. For general graphs, we show that this maximum number is $\Theta(n^2)$ for any fixed value of $k$ and, in the special case of regular graphs, we show that this asymptotic rate of growth cannot be achieved. For regular graphs, we give a general upper bound and we show its asymptotic sharpness for some values of $k$. We also discuss some connections with cages.

The locating chromatic number of composition product of graphs

Suhadi Wido Saputro

(joint work with Edy Tri Baskoro)

Let $f$ be a $k$-coloring of a connected graph $G$ and $\Pi = \{C_1, C_2, \ldots, C_k\}$ be a partition of $V(G)$ induced by $f$. For a vertex $v$ of $G$, the color code of $v$ with respect to $\Pi$ is defined as the ordered $k$-tuple $c_{\Pi}(v) = (d_G(v, C_1), d_G(v, C_2), \ldots, d_G(v, C_k))$, where $d_G(v, C_i) = \min\{d_G(v, x) | x \in C_i\}$. If all distinct vertices of $G$ have distinct color codes, then $f$ is called a locating coloring of $G$. The locating chromatic number of $G$, denoted by $\chi_L(G)$, is the smallest $k$ such that $G$ has a locating coloring with $k$ colors. In this paper, we consider a graph which is obtained by the composition product between any two graphs. The composition product of graphs $G$ and $H$, denoted by $G[H]$, is the graph with vertex set $V(G) \times V(H) = \{(a, v) | a \in V(G); v \in V(H)\}$, where $(a, v)$ adjacent with $(b, w)$ whenever $ab \in E(G)$, or $a = b$ and $vw \in E(H)$. We give a general bound of the locating chromatic number of a composition product $G[H]$ of any connected graphs $G$ and $H$. We also show that the bound is sharp.
Neighbour distinguishing gap colorings

Robert Scheidweiler
(joint work with Eberhard Triesch)

Let a graph $G = (V, E)$ with an edge labelling $f : E \rightarrow \{1, \cdots, k\}$ be given. We define the following function $l : V \rightarrow \{0, \cdots, k\}$,

\[
l(v) := \begin{cases} f(e), & \text{if } \deg_G(v) = 1 \text{ and } v \in e; \\ \max_{e \ni v} \{f(e)\} - \min_{e \ni v} \{f(e)\}, & \text{otherwise}. \end{cases}
\]

If $l(v) \neq l(w)$ for distinct vertices $v, w \in V$, we call $f$ gap vertex distinguishing or a gap-$k$-coloring, a notion defined by M. A Tahraoui, E. Duchêne, and H. Kheddouci in [1]. The minimum number $k$, such that $G$ has a gap-$k$-coloring, is called the gap chromatic number $\text{gap}(G)$ of $G$. A variant of the gap coloring problem, mentioned in [1], is to find the minimum number $k$, such that the induced function $l$ is a proper vertex coloring, i.e., $l$ distinguishes only the adjacent vertices. We define the corresponding graph parameter $\text{gap}_{ad}(G)$ of a graph $G$ and show estimations of this parameter in terms of the chromatic number. A related graph parameter, where adjacent vertices should have different sets of incident edge labels, has been studied in [2] and [3].

References


Hunting for rainbow bulls and diamonds

Ingo Schiermeyer
(joint work with Roman Soták)

In this talk we consider edge-colourings of the complete graph $K_n$. If a given subgraph $H$ contains no two edges of the same colour, then $H$ will be called a totally multicoloured (TMC) or rainbow subgraph of $K_n$ and we shall say that $K_n$ contains a TMC or rainbow $H$. For a graph $H$ and an integer $n$, let $f(n, H)$ denote the maximum number of colours in an edge-colouring of $K_n$ with no TMC $H$. The numbers $f(n, H)$ are called anti-ramsey numbers and have been introduced by Erdős, Simonovits and Sós [1].
We now define $rb(n, H)$ as the minimum number of colours such that any edge-colouring of $K_n$ with at least $rb(n, H) = f(n, H) + 1$ colours contains a TMC or rainbow subgraph isomorphic to $H$. The numbers $rb(n, H)$ will be called rainbow numbers [3].

Gorgol [2] has considered a cycle $C_k$ with a pendant edge, denoted $C_k^+$, and computed all rainbow numbers.

**Theorem 1** [2] $rb(n, C_k^+) = rb(n, C_k)$, for $n \geq k + 1$.

However, if we add two (or more) edges to a cycle $C_k$, the situation becomes surprisingly interesting.

**Theorem 2** Let $F$ be a graph of order $n \geq k \geq 3$ containing a cycle $C_k$. If $F$ has cyclomatic number $v(F) \geq 2$, then $rb(n, F)$ has no upper bound which is linear in $n$.

We first consider the graph $D = K_4 - e$, which is called the diamond. This graph contains a $C_3$ and has cyclomatic number $v(D) = 2$.

**Theorem 3** $rb(n, D) = \Theta(n^{\frac{3}{2}})$.

If $v(F) = 1$, then the situation is quite different. Let $B$ be the unique graph with 5 vertices and degree sequence $(1, 1, 2, 3, 3)$, which is called the bull. Here we have been able to compute all rainbow numbers for the bull.

**Theorem 4** $rb(5, B) = 6$ and $rb(n, B) = n + 2$ for $n \geq 6$.

**REFERENCES**


**On balanced separations of graphs**

**Jens Schreyer**

(joint work with Thomas Böhme)

The topic of our investigations is the following graph property. The question is, whether a graph $G$ can be separated by a small separator into a set of subgraphs such that no subgraph contains almost all vertices, and every subgraph is either
of a bounded size or has the same separation property as the original graph. The question arises from a computational problem in algorithmic game theory.

More formally, let \( \mathcal{G} \) be a class of graphs which is closed under subgraphs. We call the class \((\alpha, f)\)-separable for a constant \(0 < \alpha < 1\) and a function \(f \in o(n)\) if there exists a constant \(k\) such that for every graph \(G \in \mathcal{G}\) of order \(n\) either there exists a separator \(S\) such that \(|S| \leq f(n)\) and each component of \(G \setminus S\) has at most \(\alpha \cdot n\) vertices or \(n < k\). It is known, that planar graphs are \((\frac{2}{3}, \sqrt{n})\)-separable and graphs of treewidth at most \(w\) are \((\alpha, w)\) separable. We show that the converse of the latter is true in a sense, namely that every class of graphs which is \((\alpha, c)\)-separable for some constant \(c\) has to have bounded treewidth.

**Weighted \(k\)-path vertex cover**

Gabriel Semanišin
(joint work with Boštjan Brešar, Rija Erveš, Marko Jakovac, Rastislav Krivoš-Belluš, Petra Šparl)

Given a graph \(G = (V, E)\) and a positive integer \(k\), a subset \(S\) of vertices of \(G\) is called a \(k\)-path vertex cover if \(S\) intersects all paths of order \(k\) in \(G\) (in other words, each path of order \(k\) contains a vertex from \(S\)). The cardinality of a minimum \(k\)-path vertex cover is called the \(k\)-path vertex cover number of a graph \(G\), denoted by \(\psi_k(G)\). Clearly for \(k = 1\) the \(k\)-path vertex cover number corresponds to the order of a graph, and for \(k = 2\) we obtain well-known vertex cover number. The concept of \(k\)-path vertex cover was introduced in [2, 3] and later on in [1].

It is also natural to consider the weighted version of the mentioned problem, in which vertices are given weights. The problem of finding a minimum weight set such that the graph has no \(P_k\), was briefly introduced in [4]. Obviously this problem is a generalization of the Minimum Weight Cover Problem that plays a central role in the Computational Complexity Theory. It is a special case of the Vertex Deletion Problem that can be stated as follows: In a given vertex weighted graph find a minimum weight set of vertices whose deletion gives a graph satisfying a prescribed property. For instance an important special case of the Vertex Deletion Problem is the Feedback Problem: In a given graph \(G = (V, E)\) find a minimum weight set \(F\) of vertices such that the graph \(G[V \setminus F]\) induced by \(V \setminus F\) has no cycle.

In our contribution we shall present some results and algorithms concerning \(k\)-path vertex cover and weighted \(k\)-path vertex cover.

**References**

Cycles (and balls) in (planar) graphs

Jean-Sébastien Sereni

If one has a collection of cycles in a graph, how many vertices are needed to hit every cycle in the collection? Certainly, at least the number of vertex-disjoint cycles in that collection; but possibly more.

Does there exists a function, independent of the graph, that provides an upper bound on this number in terms of the maximum number of disjoint cycles in the collection?

These questions have spawn a huge amount of research. This survey talk will be focused on a selection of (old and recent) results around the corresponding duality and integrality gaps, especially for planar graphs. Of particular interest for us will be the cases where the collection is composed of all cycles, or all odd cycles, of the graph. In addition, a similar question for balls in graphs will be considered.

The main aim of the talk will be to bring to the fore a number of open problems.

Upper bounds for the facial Thue choice number of graphs

Erika Škrabuľáková
(joint work with Jens Schreyer)

Let $G$ be a plane graph and $\varphi$ a colouring of its vertices. A vertex-colouring $\varphi$ of $G$ is called facial non-repetitive if for no sequence $r_1 r_2 \cdots r_{2n}; n \geq 1$, of consecutive vertex colours of any facial path it holds $r_i = r_{n+i}$ for all $i = 1, 2, \ldots, n$. A plane graph $G$ is facial non-repetitively vertex-$l$-choosable if for every list assignment $L : V \rightarrow 2^N$ with minimum list size at least $l$ there is a facial non-repetitive vertex-colouring $\varphi$ with colours from the associated lists.

The facial Thue choice number, $\pi_{fl}(G)$ of a plane graph $G$ is the minimum number $l$, such that $G$ is facial non-repetitively vertex-$l$-choosable.
We show that \( \pi_{fl}(G) \leq 4e\Delta + 2e \) for arbitrary plane graphs \( G \) with maximum degree \( \Delta \). Moreover, we give some better bounds for special classes of plane graphs.

**Maximum induced matching of hexagonal graphs**

Petra Šparl

(joint work with Rija Erveš)

A matching \( M \) of a graph \( G = (V, E) \) is said to be an induced matching of \( G \) if no two edges in \( M \) are joined by an edge of \( E(G) \). An induced matching \( M \) is a maximum induced matching if \( M \) has the maximum size among all induced matchings in the graph. The size of a maximum induced matching in \( G \) is denoted by \( \text{mim}(G) \). It is known that the problem of determining \( \text{mim}(G) \) is NP-hard. It remains NP-hard even when restricted to bipartite or planar graphs. We investigate the size of maximum induced matching for a special subset of planar graphs, so called hexagonal graphs (i.e. graphs induced on the subsets of vertices of the triangular grid). We show that for any connected hexagonal graph \( G \), with \( |V(G)| = n \geq 2 \) it holds

\[
 n - \text{mim}(G) \leq \frac{2}{3}n + \frac{1}{3},
\]

which is equivalent to

\[
 \text{mim}(G) \geq \frac{1}{3}n - \frac{1}{3}.
\]

The bound of the above inequality is tight. Namely, there exists an example of a hexagonal graph \( G \) with \( |V(G)| = 25 \), such that the equality is attained (\( \text{mim}(G) = 8 \)). Since \( \text{mim}(G) \) can vary between different hexagonal graphs with the same number of vertices \( n = V(G) \), we are looking for the case where maximal \( n - \text{mim}(G) \) among all hexagonal graphs \( G \) with given \( n = V(G) \) is attained. We show that for any connected hexagonal graph \( G \), with \( |V(G)| = n \geq 2 \) it holds

\[
 n - \text{mim}(G) \leq \begin{cases} 
 \frac{2}{3}n & ; \quad n = 0 \text{ (mod 6)} \\
 \frac{2}{3}n + \frac{1}{3} & ; \quad n = 1 \text{ (mod 6)} \\
 \frac{2}{3}n + \frac{1}{3}(n \text{ (mod 6)}) - 2 & ; \quad 2 \leq n \text{ (mod 6)} \leq 5
\end{cases}
\]

**1-factor- and cycle covers of cubic graphs**

Eckhard Steffen

For \( k \geq 1 \), let \( m_k(G) \) be the maximum number of edges of a bridgeless cubic graph \( G \) that can be covered by the union of \( k \) 1-factors of \( G \), and \( \mu_k(G) = \)
$|E(G)| - m_k(G)$. The main focus of the paper are covers with three 1-factors. It shows that if $\mu_3(G) \neq 0$, then $2\mu_3(G)$ is an upper bound for the girth of $G$. A bridgeless cubic graph $G$ with $\mu_3(G) \leq 4$ has a Fulkerson coloring, and if $\mu_3(G) \leq 5$ or $\mu_3(G) < \text{girth}(G)$, then $G$ has three 1-factors with empty intersection. We further prove some other new upper bounds for the length of cycle covers of bridgeless cubic graphs. If $\mu_3(G) \leq 5$, then $G$ has a 3-cycle cover of length at most $\frac{4}{3}|E(G)| + 2$, and if $\mu_3(G) \leq \text{girth}(G)$, then $G$ has a 3-cycle cover of length at most $\frac{25}{36}|E(G)|$. Cubic graphs with $\mu_4(G) = 0$ have a 4-cycle cover of length $\frac{4}{3}|E(G)|$ and a 5-cycle double cover. These graphs also satisfy two conjectures of Zhang, and we also give a negative answer to a problem of Zhang.

Palindromefree facial edge coloring of plane graphs

Peter Šugerek
(joint work with Stanislav Jendroľ)

We consider a plane graph $G$. In an $l$-facial coloring, any two different vertices that lie on the same face and are at distance at most $l$ on that face receive distinct colors. Kráľ', Madaras, Škrekovski in [2] conjectured that each plane graph has an $l$-facial coloring with at most $3l + 1$ colors. It is known that every plane graph admits a 2-facial coloring using at most 8 colors, see [2]. Montassier and Raspaud in [4] improved this bound for plane graphs with large girth and proved that if $G$ is a plane graph with girth $g \geq 14$ (resp. 10, 8) then $G$ admits a 2-facial coloring using 5 colors (resp. 6, 7). We consider an edge coloring using the same conditions as the mentioned 2-facial vertex coloring, but for edges. An edge coloring is called facial palindromefree (FPF) if any three consecutive edges that lie on the same face, receive distinct colors. We present that there exists FPF edge coloring for each plane graph $G$ using at most 8 colors. We improve this bound for cubic plane graphs to 7 and moreover, we give exact bounds 6 for outerplanar graphs.

References

On the structure of Lucas cubes

Andrej Taranenko

Fibonacci and Lucas cubes are induced subgraphs of hypercubes obtained by excluding certain binary strings from the vertex set. They appear as models for interconnection networks, as well as in chemistry. We derive a characterization of Lucas cubes that is based on a peripheral expansion of a unique convex subgraph of an appropriate Fibonacci cube. This serves as the foundation for a recognition algorithm of Lucas cubes that runs in linear time.

H-force number in distance graphs

Mária Timková

A nonempty vertex set \( X \subseteq V(G) \) of a hamiltonian graph \( G \) is called an H-force set of \( G \) if every \( X \)-cycle of \( G \) (i.e. a cycle of \( G \) containing all vertices of \( X \)) is hamiltonian. The H-force number \( h(G) \) of a graph \( G \) is defined to be the smallest cardinality of an H-force set of \( G \). Let \( n \in \mathbb{N} \) and \( D \subseteq \mathbb{N} \) is a set of parameters, then distance graph is a graph with \( V(G) = \{0, 1, 2, ..., n - 1\} \) and two vertices \( i, j \) are adjacent if \( |j - i| \in D \). For a distance graph \( G_n(d_1,d_2) \) we establish an exact value of H-force number if \( \frac{d_2}{d_1} \geq 2 \), and an upper bound if \( \frac{d_2}{d_1} < 2 \).

Monochromatic triangles in three-coloured graphs

Andrew Treglown

(joint work with James Cummings, Daniel Kráľ, Florian Pfender, Konrad Sperfeld, and Michael Young)

In 1959, Goodman determined the minimum number of monochromatic triangles in a complete graph whose edge set is 2-coloured. Goodman also raised the question of proving analogous results for complete graphs whose edge sets are coloured with more than two colours. For \( n \) sufficiently large, we determine the minimum number of monochromatic triangles in a 3-coloured copy of \( K_n \). Moreover, we characterise those 3-coloured copies of \( K_n \) that contain the minimum number of monochromatic triangles.
Hypercycle systems
Zsolt Tuza
(joint work with Mario Gionfriddo and Lorenzo Milazzo)

The 3-uniform cycle of length 5 has five vertices $a, b, c, d, e$ and five edges $abc, bcd, cde, dea, eab$. Similarly, an $r$-uniform $k$-cycle has $k$ vertices arranged in a cyclic order, and $k$ edges which are the $r$-tuples formed by $r$ consecutive vertices. A cycle system $C(r, k, v)$ of order $v$ is a collection of $r$-uniform $k$-cycles on a $v$-element set, such that each $r$-tuple is an edge in precisely one of those cycles.

We study the ‘spectrum problem’ of cycle systems mostly in the case of $r = 3$ and $k = 5$; the task is to determine the values $v$ for which a $C(3, 5, v)$ exists. Necessary conditions and infinite families of constructions are presented, but the general problem remains open.

$b$-bounded circular coloring of graphs
Krzysztof Węsek

$r$-circular coloring of a graph $G$ is an assignment of open unit-length arcs on an Euclidean circle of length $r$, such that adjacent vertices get disjoint arcs. This concept can be useful to model optimization problems when we are considering tasks performed in the continuous, cyclic system and we are looking for the best schedule of tasks.

In this talk we introduce its natural modification, $b$-bounded circular coloring, motivated by the practical applications. It rises by adding of an additional condition to the basic circular coloring model: constraint corresponding to the restriction that (because of some technical limitations) we can perform at most $b$ tasks simultaneously. Namely, we require that no point of the circle (in our interpretation: a time-point in the schedule) is contained by more than $b$ arcs. $b$-bounded circular chromatic number of a graph $G$ can be defined as:

$$\chi_r^{b-bnd}(G) = \inf \{ r : G \text{ has a } b\text{-bounded } r\text{-circular coloring} \}.$$  

We present some more or less fundamental properties of $\chi_r^{b-bnd}$.

Extending choosability results to paintability
Douglas B. West
(joint work with James Carraher, Sarah Loeb, Thomas Mahoney, Gregory J. Puleo, and Mu-Tsun Tsai)

Introduced independently by Schauz and by Zhu, the Marker-Remover game is an on-line version of list coloring. Initially, each vertex has some tokens. On
each round, Marker specifies a subset $M$ of the remaining vertices, each of which loses a token. Remover then deletes a subset of $M$ forming an independent set of vertices.

Marker wins by marking a vertex with no remaining tokens. Remover wins by deleting all vertices before this occurs. The paintability of a graph is least $k$ such that Remover has a winning strategy when each vertex begins with $k$ tokens; it is at least the list chromatic number (choosability).

Ohba proved that if a $k$-chromatic graph $G$ has at most $k + \sqrt{2k}$ vertices, then $G$ is $k$-choosable. We strengthen this by showing that if $G$ has at most $k + 2\sqrt{k} - 1$ vertices, then $G$ is $k$-paintable.

Hoffman and Johnson proved that $K_{k+1,r}$ is $k$-choosable if and only if $r < k^k - (k - 1)^k$. Using a general strategy for Remover on complete bipartite graphs, we show that the threshold on $r$ where $K_{k+1,r}$ first fails to be $k$-paintable is at most roughly half of that for $k$-choosability.

The sum-paintability of a graph is the least $t$ such that Remover has a winning strategy under some distribution of $t$ tokens; it is at least the sum-choosability. We show that the sum-paintability of a graph with $n$ vertices and $m$ edges is at most $n + m$, with equality for various families of graphs. We also determine the sum-paintability of various “theta-graphs”, including $K_{2,r}$.

**On generalized Pell numbers and their relations with Fibonacci numbers**

Andrzej Włoch
(joint work with Małgorzata Wołowiec-Musial)

In the talk we present a concept of a generalization of companion Pell numbers $Q(k, n)$ in the distance sense. The numbers $Q(k, n)$ both generalize classical Pell numbers and Tribonacci numbers. We give a combinatorial representation of $Q(k, n)$ concerning counting of special subsets of 2-dimensional set $X^{(2)}$, graph interpretations and matrix generators. Graph interpretations of considered sequences are useful for proving identities. Using matrix methods different identities for considered sequences can be obtained. Furthermore we give relations of $Q(k, n)$ with classical Fibonacci numbers.

**References**

On kernels by monochromatic paths in edge-coloured digraphs

Iwona Włoch
(joint work with Urszula Bednarz)

Let $D$ be an edge-coloured digraph. A path is called monochromatic if all of its arcs are coloured alike. A set $J \subseteq V(D)$ is said to be a kernel by monochromatic paths of the edge-coloured digraph if

1. for any two different vertices $x, y \in J$ there is no monochromatic path between them and
2. for each $x \in V(D) \setminus J$ there exists monochromatic paths from $x$ to $y$, for some $y \in J$.

In this talk we study the problem of the existence of kernel by monochromatic paths in edge-colouring digraphs.

REFERENCES


Symmetries and colorings of groups

Yuliya Zelenyuk

A symmetry on a group $G$ is a mapping $G \ni x \mapsto ax^{-1}a \in G$, where $a \in G$. This notion has interesting relations to Ramsey theory and to enumerative combinatorics [1]. We will discuss some old and new results in this field [2, 3].

REFERENCES

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## Programme of the Conference

<table>
<thead>
<tr>
<th>Time</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>16:00 – 22:00</td>
<td>Registration</td>
</tr>
<tr>
<td>18:00 – 21:00</td>
<td>Dinner</td>
</tr>
<tr>
<td>Time</td>
<td>Session</td>
</tr>
<tr>
<td>--------------</td>
<td>----------------------------------------------</td>
</tr>
<tr>
<td>07:00 – 09:00</td>
<td>Breakfast</td>
</tr>
<tr>
<td>09:00 – 09:50</td>
<td>A Kostochka A. V.</td>
</tr>
<tr>
<td></td>
<td>B Schreyer J.</td>
</tr>
<tr>
<td></td>
<td>B Furmańczyk H.</td>
</tr>
<tr>
<td>09:55 – 10:15</td>
<td>A Ryjáček Z.</td>
</tr>
<tr>
<td></td>
<td>B Adiwijaya</td>
</tr>
<tr>
<td>10:20 – 10:40</td>
<td>A Ryjáček Z.</td>
</tr>
<tr>
<td></td>
<td>B Adiwijaya</td>
</tr>
<tr>
<td>10:45 – 11:15</td>
<td>Coffee break</td>
</tr>
<tr>
<td>11:15 – 11:35</td>
<td>A Král’ D.</td>
</tr>
<tr>
<td></td>
<td>B Naserasr R.</td>
</tr>
<tr>
<td>11:40 – 12:00</td>
<td>A Haluszczak M.</td>
</tr>
<tr>
<td></td>
<td>B Pettersson V. H.</td>
</tr>
<tr>
<td>12:05 – 12:25</td>
<td>A Treglown A.</td>
</tr>
<tr>
<td></td>
<td>B Kyppö J.</td>
</tr>
<tr>
<td>12:30 – 14:00</td>
<td>Lunch</td>
</tr>
<tr>
<td>15:20 – 16:10</td>
<td>A Sereni J.-S.</td>
</tr>
<tr>
<td>16:15 – 16:35</td>
<td>A Problem session</td>
</tr>
<tr>
<td>16:40 – 17:10</td>
<td>Coffee break</td>
</tr>
<tr>
<td>17:10 – 17:30</td>
<td>A Mácajová E.</td>
</tr>
<tr>
<td></td>
<td>B Przybylo J.</td>
</tr>
<tr>
<td></td>
<td>C Drgas-Burchardt E.</td>
</tr>
<tr>
<td>17:35 – 17:55</td>
<td>A Rollová E.</td>
</tr>
<tr>
<td></td>
<td>B Škrabuľáková E.</td>
</tr>
<tr>
<td></td>
<td>C Taranenko A.</td>
</tr>
<tr>
<td>18:00 – 18:20</td>
<td>A Crevals S.</td>
</tr>
<tr>
<td></td>
<td>B Mockovčiaková M.</td>
</tr>
<tr>
<td></td>
<td>C Wloch A.</td>
</tr>
<tr>
<td>18:30 – 20:00</td>
<td>Dinner</td>
</tr>
<tr>
<td>20:00 –</td>
<td>Welcome party</td>
</tr>
<tr>
<td>Time</td>
<td>Session</td>
</tr>
<tr>
<td>--------------</td>
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</tr>
<tr>
<td>07:00 – 09:00</td>
<td>Breakfast</td>
</tr>
<tr>
<td>09:00 – 09:50</td>
<td>A PACH J.</td>
</tr>
<tr>
<td>09:00 – 09:55</td>
<td>A RAUTENBACH D.</td>
</tr>
<tr>
<td></td>
<td>B WLOCH I.</td>
</tr>
<tr>
<td>09:55 – 10:15</td>
<td>A SCHIERMEYER I.</td>
</tr>
<tr>
<td></td>
<td>B ADAMUS L.</td>
</tr>
<tr>
<td>10:20 – 10:40</td>
<td>A SCHIERMEYER I.</td>
</tr>
<tr>
<td></td>
<td>B ADAMUS L.</td>
</tr>
<tr>
<td>10:45 – 11:15</td>
<td>Coffee break</td>
</tr>
<tr>
<td>11:15 – 11:35</td>
<td>A BURGER A. P.</td>
</tr>
<tr>
<td></td>
<td>B DVORÁK Z.</td>
</tr>
<tr>
<td>11:40 – 12:00</td>
<td>A VAN AARDT S.</td>
</tr>
<tr>
<td></td>
<td>B KLEŠČ M.</td>
</tr>
<tr>
<td>12:05 – 12:25</td>
<td>A FRICK M.</td>
</tr>
<tr>
<td></td>
<td>B HUDÁK D.</td>
</tr>
<tr>
<td>12:30 - 14:00</td>
<td>Lunch</td>
</tr>
<tr>
<td>15:20 – 16:10</td>
<td>A WEST D. B.</td>
</tr>
<tr>
<td>16:15 – 16:35</td>
<td>A ZELENYUK ŶU.</td>
</tr>
<tr>
<td></td>
<td>B FURUYA M.</td>
</tr>
<tr>
<td>16:40 – 17:10</td>
<td>Coffee break</td>
</tr>
<tr>
<td>17:10 – 17:30</td>
<td>A BODE J.-P.</td>
</tr>
<tr>
<td></td>
<td>B LUKOT’KA R.</td>
</tr>
<tr>
<td></td>
<td>C HOLUB P.</td>
</tr>
<tr>
<td>17:35 – 17:55</td>
<td>A FIEDOROWICZ A.</td>
</tr>
<tr>
<td></td>
<td>B NAROSKI P.</td>
</tr>
<tr>
<td></td>
<td>C ŠPARL P.</td>
</tr>
<tr>
<td>18:00 – 18:20</td>
<td>A BONAMY M.</td>
</tr>
<tr>
<td></td>
<td>B GÖRLICH A.</td>
</tr>
<tr>
<td></td>
<td>C PAVLIČ P.</td>
</tr>
<tr>
<td>18:30 - 20:00</td>
<td>Dinner</td>
</tr>
</tbody>
</table>
### Wednesday

<table>
<thead>
<tr>
<th>Time</th>
<th>Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>06:30 - 08:30</td>
<td>Breakfast</td>
</tr>
<tr>
<td>08:00 - 15:00</td>
<td>Trip</td>
</tr>
<tr>
<td>13:00 - 16:00</td>
<td>Lunch</td>
</tr>
<tr>
<td>18:30 - 20:00</td>
<td>Dinner</td>
</tr>
</tbody>
</table>

### Thursday

<table>
<thead>
<tr>
<th>Time</th>
<th>Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>07:00 – 09:00</td>
<td>Breakfast</td>
</tr>
<tr>
<td>09:00 – 09:50</td>
<td>A Brandt S.</td>
</tr>
<tr>
<td></td>
<td>The circumference of the square of a connected graph</td>
</tr>
<tr>
<td>09:55 – 10:15</td>
<td>A Neđela R.</td>
</tr>
<tr>
<td></td>
<td>Hamiltonicity of a certain class of cubic graphs</td>
</tr>
<tr>
<td></td>
<td>B Ciechacz S.</td>
</tr>
<tr>
<td></td>
<td>Distance magic labeling of product of two graphs</td>
</tr>
<tr>
<td>10:20 – 10:40</td>
<td>A Kardoś F.</td>
</tr>
<tr>
<td></td>
<td>Almost all fullerene graphs are Hamiltonian</td>
</tr>
<tr>
<td></td>
<td>B Anholcer M.</td>
</tr>
<tr>
<td></td>
<td>Group irregularity strength of graphs</td>
</tr>
<tr>
<td>10:45 – 11:15</td>
<td>Coffee break</td>
</tr>
<tr>
<td>11:15 – 11:35</td>
<td>A Li H.</td>
</tr>
<tr>
<td></td>
<td>Implicit degrees and Hamiltonian graph theory</td>
</tr>
<tr>
<td></td>
<td>B Kemnitz A.</td>
</tr>
<tr>
<td></td>
<td>d-strong total colorings of graphs</td>
</tr>
<tr>
<td>11:40 – 12:00</td>
<td>A Chiba S.</td>
</tr>
<tr>
<td></td>
<td>Hamilton cycles and Tutte cycles in claw-free graphs</td>
</tr>
<tr>
<td></td>
<td>B Scheidweiler R.</td>
</tr>
<tr>
<td></td>
<td>Neighbour distinguishing gap colorings</td>
</tr>
<tr>
<td>12:05 – 12:25</td>
<td>A Timková M.</td>
</tr>
<tr>
<td></td>
<td>H-force number in distance graphs</td>
</tr>
<tr>
<td></td>
<td>B Saputro S. W.</td>
</tr>
<tr>
<td></td>
<td>The locating chromatic number of composition product of graphs</td>
</tr>
<tr>
<td>12:30 - 14:00</td>
<td>Lunch</td>
</tr>
<tr>
<td>15:20 – 16:10</td>
<td>A Meszka M.</td>
</tr>
<tr>
<td></td>
<td>Palettes in block colorings of combinatorial structures</td>
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<td>16:15 – 16:35</td>
<td>A Rosa A.</td>
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<td>Block colorings of designs revisited</td>
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<td>1-factor- and cycle covers of cubic graphs</td>
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<td>16:40 – 17:10</td>
<td>Coffee break</td>
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<td>17:10 – 17:30</td>
<td>A Peterin I.</td>
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<td>The b-chromatic index of a graph</td>
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<td>B Lindner C.</td>
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<td>How to embed a partial odd-cycle system</td>
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<td>Generalized line graphs</td>
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<td>17:35 – 17:55</td>
<td>A Petrosyan P. A.</td>
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<td>Interval colorings of complete multipartite graphs</td>
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<td>b-bounded circular coloring of graphs</td>
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<td>Numbers of edges and vertices in magic-type graphs</td>
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<td>18:00 – 18:20</td>
<td>A Candráková B.</td>
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<td>k-regular graphs and multigraphs with the circular chromatic index close to k</td>
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<td>Proof of Caccetta-Häggkvist conjecture for oriented graphs with positive minimum out-degree and of independence number 2</td>
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<td>C Polláková T.</td>
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<td>Supermagic graphs having a saturated vertex</td>
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<td>19:00 –</td>
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<td>Time</td>
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<td>07:00 – 09:00</td>
<td>Breakfast</td>
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<td>09:55 – 10:15</td>
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<td>10:45 – 11:15</td>
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<td>12:30 - 14:00</td>
<td>Lunch</td>
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