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C&C
HIGH TATRAS, SLOVAKIA

Igor Fabrici, Mirko Horňák, Stanislav Jendroľ eds.

Workshop
Cycles and Colourings 2010

September 05 – 10, 2010 Tatranská Štrba

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Dear Participant,

welcome to the Nineteenth Workshop Cycles and Colourings. Except for the first workshop in the Slovak Paradise (Čingov 1992), the remaining seventeen workshops took place in the High Tatras (Nový Smokovec 1993, Stará Lesná 1994–2003, Tatranská Štrba 2004–2009).

The series of C&C workshops is organised by combinatorial groups of Košice and Ilmenau. Apart of dozens of excellent invited lectures and hundreds of contributed talks the scientific outcome of our meetings is represented also by special issues of journals Tatra Mountains Mathematical Publications and Discrete Mathematics (TMMP 1994, 1997, DM 1999, 2001, 2003, 2006, 2008).

The scientific programme of the workshop consists of 50 minute lectures of invited speakers and of 20 minute contributed talks. This booklet contains abstracts as were sent to us by the authors.

Invited speakers:

S. Arumugam	Kalasalingam University, Anand Nagar, Krishnankoil, India
Charles C. Lindner	Auburn University, Auburn, AL, USA
Mirka Miller	The University of Newcastle, Newcastle, Australia
André Raspaud	University of Bordeaux 1, Bordeaux, France
Akira Saito	Nihon University, Tokyo, Japan
Ingo Schiermeyer	Freiberg University of Mining and Technology, Freiberg, Germany
Douglas B. West	University of Illinois, Urbana, IL, USA

Have a pleasant and successfull stay in Tatranská Štrba (and possibly also in Zgorzelisko where the Sixth Cracow Conference on Graph Theory will take place just in the week following our workshop).

Organising Committee:

Igor Fabrici
Jochen Harant
Erhard Hexel
Mirko Horňák
Stanislav Jendroľ (chair)
František Kardoš
Dieter Rautenbach
Štefan Schrötter

Contents

The cycle spectrum of a graph

S. Arumugam

Let $G = (V, E)$ be a finite, simple undirected graph of order n and size m. Let $C(G) = \{k : G \text{ contains a cycle of length } k\}$. The set $C(G)$ is called the *cycle spectrum* of the graph G. In this survey talk we present a few fundamental results on cycle spectrum and discuss the following problems.

1. How large must the minimum degree be to force a specific number to occur in $C(G)$?
2. For any fixed integer $k \geq 1$, every graph of sufficiently large order n and more than $\frac{n^2}{4}$ edges contains a cycle of length $2k+1$. Is the corresponding result true for even integer?
3. Erdős-Gyarfás conjecture: For every graph G with minimum degree at least 3, there exists a positive integer $k \geq 2$ such that 2^k is in $C(G)$.
4. The existence of long arithmetic progressions in $C(G)$.
5. Cycle lengths congruent modulo k and Thomassen's conjecture.
6. Algorithmic applications.

Antimagicness of disconnected graphs

Martin Bača

(joint work with Yuqing Lin, Francesc A. Muntaner-Batle,
and Andrea Feňovčíková)

A *labeling* of a graph is a mapping that carries some sets of graph elements into numbers (usually the positive integers). An (a, d)-*edge-antimagic total labeling* ((a, d)-*EAT* for short) of a graph $G = (V, E)$ with p vertices and q edges is a one-to-one mapping f from $V(G) \cup E(G)$ onto the set $\{1, 2, \ldots, |V(G)|+|E(G)|\}$, such that the set of all the edge-weights, $w_f(uv) = f(u) + f(uv) + f(v), uv \in E(G)$, forms an arithmetic sequence starting from a and having a common difference d. Such a labeling is called *super* if the smallest possible labels appear on the vertices.

We mainly investigate the existence of super (a, d)-EAT labeling for disconnected graphs. We concentrate on the following problem: If a graph G is (super) (a, d)-EAT, is the disjoint union of m copies of the graph G, denoted by mG, (super) (a, d)-EAT as well?

A generalization of the concept of supermagic regular graphs

Ľudmila Bezegová
(joint work with Jaroslav Ivančo)

A graph is called *supermagic* if it admits a labelling of the edges by pairwise different consecutive positive integers such that the sum of the labels of the edges incident with a vertex is independent of the particular vertex. We define a *degree-magic* labelling of a graph which generalizes the supermagic labelling of regular graphs. We describe some properties of degree-magic graphs and characterize degree-magic multipartite graphs.

Ramsey numbers for a disjoint union of some graphs

Halina Bielak

We give the Ramsey number for a disjoint union of some G-good graphs versus a graph G generalizing the results of Stahl [4] and Baskoro et al. [1] and the previous results of the author [2,3]. Moreover, for some graphs F and H with the surplus $s(H) \geq 1$ we construct a family $\mathcal{G}$ of graphs such that F is G-good for each $G \in \mathcal{G}$.

References

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[4] S. Stahl, On the Ramsey number $r(F, K_m)$ where F is a forest, Canad. J. Math. 27 (1975), 585–589.

Vertex rainbow numbers for cube graphs

Jens-P. Bode

The *cube graph vertex rainbow number* $M(n, k)$ is the maximum number of colors in a vertex coloring of the cube graph Q_n such that no subcube Q_k is rainbow, where a graph is called *rainbow* if all its vertices have pairwise different colors.

As partial results $M(n,k)$ is determined if $k=2$, $k=n$, $k=n-1$, $k=n-2$ for $n \leq 13$, and $k=n-3$ for $n \leq 8$.

Graphs of odd girth 7 with large degree

Stephan Brandt

(joint work with Elizabeth Ribe-Baumann)

We show that every graph of order n with minimum degree $\delta > 4n/17$ and no odd cycles of length 3 or 5 is homomorphic with the Möbius ladder with 6 rungs and include the extremal graph characterization in the case of equality. The key tools used in our observations are simple characteristics of maximal odd girth 7 graphs.

Improper C-colorings of graphs

Csilla Bujtás

(joint work with E. Sampathkumar, Zsolt Tuza,
L. Pushpalatha, and R. C. Vasundhara)

We consider a new graph invariant, which combines the concepts of *improper coloring* from graph theory and *C-coloring* from hypergraph theory.

The notion of k-improper coloring (also called defective coloring) of graphs is a vertex coloring such that, for each vertex v, the neighborhood $N(v)$ contains at most k vertices having the same color as v. That is, the classical coloring constraint is relaxed up to k edges at each vertex. From the theory of 'mixed hypergraphs' we adopt the notion of 'C-coloring'. When restricted to graphs, this means that any two adjacent vertices receive the same color. Hence, we obtain the rather simple constraint that each connected component of the graph must be monochromatic, and there is not much to explore in this direction. On the other hand, the defective version of C-coloring raises many interesting questions.

For an integer $k \geq 1$, a *k-improper C-coloring* of a graph $G=(V,E)$ is a coloring $\varphi : V \to \mathbb{N}$ such that for each vertex v at most k vertices in the neighborhood $N(v)$ receive colors different from $\varphi(v)$. The *k-improper upper chromatic number* $\bar{\chi}_{k\text{-imp}}(G)$ of G is the maximum number of colors permitted in such a coloring.

We give general estimates on $\bar{\chi}_{k\text{-imp}}$ in terms of various graph invariants, e.g. minimum and maximum degree, vertex covering number, domination number and neighborhood number. Also, the analogue of the Nordhaus-Gaddum theorem is proved. Moreover, the algorithmic complexity of determining $\bar{\chi}_{k\text{-imp}}$ is considered, and structural correspondence between k-improper C-colorings and certain kinds of edge cuts is shown.

$S(2,1)$-labeling of graphs with cyclic structure

Karina Chudá

An r-$S(2,1)$-labeling of a graph G is a mapping from the vertex-set of G to the cyclic group $\mathbb{Z}_r$ such that every pair of vertices adjacent in G has labels at least 2 apart in $\mathbb{Z}_r$ and simultaneously every pair of vertices at distance 2 in G has distinct labels in $\mathbb{Z}_r$. The σ-number of a graph G is the smallest r such that G admits an r-$S(2,1)$-labeling.

In this contribution, we present our results concerning the σ-number of the Isaacs graphs, prisms and the generalized Blanuša snarks of both types. We use the cyclic structure of a graph to cover it with copies of a suitable subgraph. We determine all r-$S(2,1)$-labelings of the subgraph and form an r-$S(2,1)$-labeling of the entire graph from a concatenation of available r-$S(2,1)$-labelings of the subgraph. We apply this method directly to the Isaacs graphs and prisms and with a slight modification to the generalized Blanuša snarks since they have a single irregularity in their cyclic structure.

List colorings of K_5-minor-free graphs with special list assignments

Daniel W. Cranston
(joint work with Anja Pruchnewski, Zsolt Tuza, and Margit Voigt)

A *list assignment* L of G is a function that assigns to every vertex v of G a set (list) $L(v)$ of colors. The graph G is called *L-list colorable* if there is a coloring φ of the vertices of G such that $\varphi(v) \in L(v)$ for all $v \in V(G)$ and $\varphi(v) \neq \varphi(w)$ for all $vw \in E(G)$.

We consider the following question of Bruce Richter, where $d(v)$ denotes the degree of v in G:

Let G be a planar, 3-connected graph that is not a complete graph. Is G L-list colorable for every list assignment L with $|L(v)| = \min\{d(v), 6\}$ for all $v \in V$?

More generally, we ask for which pairs (r, k) the following question is answered in the affirmative. Let r and k be integers and let G be a K_5-minor-free r-connected graph that is not a Gallai tree. Is G L-list colorable for every list assignment L with $|L(v)| = \min\{d(v), k\}$? Recall that a *Gallai tree* is a graph G such that every block of G is either a complete graph or an odd cycle.

We study this question by considering the components of $G[S_k]$, where $S_k := \{v \in V(G) \mid d(v) < k\}$ is the set of vertices with small degree in G. We are especially interested in the minimum distance $d(S_k)$ in G between the components of $G[S_k]$.

On the k-th moment of a 1-planar graph

Jochen Harant

(joint work with Július Czap and Dávid Hudák)

For a simple 1-planar graph G and a positive integer k, an upper bound on the sum of the k-th powers of the degrees of G is proven.

On vertices enforcing a hamiltonian cycle and cycle extendability

Erhard Hexel

For a graph G and a vertex set $X \subseteq V(G)$, a cycle of G is said to be an *X-cycle* if it contains all vertices of X. A nonempty set $X \subseteq V(G)$ of a hamiltonian graph G is called a *hamiltonian cycle enforcing set* (in short an *H-force set*) of G if every X-cycle is a hamiltonian one. Moreover, $h(G)$ is defined to be the smallest cardinality of an H-force set of G and called the *H-force number* of G. These concepts have been introduced in [1] and the study of the parameter $h(G)$ has been started there.

For a hamiltonian graph G let $S(G)$ denote the set of all vertices $v \in V(G)$ for which $G - v$ is hamiltonian. Then, every H-force set contains $S(G)$ as a subset. In [2], Hendry studied the so called cycle extendable graphs, a subclass of the hamiltonian graphs. A graph G is *cycle extendable* if it contains a cycle and if for every nonhamiltonian cycle C there exists a $V(C)$-cycle C' in G such that $|V(C)| + 1 = |V(C')|$. We found that in a cycle extendable graph G which is not a cycle, $S(G)$ is the only H-force set of smallest cardinality and, that implies $h(G) = |S(G)|$.

By weakening the concept of cycle extendability we are able to characterize the class of all hamiltonian graphs G for which $h(G) = |S(G)|$ holds and to present some results in the consequence.

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Star subdivisions and connected even factors in the square of a graph

Přemysl Holub

(joint work with Jan Ekstein, Tomáš Kaiser, Liming Xiong, and Shenggui Zhang)

For any positive integer s, a $[2, 2s]$-factor in a graph G is a connected even factor with maximum degree at most $2s$. Fleischner proved that for every 2-connected graph G the square of G is hamiltonian. Gould and Jacobson conjectured that for the hamiltonicity of G^2, the connectivity condition can be relaxed for $S(K_{1,3})$-free graphs. Their conjecture was proved by Hendry and Vogler.

Moreover, Abderrezzak, Flandrin, and Ryjáček proved the following result in which graphs may contain an induced $S(K_{1,3})$ of a special type.

Theorem. Let G be a connected graph such that every induced $S(K_{1,3})$ in G has at least three edges in a block of degree at most two. Then G^2 is hamiltonian, i.e., has a $[2, 2]$-factor.

We generalize these results in such a way that if every induced $S(K_{1,2s+1})$ in a graph G has at least 3 edges in a block of degree at most two, then G^2 has a $[2, 2s]$-factor.

On properties of maximal 1-planar graphs

Dávid Hudák

A graph is called *1-planar* if it can be drawn in the plane so that each its edge contains at most one crossing. A graph G from the family $\mathcal{G}$ of graphs is *maximal* if $G+uv \notin \mathcal{G}$ for any two nonadjacent vertices $u, v \in V(G)$. We deal with selected properties of maximal 1-planar graphs (number of edges, different diagrams, local structure and hamiltonicity); the obtained results are compared to analogical results for maximal planar graphs.

College admissions and lattices

Zsuzsanna Jankó

(joint work with Tamás Fleiner)

We study an abstract model motivated by Hungarian college admissions and closely related to the stable marriage problem. In this case, an extension of stability property is 'score-stability'. Blair showed that if both sides of the matching market have so-called path-independent substitutable choice functions then stable solutions form a lattice under a natural partial order. In our model, choice functions are substituable, but not path-independent. Still we are able to prove that stable score limits form a natural lattice, using Tarski's fix point theorem.

Bounds on the broadcast chromatic number for cubic graphs

Elizabeth Jonck

(joint work with Yolande Immelman)

Goddard, Hedetniemi, Harris, and Rall introduced the broadcast chromatic number. They defined a *broadcast coloring* of order k as a function from the vertex set V to the set $\{1, ..., k\}$ such that equality between function values of u and v implies that the distance between u and v is more than the function value of u. The minimum order of a broadcast coloring is called the *broadcast chromatic number* of G, and is denoted by $\chi_b(G)$. In this talk, bounds on the broadcast chromatic number for cubic graphs are considered.

Generalized fractional total coloring of complete graphs

Gabriela Karafová

An additive hereditary property of graphs is a class of simple graphs which is closed under unions, subgraphs and isomorphism. Let $\mathcal{P}$ and $\mathcal{Q}$ be two additive and hereditary graph properties and $r, s \in \mathbb{N}$ such that $r \geq 2s$. Then $\frac{r}{s}$*-fractional* $(\mathcal{P}, \mathcal{Q})$*-total coloring* of a graph G is a mapping $f : V \cup E \to \binom{\{1,2,\ldots,r\}}{s}$ such that for any color i all vertices of color i induce a subgraph from property $\mathcal{P}$, all edges of color i induce a subgraph from property $\mathcal{Q}$ and vertices and incident edges have assigned disjoint sets of colors. The minimum value of ratio $\frac{r}{s}$ of an $\frac{r}{s}$-fractional $(\mathcal{P}, \mathcal{Q})$-total coloring of G is called *fractional* $(\mathcal{P}, \mathcal{Q})$*-total chromatic number* $\chi''_{f,\mathcal{P},\mathcal{Q}}(G)$. Let $k = \sup\{i : K_{i+1} \in \mathcal{P}\}$ and $l = \sup\{i : K_{i+1} \in \mathcal{Q}\}$. We show for a complete graph K_n that if $l \geq k + 2$ then $\chi''_{f,\mathcal{P},\mathcal{Q}}(K_n) = \frac{n}{k+1}$ for a sufficiently large n.

Acyclic edge coloring of planar graphs

František Kardoš

(joint work with Dávid Hudák, Borut Lužar, Roman Soták, and Riste Škrekovski)

An acyclic edge coloring of a graph is a proper edge coloring without bichromatic cycles. In 1978, it was conjectured that $\Delta(G)+2$ colors suffice for an acyclic edge coloring of every graph G. The conjecture has been verified for several classes of graphs, however, the best known upper bound for as special class as planar graphs are, is $\Delta + 12$.

In this talk, we present recent results on planar simple graphs which need only $\Delta(G)$ colors for an acyclic edge coloring. We show that a planar graph with girth g and maximum degree Δ admits such acyclic edge coloring if $g \geq 12$, or $g \geq 8$ and $\Delta \geq 4$, or $g \geq 7$ and $\Delta \geq 5$, or $g \geq 6$ and $\Delta \geq 6$, or $g \geq 5$ and $\Delta \geq 10$. Our results improve some previously known bounds.

Hypergraph extensions of the Erdős-Gallai Theorem

Gyula Y. Katona

(joint work with Ervin Győri and Nathan Lemons)

The Erdős-Gallai Theorem gives the maximum number of edges in a graph without a path of length k. We extend this result for Berge paths in r-uniform hypergraphs. We also find the extremal hypergraphs avoiding t-tight paths of a given length and consider this extremal problem for other definitions of paths in hypergraphs.

Minimum k-path vertex cover

Ján Katrenič

(joint work with Boštjan Brešar, František Kardoš, and Gabriel Semanišin)

Let G be a graph and let k be a positive integer. A subset of vertices $S \subseteq V(G)$ is called a *k-path vertex cover* if every path of order k in G contains at least one vertex from S. Denote by $\psi_k(G)$ the minimum cardinality of a k-path vertex cover in G. It is shown that the problem of determining $\psi_k(G)$ is NP-hard for each $k \geq 2$, while for trees the problem can be solved in linear time. We investigate upper bounds on the value of $\psi_k(G)$ and provide several estimations and exact values of $\psi_k(G)$.

[1,1,t]-colorings of complete graphs

Arnfried Kemnitz

(joint work with Massimiliano Marangio and Zsolt Tuza)

Given non-negative integers r, s, and t, an $[r, s, t]$-*coloring* of a graph $G = (V(G), E(G))$ is a mapping c from $V(G) \cup E(G)$ to the color set $\{1, \ldots, k\}$ such that $|c(v_i) - c(v_j)| \geq r$ for every two adjacent vertices v_i, v_j, $|c(e_i) - c(e_j)| \geq s$ for every two adjacent edges e_i, e_j, and $|c(v_i) - c(e_j)| \geq t$ for all pairs of incident vertices and edges, respectively. The $[r, s, t]$-*chromatic number* $\chi_{r,s,t}(G)$ of G is defined to be the minimum k such that G admits an $[r, s, t]$-coloring. We will examine $\chi_{1,1,t}(K_p)$ for complete graphs K_p.

Crossings in products of cycles and paths with other graphs

Marián Klešč

The *crossing number* $cr(G)$ of a graph G is the minimum possible number of edge crossings in a drawing of G in the plane. The investigation on the crossing number of graphs is a classical and however very difficult problem. Garey and Johnson have proved that the problem to determine the crossing number of graphs is NP-complete. Crossing numbers of some classes of graphs have been obtained. The structure of Cartesian products of graphs makes Cartesian products of special graphs one of few graph classes, for which the exact values of crossing numbers were obtained.

Let P_n and C_n be the *path* and the *cycle* of length n, respectively, and the *star* S_n be the complete bipartite graph $K_{1,n}$. There are known crossing numbers of Cartesian products of all graphs on at most 4 vertices with C_n and S_n. For the path P_n, for all graphs G of order at least five the crossing numbers of $G \times P_n$ are known.

We extend these results and we give the crossing numbers for Cartesian product of paths with some other graphs. Moreover, we will discuss some problems concerning crossing numbers of products of cycles with other graphs.

Vertex-disjoint cycles, fundamental cycles, and the maximum genus of a graph

Michal Kotrbčík
(joint work with Martin Škoviera)

In the first part of this talk we will discuss a relationship between the maximum number of vertex-disjoint cycles of a graph and its maximum genus. In particular, we show that the maximum number of vertex-disjoint cycles of G is bounded both from above and below by linear functions of the maximum genus of G and the cycle rank of G. Both bounds are tight and, as the maximum genus is computable in polynomial-time, provide an efficiently computable estimate of the maximum number of vertex-disjoint cycles.

In the second part of the talk we will focus on the set of intersection graphs of fundamental cycles $S_G = \{G\sharp T; T \text{ is a spanning tree of } G\}$, where $G\sharp T$ denotes the intersection graph of fundamental cycles of G with respect to a spanning tree T. It is known that matchings in graphs in S_G are related to the maximum genus of a graph G. We show how to construct a 3-connected graph G such that the minimum and maximum size of the maximum matching in graphs in S_G differ arbitrarily. On the other hand, it follows from a result on maximum genus that for any 4-edge-connected graph G, the graph $G\sharp T$ has a perfect matching or a matching missing exactly one vertex, depending on the parity of the number of vertices of $G\sharp T$.

Crossing numbers of some families of Cartesian products of graphs

Daniela Kravecová

The *crossing number*, $cr(G)$, of a simple graph G with vertex set V and edge set E is defined as the minimum number of crossings among all possible drawings of G in the plane. Computing the crossing number of a given graph is in general NP-complete problem. The exact values of the crossing numbers are known only for some graphs or some families of graphs. Some Cartesian products of special graphs are one of few graph classes for which the exact values of crossing numbers were obtained. There are known several exact results of the crossing numbers of the Cartesian product of a some special graphs with paths, cycles and stars.

In the talk we give several exact values of the crossing numbers for other Cartesian products of graphs.

Extending fractional precolorings

Martin Kupec

(joint work with Daniel Kráľ, Matjaž Krnc, Borut Lužar, and Jan Volec)

Theorem (Fractional coloring extension)
Let G be a graph with fractional chromatic number χ, P an independent set in G and d the minimum distance between two vertices in P. If $d \geq 4$, then every fractional $(\chi+\varepsilon)$-precoloring of P can be extended to a fractional $(\chi+\varepsilon)$-coloring of G, where ε satisfies the following inequalities:

$$\begin{array}{rccc} d = 0 \bmod 4: & \frac{1}{\chi+\varepsilon} & \geq & 1-\frac{d}{4}\varepsilon \\ d = 1 \bmod 4: & \frac{1}{\chi} & \geq & 1-\frac{d-1}{4}\varepsilon \\ d = 2 \bmod 4: & \frac{\chi-1}{\chi+\varepsilon} & \leq & 1-\frac{d-2}{4}\varepsilon \\ d = 3 \bmod 4: & \frac{(1-\varepsilon)(1-\chi)}{\chi} & \leq & \frac{d-3}{4}\varepsilon \end{array}$$

For $\chi = 2$ and $\chi \geq 3$, the value of ε is the best possible.

The cycles of extended knights

Jorma Kyppö

This presentation is focused on the extended knights. *Extended knights* is a general term used for the hyper-knights and hyper-bishops situated on the n-dimensional chessboard with varying tiling. Hyper-knights without the chessboard are also explored.

Packing chromatic number for square and hexagonal lattices

Bernard Lidický

(joint work with Martin Böhm, Jan Ekstein, Jiří Fiala, Přemek Holub, and Lukáš Lánský)

Let G be a graph. A subset of its vertices P such that distance of every pair of vertices from P is more than d is called a *packing* of width d. The *packing chromatic number* $\chi_\rho(G)$ of G is the smallest integer k such that the vertex set of G can be partitioned into packings with pairwise different widths.

It is known that χ_ρ of the square lattice is between 10 and 17. We improve the lower bound to 12. It is also known that for the hexagonal lattice $\mathcal{H}$ the $\chi_\rho(\mathcal{H}) = 7$. On the other hand for six layers of the hexagonal lattice, $\chi_\rho(\mathcal{H} \,\square\, P_6)$ is unbounded. We show that even $\mathcal{H} \,\square\, P_3$ cannot be covered by finitely many packings.

Extra perfect 2-fold cycle systems

Charles C. Lindner

(joint work with Alex Rosa and Mariusz Meszka)

A cycle system (X, C) is said to be *perfect* if every pair of vertices are connected by a path of length 2 in one of the cycles of C. Not too surprisingly a 2-fold cycle system is said to be 2-*perfect* if every pair of vertices are connected by a path of length 2 in two cycles belonging to C. Finally a 2-perfect 2-fold cycle system is said to be *extra* provided for every pair of vertices $a \neq b$ the midpoints of the two paths (a, x, b) and (a, y, b) are distinct. For example, the 2-perfect 2-fold 6-cycle system (X, C) of order 7 given by $X = \{0, 1, 2, 3, 4, 5, 6\}$ and $C = \{(0, 2, 1, 5, 3, 4), (1, 3, 2, 6, 4, 5), (2, 4, 3, 0, 5, 6), (3, 5, 4, 1, 6, 0), (4, 6, 5, 2, 0, 1), (5, 0, 6, 3, 1, 2), (6, 1, 0, 4, 2, 3)\}$ is extra. What makes an extra 2-perfect 2-fold cycle system of interest is that putting together the two paths (a, x, b) and (a, y, b) for all $a \neq b$ gives a 4-fold cycle system. For example putting together all such paths in the extra 2-perfect 2-fold 6-cycle system of under 7 given above gives the 4-fold 4-cycle system $(X, C*)$, where $C* = \{(0, 2, 1, 6), (3, 4, 6, 5), (0, 4, 3, 6), (0, 2, 6, 4), (0, 3, 2, 6), (0, 5, 4, 6), (0, 1, 5, 4), (1, 5, 6, 2), (0, 3, 1, 5), (1, 2, 4, 3), (0, 1, 4, 3), (0, 1, 6, 5), (1, 4, 2, 6), (2, 3, 5, 4), (1, 5, 3, 6), (1, 2, 5, 4), (2, 3, 6, 5), (0, 3, 5, 2), (0, 1, 3, 2), (1, 4, 6, 3), (0, 4, 2, 5)\}$.

This talk is an elementary survey on the complete solution of constructing extra 2-perfect 2-fold k-cycle systems for $k = 5$ and 6. The techniques are completely different for 5 and 6.

Small cubic graphs with large oddness

Robert Lukoťka

Oddness of a cubic graph G is the minimal number of odd cycles in a 2-factor of G. We try to find smallest graphs with given cyclic connectivity and oddness. For instance, we construct cyclically 4-edge-connected cubic graphs with $36k-26$ vertices having oddness $2k$.

Cubic graphs with 1-factor are (7,2)-edge-choosable

Edita Máčajová

A graph G is called (m, n)-*edge-choosable* if for every assignment of sets of size m to the edges of G, it is possible to choose for every edge an n-element subset from its set such that subsets chosen for any pair of incident edges are disjoint. Mohar conjectured that every cubic graph is (7,2)-edge-choosable. In 2009, Cranston and West showed that every 3-edge-colourable cubic graph is (7,2)-edge-choosable and gave a sufficient condition with the help of which they proved that many non-3-edge-colourable cubic graphs are (7,2)-edge-choosable. In this talk we show that every cubic graph which has a 1-factor is (7,2)-edge-choosable.

On minimal light sets of cycles in families of plane graphs

Tomáš Madaras

Let $\mathcal{G}$ be a family of graphs and let $\mathcal{H}$ be a finite set of graphs with the property that each graph of $\mathcal{G}$ contains a proper subgraph isomorphic to at least one member of $\mathcal{H}$. Let $\tau(\mathcal{H}, \mathcal{G})$ be the smallest integer with the property that every graph $G \in \mathcal{G}$ contains a subgraph K which is isomorphic to one of the elements in $\mathcal{H}$ such that, for every vertex $v \in V(K)$, $\deg_G(v) \leq \tau(\mathcal{H}, \mathcal{G})$. If such a finite $\tau(\mathcal{H}, \mathcal{G})$ does not exist we write $\tau(\mathcal{H}, \mathcal{G}) = +\infty$. We shall say that the set $\mathcal{H}$ is *light* in the family $\mathcal{G}$ if $\tau(\mathcal{H}, \mathcal{G})$ is finite; if $|\mathcal{H}| = 1$, we obtain the notion of *light graph* in a family of graphs.

We explore, for selected families of plane graphs, the minimal light sets (that is, the sets which are light in given family, but none their proper subset is light) comprised of cycles.

Extending directed cycles of in-tournaments

Dirk Meierling

A directed cycle C of a digraph D is *extendable* if there exists a directed cycle C' in D that contains all vertices of C and an additional one. In 1989, Hendry defined a digraph D to be *cycle extendable* if it contains a directed cycle and every non-Hamiltonian directed cycle of D is extendable. Furthermore, D is *fully cycle extendable* if it is cycle extendable and every vertex of D belongs to a directed cycle of length three. In 2001, Tewes and Volkmann extended these definitions in considering only directed cycles whose length exceeds a certain bound $3 \leq k < n$: a digraph D is *k-extendable* if every directed cycle of length t, where $k \leq t < n$, is extendable. Moreover, D is called *fully k-extendable* if D is k-extendable and every vertex of D belongs to a directed cycle of length k.

An *in-tournament* is an oriented graph such that the in-neighborhood of every vertex induces a tournament. This class of digraphs which generalizes the class of tournaments was introduced by Bang-Jensen, Huang, and Prisner in 1993. Tewes and Volkmann showed that every connected in-tournament D of order n

with minimum degree $\delta \geq 1$ is $(n - \lfloor \frac{4\delta+1}{3} \rfloor)$-extendable. Furthermore, if D is a strongly connected in-tournament of order n with minimum degree $\delta = 2$ or $\delta > \frac{8n-17}{31}$, then D is fully $(n - \lfloor \frac{4\delta+1}{3} \rfloor)$-extendable. In this talk we cover the remaining interval $3 \leq \delta \leq \frac{8n-17}{31}$.

Acyclic colouring of graphs

Peter Mihók

(joint work with Mieczysław Borowiecki and Elżbieta Sidorowicz)

A graph property $\mathcal{P}$ is any nonempty isomorphism-closed class of simple (finite or infinite) graphs. We will consider additive and hereditary graph properties i.e. classes closed under disjoint union and subgraphs.

Let a, b be positive integers, $a > b$ and $\mathcal{P}$ be an additive and hereditary graph property. A *fractional* vertex (edge) $(\mathcal{P}^{a;b})$-colouring of a graph G is a mapping ϕ of the vertex set $V(G)$ (edge set $E(G)$) of the graph G to the set of b-element subsets of $\{1, 2, \ldots, a\}$ such that for each 'colour' $i, 1 \leq i \leq a$ the subgraph $G[i]$ induced by the vertices (edges) where $i \in \phi(v)$ has the property $\mathcal{P}$. We will investigate the structure of the classes of fractionally vertex and edge $(\mathcal{P}^{a;b})$-colourable graphs for the property $\mathcal{P} = \mathcal{D}_1$ 'to be acyclic'.

Degree/diameter problem: recent advances and open problems

Mirka Miller

A well-known fundamental problem in extremal graph theory is the *degree/diameter problem*, which is to determine the largest (in terms of the number of vertices) graphs or digraphs or mixed graphs of given maximum degree, respectively, maximum out-degree, respectively, mixed degree; and given diameter. General upper bounds, called Moore bounds, exist for the largest possible order of such graphs, digraphs and mixed graphs of given maximum degree d (respectively, maximum out-degree d, resp., maximum mixed degree d) and diameter k.

The Moore bound for a directed graph of maximum out-degree d and diameter k is

$$M_{d,k} = 1 + d + d^2 + \cdots + d^k$$

It is known that digraphs of order $M_{d,k}$ (Moore digraphs) do not exist for $d > 1$ and $k > 1$. Similarly, the Moore bound for an undirected graph of degree d and diameter k is

$$M^*_{d,k} = 1 + d + d(d-1) + \cdots + d(d-1)^{k-1}$$

Undirected Moore graphs for $d > 2$ and $k > 1$ exist only when $k = 2$ and $d = 3, 7$ and possibly 57.

Mixed (or partially directed) Moore graphs of diameter $k = 2$ were first studied by Bosák. The Moore bound for mixed graphs is

$$M_{d,z,k} = 1 + d + zd + r(d-1) + \cdots + zd^{k-1} + r(d-1)^{k-1}$$

where $d = z + r$.

In recent years, there have been many interesting new results in all the three versions of the degree/diameter problem, resulting in improvements in both the lower bounds and the upper bounds on the largest possible number of vertices.

In this talk we present an overview of the current state of the degree/diameter problem, for undirected, directed and mixed graphs, and we outline several related open problems.

Open efficient domination in digraphs

Ľudovít Niepel
(joint work with Martin Knor)

Let G be a digraph. A set $S \subseteq V(G)$ is called *efficient total dominating set* if the set of open out-neighborhoods $N^-(v) \in S$ is a partition of $V(G)$. We say that digraph G is *efficiently open-dominated* if both G and its reverse digraph G^- have efficient total dominating set. We present some properties of efficiently open dominated digraphs. Special attention is given to tournaments and directed tori, being Cartesian products of directed cycles.

Completely separating systems and antimagic labeling of non-regular graphs

Oudone Phanalasy
(joint work with Martin Bača, Andrea Feňovčíková, and Mirka Miller)

A *vertex antimagic edge labeling* of a graph with q edges is a bijection from the set of edges to the set of positive integers $\{1, 2, \ldots, q\}$ such that all vertex weights are pairwise distinct, where the *vertex weight* of a vertex is the sum of the labels of all the edges incident with that vertex. A graph is *antimagic* if it has a vertex antimagic edge labeling.

In 1990, Hartsfield and Ringel conjectured that every graph with the exception of K_2 is antimagic. During the last two decades there have been many attempts to prove this conjecture.

In this talk we will describe our novel method for constructing vertex magic edge graph labeling using 'completely separating systems'.

Let $[n] = \{1, 2, \ldots, n\}$. A *completely separating system* (CSS) on $[n]$ (or (n)CSS), is a collection $\mathcal{C}$ of subsets of $[n]$ in which for each pair of elements $a \neq b \in [n]$, there are two subsets A and B of $[n]$ in $\mathcal{C}$ such that A contains a but not b and B contains b but not a.

Using completely separating systems as a tool for studying labeling of graphs, we show that there is a relationship between CSSs and antimagic labeling of graphs. Combining this relationship with various graph operations, we construct vertex antimagic edge labelings for various infinite families of non-regular graphs, thereby giving further support to the Hartsfield-Ringel conjecture.

Acyclic choosability of planar graphs

André Raspaud

Let G be a graph with vertex set $V(G)$ and edge set $E(G)$. A proper vertex coloring of G is an assignment π of integers (or labels) to the vertices of G such that $\pi(u) \neq \pi(v)$ if the vertices u and v are adjacent in G. A *k-coloring* is a proper vertex coloring using k colors. A proper vertex coloring of a graph is *acyclic* if there is no bicolored cycle in G [3]. Given a list assignment $L = \{L(v)|v \in V(G)\}$ of G, we say G is acyclically L-list colorable if there exists a proper acyclic coloring π of G such that $\pi(v) \in L(v)$ for all $v \in V(G)$. If G is acyclically L-list colorable for any list assignment with $|L(v)| \geq k$ for all $v \in V(G)$, then G is acyclically k-choosable.

Borodin et al. [2] first investigated the acyclic list coloring of planar graphs, they proved that every planar graph is acyclically 7-choosable. They also proposed the following challenging conjecture:

Conjecture. Every planar graph is acyclically 5-choosable.

If the Conjecture were true, then it would strengthen the Borodin's acyclically 5-colorable theorem [1] and the Thomassen's 5-choosable theorem [4] about planar graphs. As far as we know, the Conjecture is still open. As yet, it has been verified only for several restricted classes of planar graphs.

In this talk we will give a short survey of acyclic choosability of planar graphs and present a recent result concerning the acyclic 5-choosability of planar graphs.

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Cycles in digraphs and housing markets

Michal Repiský
(joint work with Katarína Cechlárová)

Let $G = (V, H)$ be a directed graph with loops and $P(v)$ an ordered list of arcs outgoing from $v \in V$. We interpret the vertices of G as agents, each owning one house, characteristic for him. For each agent $v \in V$, the set $\{u \in V; (v, u) \in H\}$ is the set of acceptable houses for agent v and $P(v)$ is an ordering of these houses (his preferences). The set $P = \{P(v), v \in P\}$ is called the preference profile and the pair $\mathcal{M} = (V, P)$ is a housing market.

A permutation x of V is an allocation if $(v, x(v)) \in H$ for each $v \in V$. Each allocation can include agents who are not trading, i.e. $x(v) = v$, and trading cycles (corresponding to directed cycles of G of length at least 2) with the following interpretation: if $x(v) = u$ then agent v receives the house of agent u. A cycle $C = (v_0, v_1, \ldots, v_{r-1})$ in G is a blocking cycle with respect to an allocation x if for each vertex $v_i \in C$ either $x(v_i) = v_i$ or v_{i+1} is in $P(v_i)$ written before $x(v_i)$ (indices are taken modulo r, if necessary). This means that if agents contained in C choose trading according to arcs of C, everybody will improve with respect to allocation x.

We say that an allocation x is in the core of housing market $\mathcal{M}$ if it does not admit any blocking cycle.

It is known that each housing market admits a core allocation. However, little is known about the structure of core allocations. We show that for general housing markets, it is NP-hard to decide whether the core contains allocations of some special structure (each trading cycle has length at most k, each agent is trading etc). However, the size of core can grow exponentially already for markets with a relatively simple symmetric structure.

Circulants as signatures of cyclic Steiner triple systems

Alexander Rosa
(joint work with Mariusz Meszka)

Let $(V, \mathcal{B})$ be a cyclic Steiner triple system of order v (STS(v)), $v \equiv 1 \pmod{6}$, with block orbits $O_1, O_2, \ldots, O_{\frac{v-1}{6}}$ where the differences in O_i are a_i, b_i, c_i. A *signature of* $(V, \mathcal{B})$ is a circulant $C(v; S)$ of degree $\frac{v-1}{3}$ with connection set $S = \{s_1, s_2, \ldots, s_{\frac{v-1}{6}}\}$ where $s_i \in \{a_i, b_i, c_i\}$.

A circulant $C(v; S)$ of degree $\frac{v-1}{3}$ with $v \equiv 1 \pmod{6}$ and connection set S is a *signature circulant* if it is isomorphic to a signature of some cyclic STS(v); otherwise it is a *non-signature*. We show the existence of non-signatures for all $v \equiv 1 \pmod{6}$, $v \geq 13$, and examine the number of signatures and non-signatures as the order increases.

Forbidden subgraphs generating a finite set

Akira Saito
(joint work with Michael D. Plummer and Jun Fujisawa)

Since Bedrossian (1991) and Faudree-Gould (1997) determined the pairs of forbidden subgraphs that force the existence of a hamiltonian cycle in a 2-connected graph, the relationship between forbidden subgraphs and hamiltonian properties has long been studied. In this research, given a property P of a graph, we investigate the sets of graphs $\mathcal{H}$ such that every connected (or 2-connected) $\mathcal{H}$-free graph, possibly except for a finite number of exceptions, satisfies P. Here, we allow a finite number of exceptions to handle sporadic exceptions, which often occur in the study. These sporadic exceptions are not essential, and disappear if we raise the order of graphs in consideration. However, we have one problem: what happens if the class of connected $\mathcal{H}$-free graphs is finite? If it occurs, we can simply declare that all the graphs in the class are 'finite number of exceptions', and claim that all the others, which are empty, satisfy the given property P. But this class does not give any insight into a particular property P since it satisfies every property with a finite number of exceptions. It obscures the whole picture of the research.

With this background in mind, we discuss the sets of connected graph $\mathcal{H}$ such that the class of connected (or 2-connected) $\mathcal{H}$-free graphs is finite. After some preparations, we will first explore the problem in the domain of connected graphs and see the complete answer, which has been published in Diestel's textbook. Then we will look into the problem in the domain of the graphs of higher connectivity where the problem becomes harder. At the end of the talk, we pose several open problems.

Matchings in balanced hypergraphs

Robert Scheidweiler

(joint work with Eberhard Triesch)

We investigate the class of balanced hypergraphs, a common generalization of bipartite graphs due to Berge. By means of coloring and regularity properties of these hypergraphs we estimate the cardinality of maximum matchings (with respect to contained vertices and edges).

Moreover we generalize the Gallai-Edmonds decomposition for this class of hypergraphs in three different ways. First we consider an arbitrary weight function. Second, we analyze the weight function, defined by the number of contained vertices. Finally we take a look at the decomposition for maximum cardinality matchings. In addition we obtain a short and combinatorial proof of Hall's theorem for balanced hypergraphs based on the decompositions.

Rainbow numbers and minimum rainbow subgraphs

Ingo Schiermeyer

In this talk we consider edge colourings of graphs. A subgraph H of a graph G is called *rainbow subgraph*, if all its edges are coloured distinct.

In the first part we will survey the computation of rainbow numbers. For given graphs G, H the *rainbow number* $rb(G, H)$ is the smallest number m of colours such that if we colour the edges of G with at least m different colours, then there is always a totally multicoloured or rainbow copy of H. For various graph classes of H we will list the known rainbow numbers if G is the complete graph and report about recent progress on the computation of rainbow numbers. Finally, new results on the rainbow numbers $rb(Q_n, Q_k)$ for the hypercube Q_n will be presented.

The generation of genome populations in bioinformatics can be solved by computing *Minimum Rainbow Subgraphs*. In the second part we will report about the Minimum Rainbow Subgraph problem (MRS):

Given a graph G, whose edges are coloured with p colours. Find a subgraph $H \subseteq G$ of G of minimum order $r^*(G)$ with $|E(H)| = p$ such that each colour occurs exactly once.

We will discuss several complexity results and show lower and upper bounds for $r^*(G)$. Finally, we will present some recent polynomial time approximation algorithms for the MRS problem.

Extending Bondy's theorem for graphs with small minimum degree

Anika Schwarz

In 1980 Bondy proved the following sufficient condition for hamiltonicity:

Let G be a k-connected graph of order $n \geq 3$. If the degree sum of every $k+1$ pairwise nonadjacent vertices is at least $\frac{1}{2}((k+1)(n-1)+1)$, then G is hamiltonian.

It is possible to allow some independent $(k+1)$-sets in G violating the degree condition of Bondy's theorem but still implying the hamiltonicity of G.

In particular, we show that a k-connected graph of order at least 3 with minimum degree δ is hamiltonian if the number of exceptional $(k+1)$-sets is at most $\delta - k$.

Nowhere-zero flows on bidirected eulerian graphs

Martin Škoviera

(joint work with Edita Máčajová)

A *bidirected graph* is a graph in which each edge is divided into two half-edges and each half-edge has an independent orientation. A *nowhere-zero k-flow* on a bidirected graph is an assignment of a value from the set $\{\pm 1, \pm 2, \ldots, \pm(k-1)\}$ to each edge of in such a way that for every vertex v the sum of the values on the half-edges directed to v equals the sum of the values on the half-edges directed away from v.

Bidirected graphs and their flows first appeared in late 1960s in a work of Youngs related to the solution of the Heawood map colouring problem. They were independently introduced by Edmonds and Johnson in 1970 to express algorithms for matchings. Nevertheless, the first deeper study of flows on bidirected graphs was undertaken by Bouchet in 1983 with motivations from topological graph theory. Bouchet also proposed the conjecture that every bidirected graph that has nowhere-zero flow has a nowhere-zero 6-flow.

In this talk we present a result inspired by the well-known fact that every (directed) eulerian graph has a nowhere-zero 2-flow. We show that every bidirected eulerian graph that has a nowhere-zero flow has a nowhere-zero 4-flow. While bidirected eulerian graphs with a nowhere-zero 2-flow are easy to describe, a characterisation of those that admit a 3-flow seems to be difficult and remains open.

Non-repetitive list edge-colourings of graphs

Erika Škrabuľáková
(joint work with Jens Schreyer)

Let G be a plane graph and $L : E(G) \to 2^{\mathbb{N}}$ be a list assignment. An edge-colouring φ of G is a *non-repetitive list edge-colouring* of G if for no sequence $r_1, r_2, r_3, \ldots, r_{2n}$ of colours of the edges of some path of G it holds $r_i = r_{n+i}$ for all $i = 1, 2, 3, \ldots, n$. If a graph G is non-repetitively list edge-colourable for every list assignment L with list size at least k, we call G *non-repetitively edge k-choosable.* The smallest number k such that G is non-repetitively edge k-choosable is called the *list Thue chromatic index* of G and is denoted by $\pi'_l(G)$. We show several ideas how to find non-repetitive list edge-colourings of graphs and give some bounds for the list Thue chromatic index of G when some conditions are fulfilled.

Fractional and circular 1-defective colorings of outerplanar graphs

Roman Soták
(joint work with Zuzana Farkasová)

We consider fractional $(\frac{k}{q}, d)$-defective coloring of graphs which is an assignment of q-element subsets of k-element set to vertices of a graph G in such a way that every vertex has at most d defects (the *defect* means nonempty intersection of color sets assigned to neighbouring vertices). This notion is similar to defective $(\frac{k}{q}, d)$-circular coloring introduced by Klostermeyer in [1], where q-element subsets in coloring must be consecutive. He proved that each outerplanar graph G with no adjacent triangles (i.e., no 3-cycles sharing an edge) is circular $(\frac{5}{2}, 1)$-defective colorable. We give a counterexample for this result. Moreover, we prove that for each outerplanar graph with no 3-circuits sharing common vertex, the value of fractional 1-defective chromatic number is at most $\frac{7}{3}$ and this bound is the best possible.

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3-consecutive edge colorings of graphs

Zsolt Tuza

(joint work with Csilla Bujtás, Charles Dominic, L. Pushpalatha, and E. Sampathkumar)

Three edges e_1, e_2 and e_3 in a graph G are consecutive if they form a path (in this order) or a cycle of length 3. The *3-consecutive edge coloring number* $\chi'_{3c}(G)$ of G is the maximum number of colors permitted in a coloring of the edges of G such that if e_1, e_2 and e_3 are consecutive edges in G then e_1 or e_3 receives the color of e_2.

We give general bounds on χ'_{3c} in terms of various graph invariants, and also point out a close relation between 3-consecutive edge colorings and a certain kind of vertex cuts. Algorithmically, the distinction between $\chi'_{3c} = 1$ and $\chi'_{3c} \geq 2$ is intractable, while efficient algorithms can be designed to determine χ'_{3c} on some particular graph classes.

We also consider briefly the vertex-coloring version of the problem (i.e., where the middle vertex v_2 of any path $v_1v_2v_3 \subset G$ of length 2 is required to have the same color as one of the ends v_1, v_3) and show that a really surprising relation is valid concerning 3-consecutive colorings of vertices and edges.

Weights of induced subgraphs in $K_{1,r}$-free graphs

Margit Voigt

(joint work with Anja Pruchnewski)

Let H be a subgraph of a given graph G. The weight

$$w(H) = \sum_{v \in V(H)} d_G(v)$$

is the sum of the degrees of all vertices of H in G.

Investigations of this parameter are initiated by the beautiful result of Kotzig in 1955 who proved that every 3-connected planar graph contains an edge of weight at most 13.

Meanwhile there are many results for planar graphs and graphs embedded on general surfaces concerning the existence of specified subgraphs with bounded weight.

In 2001 Jendroľ and Schiermeyer gave the answer to a question of Erdős. They determined the integer $W(n, m)$ such that every graph with n vertices and m edges contains an edge of weight at most $W(n, m)$.

Harant et al. started to investigate another kind of question. They asked about a bound f depending on some parameters of G and H such that for **every** induced subgraph in G isomorphic to H it holds $w(H) \leq f$. They obtained bounds for paths and cycles in $K_{1,r}$-free graphs.

We generalize these results and give bounds for k-colorable induced subgraphs in $K_{1,r}$-free graphs. Moreover, several sharpness examples will be presented.

Independent sets in cubic graphs with large girth

Jan Volec

(joint work with Daniel Kráľ and František Kardoš)

We show that every (sub)cubic n-vertex graph with sufficiently large girth has an independent set of size $0.4352n$. As a corollary of our method, we obtain that the fractional chromatic number of cubic graphs of large girth is at most 2.2978. The bound on the independence number also translates to random cubic graphs.

Two theorems on long cycles

Douglas B. West

(joint work with Suil O, Hehui Wu, and Reza Zamani)

The Chvátal-Erdős Theorem states that every graph whose connectivity is at least its independence number has a spanning cycle. In 1976, Fouquet and Jolivet conjectured an extension: If G is an n-vertex k-connected graph with independence number a, and $a \geq k$, then G has a cycle with length at least $\frac{k(n+a-k)}{a}$. We prove this conjecture.

If time permits, we will also present a theorem on spanning cycles in a balanced bipartite graph G with n vertices, giving a sharp threshold for the existence of a spanning cycle containing a specified linear forest F with k edges. The sufficient condition is that any two nonadjacent vertices in opposite partite sets have degree-sum at least $n/2 + \lceil k/2 \rceil + \epsilon$, where $\epsilon = 1$ if all components of F have even length or F has at most two odd components and none even, and $\epsilon = 0$ otherwise. The threshold on the degree-sum is sharp when $n \geq 3k$.

(2)-pancyclic graphs

Carol T. Zamfirescu

In this talk, graphs will always be simple (i.e. without multiple edges or loops), undirected, finite, and connected.

In the well-known book on graph theory by J. A. Bondy and U. S. R. Murty [1] we find a series of 50 open problems, among which problem number 10 shall be the initial point of our investigation: Determine all graphs having exactly one cycle of each length p, $3 \leq p \leq n$, where n is the order of the graph (such graphs are called *uniquely pancyclic*). [1] attributes this problem to R. C. Entringer, who formulated it in 1973.

Constructing the four smallest uniquely pancyclic graphs (they are of order 3, 5, 8, and 8) is an easy task. In 1986, Y. Shi [4] constructed three further such graphs (each of order 14), conjecturing that there are no uniquely pancyclic graphs other than these seven. This problem is still widely open, and only recently K. Markström [3] confirmed Shi's conjecture for $n \leq 59$.

We discuss the class of (2)-*pancyclic* graphs, which are graphs of order n having exactly two cycles of length p for all p fulfilling $3 \leq p \leq n$. Very little is known concerning these graphs. We provide examples of such graphs (most of which were constructed by G. Exoo [2]), establish their existence or non-existence for all orders up to 11, and provide all non-isomorphic (2)-pancyclic graphs of smallest order. We also give bounds on the vertex-degrees in such graphs, discuss on how many cycles a given edge may occur, and prove a lower bound for the order of non-Eulerian (2)-pancyclic graphs. Finally, we introduce (see also [5]) r-(2)-*pancyclic* graphs, which are graphs of order n featuring exactly two cycles of each length p with $r \leq p \leq n$, and construct an infinite family of such graphs for non-trivial r.

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Programme of the Conference

Sunday	
16:00 - 22:00	Registration
18:00 - 21:00	Dinner

Monday		
07:30 - 08:30	Breakfast	
08:45 - 09:35	Saito A.	Forbidden subgraphs generating a finite set
09:40 - 10:00	Škoviera M.	Nowhere-zero flows on bidirected eulerian graphs
10:05 - 10:25	Hexel E.	On vertices enforcing a hamiltonian cycle and cycle extendability
10:25 - 10:55	Coffee break	
10:55 - 11:15	Tuza Zs.	3-consecutive edge colorings of graphs
11:20 - 11:40	Bujtás Cs.	Improper C-colorings of graphs
11:45 - 12:05	Madaras T.	On minimal light sets of cycles in families of plane graphs
12:10 - 12:30	Lidický B.	Packing chromatic number for square and hexagonal lattices
12:30 - 13:00	Lunch	
14:45 - 15:35	Schiermeyer I.	Rainbow numbers and minimum rainbow subgraphs
15:40 - 16:00	Bača M.	Antimagicness of disconnected graphs
16:05 - 16:25	Bezegová Ľ.	A generalization of the concept of supermagic regular graphs
16:25 - 16:55	Coffee break	
16:55 - 17:15	Schwarz A.	Extending Bondy's theorem
17:20 - 17:40	Meierling D.	Extending directed cycles of in-tournaments
17:45 - 18:05	Chudá K.	$S(2,1)$-labeling of graphs with cyclic structure
18:10 - 18:30	Katrenič J.	Minimum k-path vertex cover
18:30 - 19:00	Dinner	
20:00 -	Welcome party	

Tuesday		
07:30 - 08:30	Breakfast	
08:45 - 09:35	Lindner C.	Extra perfect 2-fold cycle systems
09:40 - 10:00	Brandt S.	Graphs of odd girth 7 with large degree
10:05 - 10:25	Mihók P.	Acyclic colouring of graphs
10:25 - 10:55	Coffee break	
10:55 - 11:15	Kemnitz A.	$[1, 1, t]$-colorings of complete graphs
11:20 - 11:40	Rosa A.	Circulants as signatures of cyclic Steiner triple systems
11:45 - 12:05	Bielak H.	Ramsey numbers for a disjoint union of some graphs
12:10 - 12:30	Niepel Ľ.	Open efficient domination in digraphs
12:30 - 13:00	Lunch	
14:45 - 15:35	Raspaud A.	Acyclic choosability of planar graphs
15:40 - 16:00	Klešč M.	Crossings in products of cycles and paths with other graphs
16:05 - 16:25	Kravecová D.	Crossing numbers of some families of Cartesian products of graphs
16:25 - 16:55	Coffee break	
16:55 - 17:15	Bode J.-P.	Vertex rainbow numbers for cube graphs
17:20 - 17:40	Scheidweiler R.	Matchings in balanced hypergraphs
17:45 - 18:05	Volec J.	Independent sets in cubic graphs with large girth
18:10 - 18:30	Hudák D.	On properties of maximal 1-planar graphs
18:30 - 19:00	Dinner	
20:00 - 21:00	Videopresentation C&C 2009	

Wednesday	
07:30 - 08:30	Breakfast
08:30 - 16:00	Trip
19:00 - 20:00	Dinner

Thursday		
07:30 - 08:30	Breakfast	
08:45 - 09:35	West D. B.	Two theorems on long cycles
09:40 - 10:00	Voigt M.	Weights of induced subgraphs in $K_{1,r}$-free graphs
10:05 - 10:25	Jonck E.	Bounds on the broadcast chromatic number for cubic graphs
10:25 - 10:55	Coffee break	
10:55 - 11:15	Katona Gy. Y.	Hypergraph extensions of the Erdős-Gallai Theorem
11:20 - 11:40	Soták R.	Fractional and circular 1-defective colorings of outerplanar graphs
11:45 - 12:05	Máčajová E.	Cubic graphs with 1-factor are (7,2)-edge-choosable
12:10 - 12:30	Holub P.	Star subdivisions and connected even factors in the square of a graph
12:30 - 13:00	Lunch	
14:45 - 15:35	Miller M.	Degree/diameter problem: recent advances and open problems
15:40 - 16:00	Kardoš F.	Acyclic edge coloring of planar graphs
16:05 - 16:25	Zamfirescu C.	(2)-pancyclic graphs
16:25 - 16:55	Coffee break	
16:55 - 17:15	Cranston D. W.	List colorings of K_5-minor-free graphs with special list assignments
17:20 - 17:40	Škrabuľáková E.	Non-repetitive list edge-colourings of graphs
17:45 - 18:05	Karafová G.	Generalized fractional total coloring of complete graphs
18:10 - 18:30	Lukoťka R.	Small cubic graphs with large oddness
19:00 -	Farewell party	

Friday		
07:30 - 08:30	Breakfast	
08:45 - 09:35	Arumugam S.	The cycle spectrum of a graph
09:40 - 10:00	Kyppö J.	The cycles of extended knights
10:05 - 10:25	Phanalasy O.	Completely separating systems and antimagic labeling of non-regular graphs
10:25 - 10:55	Coffee break	
10:55 - 11:15	Jankó Zs.	College admissions and lattices
11:20 - 11:40	Kotrbčík M.	Vertex-disjoint cycles, fundamental cycles, and the maximum genus of a graph
11:45 - 12:05	Kupec M.	Extending fractional precolorings
12:10 - 12:30	Repiský M.	Cycles in digraphs and housing markets
12:30 - 13:00	Lunch	