C&C

HIGH TATRAS, SLOVAKIA

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Workshop

Cycles and Colourings 2011

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Dear Participant,

welcome to the Twentieth Workshop Cycles and Colourings. Except for the first workshop in the Slovak Paradise (Čingov 1992), the remaining eighteen workshops took place in the High Tatras (Nový Smokovec 1993, Stará Lesná 1994–2003, Tatranská Štrba 2004–2010). This year we decided to move to the hotel Atrium in Nový Smokovec. Among other things this will enable us to show to participants other interesting places of the High Tatras National Park.

The series of C&C workshops is organised by combinatorial groups of Košice and Ilmenau. Apart of dozens of excellent invited lectures and hundreds of contributed talks the scientific outcome of our meetings is represented also by special issues of journals Tatra Mountains Mathematical Publications and Discrete Mathematics (TMMP 1994, 1997, DM 1999, 2001, 2003, 2006, 2008).

To commemorate the anniversary of our event this year we arranged again to prepare a special issue of DM. For those who plan to submit a paper to the forthcoming SI we recall that submissions should be made using the Elsevier Editorial System (http://ees.elsevier.com/disc/, where the Special Issue CC 2011 is to be selected as the Article Type. The opening of the submission procedure is expected during the month of September and 30th November is defined as the deadline.

The scientific programme of the workshop consists of 50 minute lectures of invited speakers and of 20 minute contributed talks. This booklet contains abstracts as were sent to us by the authors.

Invited speakers:

Pavol Hell,	Simon Fraser University, Burnaby, Canada
Matthias Kriesell,	University of Southern Denmark, Odense, Denmark
Hao Li,	University Paris-Sud, Orsay, France
Jaroslav Nešetřil,	Charles University, Prague, Czech Republic
Katsuhiro Ota,	Keio University, Yokohama, Japan
Zdzisław Skupień,	AGH University of Science and Technology, Cracow, Poland
Zsolt Tuza,	Hungarian Academy of Sciences, Budapest, Hungary

Have a pleasant and successfull stay in Nový Smokovec.

Organising Committee:

Igor Fabrici	Stanislav Jendrol'
Jochen Harant	František Kardoš
Erhard Hexel	Štefan Schrötter
Mirko Horňák	Roman Soták

Contents

Some sufficient conditions of a graph to be in C_f2

Adiwijaya
(joint work with A.N.M. Salman, Oriol Serra,
Djoko Suprijanto, and Edy Tri Baskoro)

An f-coloring of graph $G(V, E)$ is a generalized edge-coloring such that every vertex v in V has at most $f(v)$ edges colored with a same color. The minimum number of colors needed to f-color G is called an f-chromatic index of G, denoted by $\chi'_f(G)$. Any graph G has f-chromatic index equal to $\Delta_f(G)$ or $\Delta_f(G) + 1$, where $\Delta_f(G) = \max_{v \in V}\{\lceil d(v)/f(v) \rceil\}$. If $\chi'_f(G) = \Delta_f(G)$, then G is of C_f1; otherwise G is of C_f2. A problem in the f-coloring is how to determine $\chi'_f(G)$ of a given graph G. It arises in many applications, including the network design problem, the scheduling problem, and the file transfer problem in a computer network. In 2006, Zhang and Liu gave the classification of complete graphs based on f-colorings. Moreover, Zhang, Wang, and Liu (2008) gave the suficient condition for a regular graph to be in C_f2. In this paper, we give some suficient conditions for a graph to be in C_f2. One of the results is a generalization of a theorem by Zhang et al. by dropping the condition of regularity of a graph. Moreover, we show that, when f is constant and a divisor of $(n-1)$, a maximal subgraph of the complete graph K_n which is in class C_f1 has precisely $\binom{n}{2} - \Delta_f(K_n)/2$ edges.

On super edge magic and edge antimagic labeling of $C_n(n-2)$ and C_n^{n-3} graphs

Gohar Ali

For a graph $G = (V, E)$, a bijection

$$f : V(G) \cup E(G) \to \{1, 2, \ldots, |V(G)| + |E(G)|\}$$

is called (a, d)-EAT labeling of G if the edge-weights

$$w(xy) = f(x) + f(y) + f(xy),\ xy \in E(G),$$

form an arithmetic progression starting from a and having common difference d. An (a, d)-EAT labeling is called super (a, d)-EAT labeling if $f(V) = \{1, 2, \ldots, |V(G)|\}$.

We study super (a, d)-EAT labeling of two types of graphs, namely $C_n(n-2)$ and C_n^{n-3}, i.e. odd cycles with $n-2$ pendent edges and $n-3$ chords respectively.

Progress in mixed hypergraph coloring for finite planes

Gábor Bacsó
(joint work with Tamás Héger, Tamás Szőnyi, and Zsolt Tuza)

For a finite projective plane Π, let $\bar{\chi}(\Pi)$ denote the maximum number of classes in a partition of the point set, such that each line has at least two points in the same partition class.

In [1], for general planes, not only for Galois planes, it was proved that the best possible estimate in terms of the orders of projective planes is $q^2 - q - \Theta(\sqrt{q})$, which is tight apart from a multiplicative constant in the third term $\sqrt{q}$:

- For q large enough, $\bar{\chi}(\Pi) \leq q^2 - q - \sqrt{q}/2 + o(\sqrt{q})$ holds for every projective plane Π of order q, not only for Galois planes.

Our results asymptotically solved a ten-year-old open problem in the coloring theory of mixed hypergraphs, where $\bar{\chi}(\Pi)$ is termed the upper chromatic number of Π.

Recently some stronger estimations have been obtained on $\bar{\chi}$ for Galois planes. In some cases, even sharp estimations are at hand.

References

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A construction of balanced degree-magic graphs

Ľudmila Bezegová

A graph is called supermagic if it admits a labelling of the edges by pairwise different consecutive positive integers such that the sum of the labels of the edges incident with a vertex is independent of the particular vertex. A graph is called degree-magic if it admits a labelling of the edges by integers $1, 2, \ldots, |E(G)|$ such that the sum of the labels of the edges incident with any vertex v is equal to $\frac{1}{2}(|E(G)| + 1)\deg(v)$. A degree-magic labelling of a graph generalizes the supermagic labelling of regular graphs. We used a degree-magic labelling for construction of supermagic graphs. Construction of some balanced degree-magic complements of bipartite graphs are presented.

Ramsey numbers for graphs versus disjoint union of some small graphs

Halina Bielak

The graph H is G-good if the Ramsey number for the pair of graphs G and H is expressed as follows: $R(G,H) = (\chi(G)-1)(|V(H)|-1)+s(G)$, where $\chi(G)$ is the chromatic number of G and $s(G)$ is the minimum cardinality of colour classes over all chromatic colourings of $V(G)$.

We give the Ramsey number for a disjoint union of some G-good graphs versus a graph with components isomorphic to G generalizing the results of Stahl [5], Lin [4], and the previous results of the author [1, 2]. Moreover we extend some result of Chvátal and Harary [3].

References

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Minimum size of rainbow k-connected graphs of given order

Jens-P. Bode

(joint work with Heiko Harborth)

A graph G is called rainbow k-connected if there exists a coloring of the edges of G with at most k colors such that any two vertices are connected by a path with edges of pairwise different colors. It is asked for the minimum number $t(n,k)$ of edges of a rainbow k-connected graph with n vertices. New upper bounds of $t(n,k)$ are given and $t(n,3)$ is determined completely.

Local ranking of graphs

Mieczysław Borowiecki

Let $G = (V, E)$ be a graph. A *vertex ranking* (*ranking* for short) of G is a vertex colouring by a linear ordered set of colours such that for every path in the graph with end-vertices of the same colour there is a vertex on this path with a higher colour. If this set of colours is cardinality k, then a vertex ranking is a *k-ranking*. The *ranking number* $\chi_r(G)$ of a graph G is defined to be the smallest integer k for which the graph G admits a k-ranking. Note that adjacent vertices have different colours in any k-ranking, thus any k-ranking is a proper k-colouring. It implies that $\chi_r(G)$ is bounded below by the chromatic number $\chi(G)$.

A proper k-colouring c of G is called a *local k-ranking* of G if for every vertex $v \in V$ the colouring c restricted to the subgraph $G[N[v]]$ is a ranking. The local ranking number of G, denoted by $\chi_{lr}(G)$, is the smallest value k for which the graph G has a local k-ranking. Since each ranking is a local ranking, then it is clear that $\chi(G) \leq \chi_{lr}(G) \leq \chi_r(G)$.

In a talk some properties of a local k-ranking, exact values and bounds of $\chi_{lr}(G)$ and relations to some other parameters of graphs will be given.

Induced cycles in triangle graphs

Csilla Bujtás

(joint work with S. Aparna Lakshmanan and Zsolt Tuza)

The triangle graph $\mathcal{T}(G)$ of a graph G is the graph whose vertices represent the triangles of G, and two vertices of $\mathcal{T}(G)$ are adjacent if the corresponding two triangles share an edge in G. This quite natural notion was introduced first more than 20 years ago by the third author [6] and studied in some papers (see, e.g., [1, 2, 3]), but the characterization of triangle graphs is still an open problem.

In the talk, we identify graphs whose triangle graphs are cycles. Moreover, we solve the similar but different problem of characterizing graphs whose triangle graphs contain an induced C_n for some $n \geq 4$. As a consequence, graphs G with chordal $\mathcal{T}(G)$, and also with perfect $\mathcal{T}(G)$ will be characterized.

These results yield a further graph class on which the longstanding conjecture of the third author [4, 5] on packings and coverings of triangles of a graph is true.

References

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Circumferences of 2-factors in claw-free graphs

Shuya Chiba

(joint work with Roman Čada)

We consider only finite graphs without loops. Let $\delta(G)$ be the minimum degree of a graph G. For a graph G, we denote by $c(G)$ the length of a longest cycle of G. We call $c(G)$ the *circumference* of G. A graph G is said to be *claw-free* if G has no induced subgraph isomorphic to $K_{1,3}$. A 2-*factor* is a spanning subgraph in which every component is a cycle.

It is an well-known conjecture that every 4-connected claw-free graph is Hamiltonian [3]. Since we can regard a graph with large circumference as "close" to Hamiltonian, there are many results concerning the circumferences of claw-free graphs. Matthews and Sumner [4] showed that if G is a 2-connected claw-free graph of order n, then $c(G) \geq \min\{2\delta(G) + 4, n\}$.

On the other hand, since a Hamilton cycle is a connected 2-factor, there are many results on 2-factors of claw-free graphs. For instance, it is known that a moderate minimum degree condition already guarantees the existence of a 2-factor in claw-free graphs. Choudum and Paulraj [1] and Egawa and Ota [2] showed that every claw-free graph G with $\delta(G) \geq 4$ has a 2-factor, and Yoshimoto [5] showed that every 2-connected claw-free graph G with $\delta(G) \geq 3$ has a 2-factor.

Our motivation is to consider the lower bound of circumferences of 2-factors in claw-free graphs, and we show that if G is a 2-connected claw-free graph of order n with $\delta(G) \geq 7$, then G has a 2-factor F such that $c(F) \geq \min\{2\delta(G)+4, n\}$.

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The complexity of risk-free marriage

Konrad K. Dąbrowski

(joint work with Marc Demange and Vadim V. Lozin)

Many graph-theoretic problems are difficult to solve in general. However, this is not the end of the story. Sometimes we only want to solve the problem on certain types of graphs. This gives us some extra knowledge about the structure of the graph, which we can use to our advantage. We introduce a parameter, which encodes some of this structure. If we are lucky, we find that this parameter somehow restricts all the "non-polynomial behaviour" of the problem. In this case we say that the problem is fixed-parameter tractable.

I will give some examples where this approach works, using some Ramsey-theoretic results. In particular, I will talk about the risk-free marriage problem (also known as the induced matching problem). This is the problem of finding an induced subgraph H in a graph G, such that all the vertices in H have degree exactly 1 and H is as large as possible. The talk will focus on graph-theoretic proofs and will be very light on algorithmic details.

On the crossing numbers of Cartesian products

Emília Draženská

The *crossing number*, $cr(G)$, of a graph G is the minimum number of pairwise intersections of edges in a drawing of G in the plane. Computing the crossing number of a given graph is, in general, an elusive problem. There are known several exact results on the crossing numbers of the Cartesian product of small graphs with paths, cycles and stars.

In the talk we extend these results. We give lower or upper bounds or the exact values of the crossing numbers of Cartesian products of another specific graphs.

5-choosability of near-planar graphs

Zdeněk Dvořák

(joint work with Bernard Lidický, Bojan Mohar, and Luke Postle)

We give some results on coloring graphs from lists of size 5. In particular, we show that if G can be drawn in plane so that the distance between every pair of crossings is at least 30, then G can be properly colored from any such lists. Furthermore, we give some partial results for other kinds of irregularities, e.g., precolored vertices.

$\{1, 2, 3\}$-weight colorability of claw-free graphs

Hossein Esfandiari

(joint work with Pouria Salehi Nowbandegani)

An edge-weighting vertex coloring of a graph is an edge-weight assignment such that the accumulated weights as the vertices yields a proper vertex coloring. If such an assignment from a set S exists, we say that graph is S-weight colorable. It is conjecture that every graph with no isolated edge is $\{1, 2, 3\}$-weight colorable [1]. Karoński et al. [2] proved this conjecture for 3-colorable graphs.

Here we proved the conjecture for 4-regular, claw-free, and C_4-free graphs.

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Acyclic chromatic indices of graphs

Anna Fiedorowicz

(joint work with Mariusz Hałuszczak)

An acyclic edge k-colouring of a graph G is defined as a proper edge k-colouring of G such that for every pair of distinct colours i and j, the subgraph induced in G by all the edges which have either colour i or j is acyclic. The minimum number k of colours such that G has an acyclic edge k-colouring is called an acyclic chromatic index of G.

There is a conjecture, stated by Fiamčík in 1978 and later restated by Alon, Sudakov, and Zaks in 2001, which says that for any graph G, its acyclic chromatic index does not exceed $\Delta(G) + 2$. This conjecture has been verified by now only for some special classes of graphs.

In this talk, we show that if G is a plane graph such that for $i \in \{3, 4\}$, no two i-faces of G touch each other, then G has an acyclic edge colouring with at most $\Delta(G) + 2$ colours. We also determine the acyclic chromatic index of the class of fully subdivided graphs.

Week odd 3-colorings of planar graphs

Frank Göring

(joint work with Igor Fabrici)

We prove that every nonempty simple plane graph has a coloring of its vertex set with colors red, blue, and black such that each face is incident with at most one red vertex and each face not incident with a red vertex is incident with exactly one blue vertex. This solves a problem posed by Stano Jendroľ in Elgersburg this year.

On Ramsey $(K_{1,m}, \mathcal{G})$-minimal graphs

Mariusz Hałuszczak

(joint work with Marta Borowiecka-Olszewska)

Let F be a graph and let $\mathcal{G}$,$\mathcal{H}$ denote nonempty families of graphs. We write $F \to (\mathcal{G}, \mathcal{H})$ if in any 2-coloring of the edges of F, with red and blue, there is a red subgraph isomorphic to some graph from $\mathcal{G}$ or a blue subgraph isomorphic to some graph from $\mathcal{H}$. The graph F is said to be a $(\mathcal{G}, \mathcal{H})$-minimal graph if $F \to (\mathcal{G}, \mathcal{H})$ and $F - e \not\to (\mathcal{G}, \mathcal{H})$ for $e \in E(F)$. The set of all $(\mathcal{G}, \mathcal{H})$-minimal graphs (up to isomorphism) is called the Ramsey set $\Re(\mathcal{G}, \mathcal{H})$.

We present a procedure, which on the basis of the set of some special graphs, generates an infinite family of $(K_{1,m}, \mathcal{G})$-minimal graphs, where $m \geq 2$ and $\mathcal{G}$ is a family of 2-connected graphs. Moreover, graphs obtained by this procedure can be obtained in linear time with respect to theirs order. In particular, we present how to obtain infinite Ramsey sets $\Re(K_{1,m}, K_n)$, $\Re(K_{1,m}, K_{p,q})$, for every $m, p, q \geq 2$ and $n \geq 3$, and the Ramsey set $\Re(K_{1,m}, C_n)$, for $m \geq 2$ and $n \in [4, 6]$. We show minimal graphs with respect to the number of vertices, which belong to the family $\Re(K_{1,m}, K_n)$, for $m, n \geq 3$. We also present graphs which can be used to the construction of infinitely many graphs belonging to the Ramsey set $\Re(K_{1,m}, \mathcal{G})$, where $m \geq 2$ and $\mathcal{G}$ is some family of 2-connected graphs consisting of more than one graph.

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Variants of interval graphs

Pavol Hell

(joint work with Arash Rafiey, Tomás Feder, and Jing Huang)

I will discuss recent results classifying the complexity of list homomorphism and of minimum cost homomorphism problems, by certain combinatorial conditions. These conditions in turn suggest interesting classes of graphs and especially of digraphs, analogous to well studied graph classes such as interval and unit interval graphs.

The packing chromatic number of distance graphs

Přemysl Holub

(joint work with Jan Ekstein, Bernard Lidický, and Olivier Togni)

The packing chromatic number $\chi_\rho(G)$ of a graph G is the smallest integer k such that vertices of G can be partitioned into disjoint classes $X_1, \ldots, X_k$ where vertices in X_i have pairwise distance greater than i. We study the packing chromatic number of infinite distance graphs $G(\mathbb{Z}, D)$, i.e. graphs with the set $\mathbb{Z}$ of integers as vertex set and in which two distinct vertices $i, j \in \mathbb{Z}$ are adjacent if and only if $|i - j| \in D$.

In this paper we focus on distance graphs with $D = \{s, t\}$, $s, t \in \mathbb{N}$, $s < t$. For sufficiently large s and t we show that $\chi_\rho(G(\mathbb{Z}, D)) \leq 35$ when s and t are both odd t and $\chi_\rho(G(\mathbb{Z}, D)) \leq 56$ otherwise. For $s = 1$ we also give a lower bound 12 for $t \geq 9$ and tighten several gaps for $\chi_\rho(G(\mathbb{Z}, D))$ with small t.

1-planarity of complete multipartite graphs

Dávid Hudák

(joint work with Július Czap)

A graph is called *1-planar* if it can be drawn in the plane so that each edge is crossed by at most one other edge. The family of 1-planar graphs shows some fundamental differences when compared to planar graphs. 1-planar graphs are not preserved under edge contractions, hence, 1-planar graphs are not minor closed. Furthermore, Korzhik and Mohar in [1] showed that, for large enough n, there are exponentially many non-isomorphic minimal non-1-planar graphs on n vertices; this is in sharp contrast with the planar case. All these results indicate that probably there exists no characterization of 1-planar graphs by Kuratowski type theorem using a finite number of forbidden topological minors.

From these observations, it follows that to prove that a certain graph is not 1-planar, one may try to find a small non-1-planar subgraph. In this talk we give the full characterization of (non-) 1-planar complete multipartite graphs, which help us to determine 1-planarity of graphs.

References

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On selfcentric graphs

Jana Hurajová

(joint work with Silvia Gago and Tomáš Madaras)

The centrality index of vertices of a graph is isomorphism invariant real-valued function that measures the importance of the selected vertex within the whole graph. The most frequently used centrality indices are vertex degree, eccentricity, the sum of all distances from a vertex, and the betweenness centrality, which is defined in [1] as the relative number of shortest paths between all pairs of vertices passing through given vertex.

We study the selfcentric graphs (that is, the graphs whose vertices have the same centrality) focusing on betweenness-selfcentric graphs, their properties and constructions.

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On computing the minimum 3-path vertex cover and dissociation number of graphs

František Kardoš

(joint work with Ján Katrenič and Ingo Schiermeyer)

The dissociation number of a graph G is the number of vertices in a maximum size induced subgraph of G with vertex degree at most 1. A k-path vertex cover of a graph G is a subset S of vertices of G such that every path of order k in G contains at least one vertex from S. The minimum 3-path vertex cover is a dual problem to the dissociation number. For this problem we present an exact algorithm with a running time of $O^*(1.5171^n)$ on a graph with n vertices. We also provide a polynomial time randomized approximation algorithm with an expected approximation ratio of $\frac{23}{11}$ for the minimum 3-path vertex cover.

Grundy numbers of strong products of graphs

Arnfried Kemnitz

(joint work with Jens-P. Bode and Christoph Hillert)

A proper k-coloring c of a graph G is called Grundy k-coloring if for each vertex u and all colors i with $1 \leq i < c(u)$ there exists an adjacent vertex v with $c(v) = i$. The maximum number k of colors for which a Grundy k-coloring using all k colors exists is the Grundy number $\Gamma(G)$ of G.

The strong product of graphs $G = (V_G, E_G)$ and $H = (V_H, E_H)$ has as vertex set the Cartesian product $V_G \times V_H$ of the vertex sets of G and H. Two vertices (u, v) and $(\bar{u}, \bar{v})$ of $V_G \times V_H$ are adjacent if and only if $u = \bar{u}$ and $v\bar{v} \in E_H$ or $v = \bar{v}$ and $u\bar{u} \in E_G$ or $u\bar{u} \in E_G$ and $v\bar{v} \in E_H$.

We will give results on the Grundy number of strong products of paths and cycles.

On the crossing numbers of join products with stars

Marián Klešč

The *crossing number* $cr(G)$ of a graph G is the minimum possible number of edge crossings in a drawing of G in the plane. The investigation on the crossing numbers of graphs is a classical and however very difficult problem. The problem of reducing the number of crossings was therefore not only studied by the graph theory community, but also by VLSI communities and computer scientists.

It has been long-conjectured by Zarankiewicz [4] that the crossing number of the complete bipartite graph $K_{m,n}$ equals $\lfloor\frac{m-1}{2}\rfloor\lfloor\frac{m}{2}\rfloor\lfloor\frac{n-1}{2}\rfloor\lfloor\frac{n}{2}\rfloor$. This conjecture has been verified by Kleitman [1] for $\min\{m,n\} \leq 6$. Let G and H be two disjoint graphs. The *join product* of G and H, denoted by $G+H$, is obtained from vertex-disjoint copies of G and H by adding all possible edges between $V(G)$ and $V(H)$. For $|V(G)| = m$ and $|V(H)| = n$, the edge set of $G+H$ is the union of disjoint edge sets of the graphs G, H, and the complete bipartite graph $K_{m,n}$.

In [2] there are established crossing numbers for join of two paths, join of two cycles, and for join of path and cycle. In [3], the crossing numbers of join products of all graphs of order at most four with paths are collected. The same was done also for the cycles. We extend these results and we start to collect crossing numbers for join products of stars with other graphs.

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Triangular embeddings of K_n in non-orientable surfaces

Martin Knor

(joint work with Mike Grannell)

Establishing the existence of a minimum genus surface embedding of each complete graph K_n was a crucial step in Ringel and Youngs' solution of the famous Heawood map colouring problem for surfaces of positive genus. For some residue classes modulo 12 such embeddings necessarily have all their faces triangular. Until 1999 the maximum number of known nonisomorphic triangular embeddings of K_n was a mere three. Then Korzhik and Voss established a lower bound of the form 2^{an} for all sufficiently large n. At the same time Bonnington, Grannell, Griggs, and Širáň proved that, for linear class of n, there are at least 2^{an^2} such triangular embeddings for some constant $a > 0$. Finally, in 2008 Grannell and Griggs proved that there are at least n^{an^2} triangular embeddings of K_n for an infinite class of n. We remark that a trivial upper bound is $n^{n^2/3}$.

In this talk we describe a face two-colourable triangular embedding of $K_{m,m,m}$ in which triangles of one colour class form a Cayley table of dihedral group D_m, $m \equiv 2$ or $10 \pmod{12}$. Using this embedding we establish a lower bound n^{an^2} for triangular embeddings of K_n in a non-orientable surface for currently the best constant a and for linear class of n.

Enforced hamiltonian cycles in two classes of graphs

Mária Kopperová

A nonempty vertex set $X \subseteq V(G)$ of a hamiltonian graph G is called an *H-force* set of G if every *X-cycle* of G (i.e. a cycle of G containing all vertices of X) is hamiltonian. The *H-force* number $h(G)$ of a graph G is defined to be the smallest cardinality of an *H-force* set of G. We established exact values of this parameter for two classes of graphs, namely generalized dodecahedra and circulant graphs.

Points covered by many simplices

Daniel Kráľ

(joint work with Lukáš Mach and Jean-Sébastien Sereni)

Boros and Füredi (for $d = 2$) and Bárány (for arbitrary d) proved that there exists a constant $c_d > 0$ such that for every set P of n points in R^d in general position, there exists a point of R^d contained in at least $c_d\binom{n}{d+1}$ $(d+1)$-simplices with vertices at the points of P. For $d = 2$, the optimum value of c_d is $2/9$. For $d = 3$ the currently best lower bound is 0.0633 and the upper bound is 0.0938. Based on a topological method of Gromov and a related work of Matoušek and Wagner, we consider a reformulation of the problem in language of extremal graph theory and improve the lower bound to 0.0751.

On the crossing numbers of products of some special graphs

Daniela Kravecová

(joint work with Marián Klešč)

The *crossing number* $cr(G)$ of a simple graph G with $|V|$ vertices and $|E|$ edges is defined as the minimum number of crossings among all possible projections of G on the $\mathbb{R}^2$ plane. The investigation on the crossing number of graphs is a classical and however very difficult problem. The exact values of the crossing numbers are known only for some families of graphs. Some Cartesian products of special graphs are one of few graph classes for which the exact values of crossing numbers were obtained. There are known several exact results of the crossing numbers of the Cartesian product of a small graphs with paths, cycles and stars.

In the talk, we extend the earlier results by giving the crossing numbers for some new classes of graphs.

Disjoint cycles and dicycles in digraphs

Matthias Kriesell

(joint work with Jørgen Bang-Jensen, Alessandro Maddaloni, and Sven Simonsen)

We study the following problem: Given a digraph D, decide if there is a cycle B in D and a cycle C in its underlying undirected graph $UG(D)$ such that $V(B) \cap V(C) = \emptyset$. Whereas for both *unmixed* versions of the problem, i.e. deciding whether a graph or digraph, respectively, admits two disjoint cycles, there are polytime algorithms, this turned out to be NP-complete — which we think is contraintuitive.

Nevertheless, one can decide the existence of B, C in polynomial time under the additional assumption that D is strongly connected. Our methods actually find B, C in polynomial time if they exist. The behaviour of the problem as well as our solution depend on the *cycle transversal number* $\tau(D)$ of D, i.e. the smallest cardinality of a set T of vertices in D such that $D - T$ is acyclic: If $\tau(D) \geq 3$ then we employ McCuaig's framework on intercyclic digraphs to (always) find these cycles. If $\tau(D) = 2$ then we characterize the digraphs for which the answer is "yes" by using topological methods relying on Thomassen's theorem on 2-linkages in acyclic digraphs. For the case $\tau(D) \leq 1$ we provide an algorithm independent from any earlier work.

Heavily based on these results on strongly connected digraphs we were able to construct polytime algorithms for the general problem restricted to the case that $\tau(D) \neq 1$ and also to the case that $\tau(D) = 1$ and the number of cycle transversals is bounded by some constant.

Complexity of $\lambda - L(p, q)$-labelling

Martin Kupec

We study the complexity of the $\lambda - L(p, q)$-labelling problem for fixed λ, p, and q. The task is to assign vertices of a graph labels from the set $\{0, \ldots, \lambda\}$ such that labels of adjacent vertices differ by at least p while vertices with a common neighbour have different labels. We use two different reductions, one from the NAE-3SAT and the second one from the edge precoloring extension problem. Combination of those reductions results in the following characterization:

$p \geq 3q$	$\lambda \in [0, p+2q)$	polynomial
	$\lambda \in [p+2q, p+3q)$	polynomial
	$\lambda \in [p+3q, \infty)$	NP-C
$p \geq 2q, p < 3q$	$\lambda \in [0, p+2q)$	polynomial
	$\lambda \in [p+2q, \max\{2p, p+2q\})$	polynomial
	$\lambda \in [\max\{2p, p+2q\}, p+3q)$	NP-C
	$\lambda \in [p+3q, \infty)$	NP-C
$p \geq q, p < 2q$	$\lambda \in [0, p+2q)$	polynomial
	$\lambda \in [p+2q, p+3q)$	NP-C

Odd and even cycles in digraphs of small strategy games

Jorma Kyppö

The main focus of this presentation is the exploration of winning strategies of some small board games. The progress of the games is perfectly modeled by means of the digraphs containing both, the odd and even cycles. Thus the information about the game situation may be provided by the codes situated in nodes. Some of the subgraphs are enumerated by a unique positive integer. The method of enumeration is based on the adjacency matrix of a graph. The possibilities of generalizing the achieved results to the more complicated games have been explored.

Degrees and neighborhoods of vertices and Hamiltonicity of graphs

Hao Li

Dirac's theorem and Ore's theorem, that gives minimum degree condition and condition of minimum degree-sum of any pair of nonadjacent vertices for Hamiltonian graphs respectively, are basic results in Hamiltonian graph theory. In this talk, we will introduce some results that generalize Dirac's theorem and Ore's theorem. These results give sufficient conditions of Hamiltonian graphs by using degrees and neighborhoods of more independent vertices. Implicit degrees of vertex were defined by Zhu, Li, and Deng in 1989. We also talk about some new results on implicit degrees of vertices and Hamiltonicity.

New Ore's type results on hamiltonicity and existence of paths of given length in graphs

Nicolas Lichiardopol

The well-known Ore's theorem (see [2]), states that a graph G of order n such that $d(x)+d(y) \geq n$ for every pair $\{x, y\}$ of non-adjacent vertices of G, is Hamiltonian. In this paper, we considerably improve this theorem by proving that in a graph G of order n and of minimum degree $\delta \geq 2$, if there exist at least $n-\delta$ vertices x of G so that the number of the vertices y of G non-adjacent to x and satisfying $d(x)+d(y) \leq n-1$ is at most $\delta-1$, then G is Hamiltonian. We will see that our result is pertinent relatively to the so called "Extended Ore's theorem" (see [1]) and to the Pósa's Theorem (see [3]). We give also a new result of the same type, ensuring the existence of a path of given length.

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Possible values of the circular chromatic index of a snark

Robert Lukoťka

(joint work with Ján Mazák)

For a cubic graph the set of known attainable values of circular chromatic index is $\langle 3, 3+1/3\rangle \cup \{3+k/(3k-1) | k \in \mathbb{N}\} \cup \{3+2/3, 4\}$. First we discuss current progress with generalizations of results for cubic graphs to graphs of maximal degree more than three. Later we concentrate us on cubic graphs of higher connectivity and girth. We show that it is a problem to avoid 2-edge cuts in current known construction methods. However we show that there exist a 4-edge-connected graph with girth at least 5 with circular chromatic index c for each $c \in \langle 3, 3+2/9\rangle$. We also obtain several classes of graphs with circular chromatic indices in $(3+2/9, 3+1/3)$.

On doubly light triangles in plane graphs

Tomáš Madaras
(joint work with Peter Hudák)

In [1], Borodin proved that each normal plane map of minimum degree 5 contains a triangular face such that the sum of degrees of its vertices (its weight) is at most 17 (the bound 17 being best possible). We strengthen this result by showing that, in each plane graph of minimum degree 5, there exists a triangular face of weight at most 17 which is incident to three faces with the sum of their sizes at most 13; this bound is sharp. Furthermore, we show that, under the additional requirement of minimum edge weight 11, there exists a triangular face of weight 17 surrounded by faces whose sum of sizes does not exceed 34, the bound 34 again being sharp. Similar results concerning other families of plane graphs are discussed as well.

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Vertex-distinguishing edge colorings of circulant graphs

Martina Mockovčiaková
(joint work with Roman Soták)

Let $\varphi : E \to \{1, 2, \ldots, k\}$ be a proper edge coloring of a graph $G = (V, E)$. The set of colors of edges incident to vertex $v \in V$ is called the *color set* of vertex v and denoted by $S(v)$. The coloring φ is *vertex-distinguishing coloring* if $S(u) \neq S(v)$ for any two distinct vertices $u, v \in V$. The *vertex-distinguishing chromatic index* or *strong chromatic index* of a graph G denoted by $\chi'_s(G)$ is the minimum number of colors in such coloring of G.

A *d-strong edge coloring* of graph G is a proper edge coloring that distinguishes any two distinct vertices u and v with distance $d(u, v) \leq d$. The minimum number of colors of d-strong edge coloring of graph G is called *d-strong chromatic index of* G and denoted by $\chi'_d(G)$.

We present some general results on vertex-distinguishing edge coloring of circulant graphs, we determine exact values of d-strong chromatic index for circulant graphs $C_n(1, 2)$ for $d = 1$ and 2 and we also prove that the difference between lower bound for d-strong chromatic index and value of this index can be arbitrarily large.

Six-decompositions of snarks

Roman Nedela
(joint work with Ján Karabáš and Edita Máčajová)

A snark is a cubic graph with no 3-edge-colouring. In 1996, Nedela and Škoviera proved the following theorem: Let G be a snark with an n-edge-cut, $n \geq 2$, whose removal leaves two 3-edge-colorable components M and N. Then both M and N can be completed to two snarks $\tilde{M}$ and $\tilde{N}$ of order not exceeding that of G by adding at most $\Phi(n)$ vertices, where the number $\Phi(n)$ only depends on n. The known values of the function $\Phi(n)$ are $\Phi(2) = 0$, $\Phi(3) = 1$, $\Phi(4) = 2$ (Goldberg, 1981), and $\Phi(5) = 5$ (Cameron, Chetwynd, Watkins, 1987). The value $\Phi(6)$ is not known and is apparently difficult to calculate. Our paper is aimed attacking the problem of determining $\Phi(6)$ by investigating the structure and colour properties of potential complements in 6-decompositions of snarks. In 1979, Jaeger conjectured that there are no 7-cyclically-connected snarks. Hence $\Phi(6)$ is the last important value to determine.

Existence & counting

Jaroslav Nešetřil
(joint work with Patrice Ossona de Mendez)

We show how for sparse classes of graphs (such as Nowhere dense classes) one can define asymptotic degree of freedom. This integral parameter in fact characterizes Nowhere dense classes by counting of subgraphs.

Locating and identifying codes in circulant networks

Ľudovít Niepel
(joint work with Mohammad Ghebleh)

A set D of vertices in a graph $G = (V, E)$ is a locating-dominating set (LDS) if for every two vertices u, v of $V - D$ the sets $N(u) \cap D$ and $N(v) \cap D$ are non-empty and different. The locating-domination number $\gamma_l(G)$ is the minimum cardinality of a LDS of G. A set D' of vertices in a graph $G = (V, E)$ is an identifying-dominating set (IDS) if for every two vertices u, v of V the sets $N(u) \cap D'$ and $N(v) \cap D'$ are non-empty and different. The identifying-domination number $\gamma_i(G)$ is the minimum cardinality of an IDS of G. These sets of vertices are used to locate or detect faulty nodes in a computer network. We study locating and identifying dominating sets in circulant graphs. We establish lower and upper bounds on both locating-domination and identifying-domination numbers in the circulant graphs $C_n(1, d)$.

On vertex stability of complete k-partite graphs

Mateusz Nikodem
(joint work with Sylwia Cichacz, Agnieszka Görlich, and Andrzej Żak)

We say that graph G is H-*vertex-stable* (shortly H-stable) if after removing any of its vertex, it still contains H as a subgraph. For given H we are interested in finding H-stable graph G of minimal size. The size of such a graph G is denoted by $\mathrm{stab}(H)$. For each graph H the following is satisfied,

$$||H|| + \Delta_H \leq \mathrm{stab}(H) \leq ||H|| + |H|. \qquad (*)$$

Dudek and Żak considered the problem for complete bipartite graphs [1] and characterized all $K_{m,n}$-stable graphs of minimal size. In particular, if $H = K_{m,n}$ then, dependently on m and n, the value of $\mathrm{stab}(H)$ achieves exactly one of the lower or the upper bound of $(*)$, i.e. $\mathrm{stab}(H) \in \{||H|| + \Delta_H, ||H|| + |H|\}$ (no in-between values).

In this talk we will consider the generalization of problem to complete k-partite graphs H (with $k \geq 2$). In this case the property that $\mathrm{stab}(H) \in \{||H|| + \Delta_H, ||H|| + |H|\}$ is still satisfied.

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The empire problem in even embeddings on closed surfaces

Kenta Noguchi

Let M be a map on a closed surface F^2 and suppose that each country of the map have at most r disjoint connected regions. Such a map is called an r-pire map on F^2. We call a graph on F^2 an even embedding if it has no faces of boundary length odd. We consider the r-pire maps whose underlying graphs are even embedding on F^2 and prove that it can be properly colored with $n_{\varepsilon,r} = \lfloor (4r + 1 + \sqrt{(4r + 1)^2 - 16\varepsilon})/2 \rfloor$ colors. Moreover, we conjecture that this is best possible except for the cases $(\varepsilon, r) = (2, 1), (0, 1), (-2, 1)$. We prove it for the cases

(1) $\varepsilon = 2, 1, 0$ with $r \geq 2$,
(2) F^2 is orientable, r is even and $n_{\varepsilon,r} \equiv 1 \pmod 8$,
(3) F^2 is orientable, r is odd and $n_{\varepsilon,r} \equiv 5 \pmod 8$,
(4) F^2 is nonorientable and $n_{\varepsilon,r} \equiv 1 \pmod 4$.

Minimum degree condition for forests

Katsuhiro Ota

It is an easy exercise to show that a graph with minimum degree at least k contains every tree on $k+1$ vertices as a subgraph. We consider a natural generalization of this result. For a given forest H, what is the minimum value of k (depending on H) such that any graph with minimum degree at least k contains H as a subgraph? Formally, we consider the following function:

$$\delta(n, H) \;=\; \min\{k \,:\, |V(G)| = n \text{ and } \delta(G) \geq k \text{ imply } H \subset G\}.$$

The exercise shows that if H is a tree, then $\delta(n, H) \leq |V(H)| - 1$, and it is easy to see that for each tree H, the equality holds for infinitely many values of n.

From the result on trees, it is easy to see that $\delta(n, H) \leq |V(H)| - 1$ for any forest H. Brandt (1994) improved this observation by showing that $\delta(n, H) \leq |E(H)|$. However, this bound is not tight in general. For example, if $H = K_{1,k} \cup K_{1,k-1} \cup \cdots \cup K_{1,1}$, then we can show that $\delta(n, H) \leq k$ for large n, while $|E(H)| = k(k+1)/2$. In this talk, we shall pose a conjecture which will be sharp for each forest H, and verify it for several special classes of forests.

Recent results on the hamiltonicity of graphs on surfaces

Kenta Ozeki
(joint work with Ken-ichi Kawarabayashi)

In this talk, we will show some results on the hamiltonicity of graphs on surfaces. Tutte [6] proved that every 4-connected plane graph has a hamiltonian cycle. Extending Tutte's technique, Thomassen [5] proved that every 4-connected plane graph is in fact hamiltonian-connected, i.e., there is a hamiltonian path connecting any two prescribed vertices.

Beginning with these results, many researchers have considered the hamiltonicity of graphs on non-spherical surfaces. With some additional techniques to the above results and new ideas, Thomas and Yu [4] proved that every edge in a 4-connected projective-planar graph is contained in a hamiltonian cycle. In this talk, we show the following result, which give a positive answer to a conjecture by Dean [1]. Note that the following theorem extends the result due to Thomas and Yu.

Theorem. Every 4-connected graph embedded on the projective plane is hamiltonian-connected.

For graphs on the torus, we have a famous conjecture by Grünbaum [2] and Nash-Williams [3]; every 4-connected graph on the torus has a hamiltonian cycle. In this talk, we will also mention recent results around this conjecture.

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Online Ramsey games on planar graphs

Šárka Petříčková

An online Ramsey game is a game between Builder and Painter, alternating in turns. In each turn, Builder draws an edge, and Painter colors it blue or red. The goal of Builder is to force Painter to create a monochromatic copy of a fixed graph, while Painter tries to avoid it. The only limitation for Builder is that after each of his moves, the resulting graph has to belong to some predefined class of graphs. It was conjectured by Grytczuk et al. [1] that playing on the class of planar graphs, Builder can force a graph G if and only if G is outerplanar. Here we show that the left-to-right implication does not hold while the right-to-left implication does.

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Join products with crossing number one

Jana Petrillová

Let G_1 and G_2 be simple graphs with vertex sets $V(G_1)$ and $V(G_2)$, and edge sets $E(G_1)$ and $E(G_2)$. The join product of two graphs G_1 and G_2, denoted by $G_1 + G_2$, is obtained from vertex-disjoint copies of G_1 and G_2 by adding all edges between $V(G_1)$ and $V(G_2)$. For $|V(G_1)| = m$ and $|V(G_2)| = n$, the edge set of $G_1 + G_2$ is the union of disjoint edge sets of the graphs G_1, G_2, and the complete bipartite graph $K_{m,n}$.

Kulli at al. started to study line graphs with crossing number one. Later Kulli and Muddebihal gave the characterization for all pairs of graphs for which the crossing number of their join product is zero. In this talk, we give the necessary and sufficient conditions for all pairs of graphs G_1 and G_2 for which the crossing number of their join product $G_1 + G_2$ is one.

On magic joins of graphs

Tatiana Polláková

A graph is called magic (supermagic) if its admits a labeling of the edges by pairwise different (and consecutive) integers such that the sum of the labels of the edges incident with a vertex is independent of the particular vertex. We will deal with magic joins of graphs and we will establish some conditions for magic joins of graphs to be supermagic.

Covering signed graphs with cycles

Edita Rollová

(joint work with Edita Máčajová, André Raspaud, and Martin Škoviera)

Cycle covers of graphs have been extensively studied for more than 20 years because of their close relationship to several areas of graph theory. In this talk we study cycle covers in signed graphs, graphs, where each edge has a sign – either $+1$ or -1. For this purpose the concept of a cycle cover has to be appropriately modified in order to reflect the structure of a signed graph. We cover signed graphs with signed circuits, which are of two kinds: balanced circuits (circuits with even number of negative edges) and bicircuits (two unbalanced circuits joined with a path). These correspond to circuits of the signed graphical matroid. Our main result states that every flow-admissible signed graph G has a cycle cover with the total length at most $(9 + 1/6)|E(G)|$.

Triple metamorphosis of twofold triple systems

Alexander Rosa
(joint work with Curt Lindner and Mariusz Meszka)

The concept of a metamorphosis of block designs, due to Lindner, has been dealt with in many papers. Typically, for a subgraph G' of G, each block of a G-design of order n and index λ is modified by deleting the edges of $G \setminus G'$, and then reassembling the totality of deleted edges into G'-blocks, so as to form, together with the modified block of the original G-design, a new G'-design of order n and index λ'. One such instance is the metamorphosis of a simple twofold triple system of order n, $\mathrm{TS}(n,2)$, into a twofold 4-cycle system of order n, $4\mathrm{C}(n,2)$. The spectrum for $\mathrm{TS}(n,2)$ having a metamorphosis into $4\mathrm{C}(n,2)$ has previously been shown to be the set $n \equiv 0,1,4$ or 9 $(mod\ 12)$, $n \geq 9$. Here we extend the concept of a metamorphosis to that of a triple metamorphosis of a $\mathrm{TS}(n,2)$ into a $4\mathrm{C}(n,2)$. This consists of three distinct metamorphoses satisfying an additional condition. We show that the necessary conditions for the existence of a triple metamorphosis of a $\mathrm{TS}(n,2)$ into a $4\mathrm{C}(n,2)$, namely $n \equiv 0,1,4$, or 9 $(mod\ 12)$, are also sufficient, with one exception ($n = 9$) and one possible exception ($n = 12$).

On the vertex suppression in 3-connected graphs

Ondřej Rucký
(joint work with Jernej Azarija, Tomáš Kaiser, Matjaž Krnc,
Šárka Petříčková, and Riste Škrekovski)

Kriesell [1] defines the operation of *suppressing* a vertex v in a graph G as joining each pair of nonadjacent neighbors of v by an edge and subsequently deleting v. He uses $G -- v$ to denote the graph obtained from $G - v$ by suppressing vertices of degree at most 2 as long as possible (it is shown to be well defined). The main result of [1] is that if G is 3-connected, it has a vertex x such that $G -- x$ is 3-connected unless G is a $K_{3,3}$, a $K_2 \times K_3$, or a wheel $K_1 * C_\ell$ for some $\ell \geq 3$. Further, the following two conjectures extending this result are stated in the paper: every 3-connected graph G not isomorphic to $K_{3,3}$, $K_2 \times K_3$, nor $K_1 * C_\ell$ for any $\ell \geq 3$ has

(1) a vertex x such that $G -- x$ is 3-connected and $|V(G -- x)| \geq |V(G)|/2 + 1$,

(2) three distinct vertices x_1, x_2, and x_3 such that each of $G -- x_i$ is 3-connected.

Apart from certain trivial counterexamples, we confirm the second conjecture using the *Y-Δ transformation* and results of Veldman [2]. We believe that a slightly modified version of the first conjecture is also true, but there are still details to be settled in order to obtain a complete proof.

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Closures for strong hamiltonian properties

Zdeněk Ryjáček
(joint work with Petr Vrána)

The closure for hamiltonicity introduced in [1] is known to turn a claw-free graph into the line graph of a triangle-free graph while preserving many Hamilton-type graph properties; however, it turns out that many properties stronger than hamiltonicity (such as Hamilton-connectedness or 1-Hamilton-connectedness) are not preserved.

In the talk we show recently introduced closure concepts that preserve the above mentioned stronger properties and still turn a claw-free graph into a line graph (but possibly of a multigraph which can contain triangles).

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Gap k-colorings of trees and cycles

Robert Scheidweiler
(joint work with Eberhard Triesch)

Let $G = (V, E)$ be a graph, $k \in \mathbb{N}$, and $f : E \to \{1, \dots, k\}$ a labelling of its edges. We define

$$l : V \to \mathbb{N},\ l(v) = \begin{cases} f(e), & \text{if } \deg_G(v) = 1 \text{ and } v \in e \\ \max\limits_{e \ni v}\{f(e)\} - \min\limits_{e \ni v}\{f(e)\}, & \text{otherwise.} \end{cases}$$

If $l(v) \neq l(w)$ for distinct vertices $v, w \in V$, we call f gap vertex distinguishing or a gap-k-coloring, a notion defined in [1]. We investigate the gap chromatic number $\text{gap}(G)$ of G, i.e., the minimum number k, for which G has a gap-k-coloring.

At the first step, we study the gap chromatic number of trees. After that we analyze the gap chromatic number of graphs, which consist of disjoint cycles. Combining these investigations with the results of [1], we are able to give upper bounds for the gap chromatic number of several classes of graphs with 2-edge-connected components.

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Greedy is good to approximate minimum rainbow subgraphs

Ingo Schiermeyer

(joint work with Stephan Matos Camacho)

We consider the Minimum Rainbow Subgraph problem (MRS): Given a graph G of order n, whose edges are coloured with p colours. Find a subgraph $F \subseteq G$ of minimum order and with p edges such that each colour occurs exactly once. This problem is NP-hard and APX-hard, even for graphs with maximum degree $\Delta = 2$ [3].

If we do not consider edge colourings, the analogous problem is known as the $(t, f(t))$ dense subgraph problem ($(t, f(t))$-DSP), which asks whether there is a t-vertex subgraph of a given graph G which has at least $f(t)$ edges. When $f(t) = \binom{t}{2}$, $(t, f(t))$-DSP is equivalent to the well-known t-clique problem (cf. [1]). Maximum clique and maximum independent set are both hard to approximate within $n^{1-\epsilon}$ in polynomial time.

The minimum-degree greedy algorithm, or Greedy for short, is a simple and well-studied method for finding independent sets in graphs. Halldórsson and Radhakrishnan [2] have shown that it achieves a performance ratio of $\frac{\Delta+2}{3}$ for approximating independent sets in graphs with degree bounded by Δ.

In this talk we will show that the Greedy algorithm for the MRS problem has an approximation ratio of $\frac{\Delta}{2} + \frac{\ln \Delta + 1}{2}$ for graphs with maximum degree Δ. If the average degree d of a minimum rainbow subgraph is known, then the approximation ratio is $\frac{d}{2} + \frac{\ln\lceil d \rceil + 1}{2}$. These results improve the best known previous approximation ratios for the MRS problem [4].

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The facial Thue choice index of plane graphs

Jens Schreyer

(joint work with Erika Škrabuľáková)

A repetition is a sequence of symbols $r_1, r_2, \ldots, r_{2n}$ such that for all $1 \leq i \leq n$ it holds $r_i = r_{n+i}$. Let G be a simple plane graph. We say that φ is a *non-repetitive edge-colouring of the graph* G if for any simple path on edges $e_1, e_2, \ldots, e_{2n}$ in G the associated sequence of colours $\varphi(e_1), \varphi(e_2), \ldots, \varphi(e_{2n})$ is not a repetition. A *facial non-repetitive edge-colouring* of G is an edge-colouring such that any *facial path* in G is coloured non-repetitively. Moreover if the colour of every edge $e \in E(G)$ is chosen only from a list assigned to e we call φ a facial non-repetitive list edge-colouring of the graph G. The facial Thue choice index of G, $\pi'_{fl}(G)$, is the minimum number such that for every list assignment $L : E(G) \to 2^{\mathbb{N}}$ with list length at least $\pi'_{fl}(G)$ the graph is facial non-repetitively edge-colourable with colours from the corresponding lists. We show that for an arbitrary plane graph G the facial Thue choice index is at most 17. We also give examples of families of plane graphs where better upper bounds are achieved.

Graphs with relatively constant metric dimensions

Rinovia Simanjuntak

(joint work with Hilda Assiyatun, Herolistya Baskoroputro, Hazrul Iswadi, Yudi Setiawan, and Saladdin Uttunggadewa)

For an ordered set W=$\{w_1, w_2, \cdots, w_k\}$ of vertices and a vertex v in a connected graph G, the representation of v with respect to W is the vector $r(v|W) = (d(v, w_1), d(v, w_2), \cdots, d(v, w_k))$. A set R is called a resolving set of G if for every vertex v of G, its representation with respect to R is unique. A resolving set of G is called basis of G if it has minimum cardinality among all resolving sets of G. The metric dimension of G, $dim(G)$, is the cardinality of a basis of G.

Here we consider two families of graphs with "relatively constant" metric dimension, that is, graphs whose orders are determined by two or more parameters, however only one of them contributes to the dimensions of the graphs.

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Cycle bases, matchings, and the maximum genus of a graph

Martin Škoviera

(joint work with Michal Kotrbčík)

The talk focuses on the interplay between the maximum genus of a graph and the structure of its cycle space. We show that the matching number of the intersection graph of a cycle basis is independent of the basis precisely when the graph is upper-embeddable (i.e., embeds in the orientable surface of genus $\lfloor \beta/2 \rfloor$ where β is its cycle rank), and completely describe the range of matching numbers of intersection graphs when the graph is not upper-embeddable. We also discuss specific properties of intersection graphs of cycle bases formed from fundamental cycles with respect to a given spanning tree.

On the Thue choice number of graphs

Erika Škrabuľáková

(joint work with Jens Schreyer)

A sequence is called *non-repetitive* if no of its subsequences forms a repetition (a sequence $r_1, r_2, \ldots, r_{2n}$ such that $r_i = r_{n+i}$ for all $1 \leq i \leq n$). Let G be a graph whose vertices are coloured. A colouring φ of the graph G is *non-repetitive* if the sequence of colours on any path in G is non-repetitive. The *Thue chromatic number*, denoted by $\pi(G)$, is the minimum number of colours of a non-repetitive colouring of G.

Moreover, if the colour of every vertex v is chosen only from a list $L(v)$ of colours assigned to the vertex v we speak about a *non-repetitive list colouring* φ_L of the graph G with list assignment L. If the graph G is non-repetitively list colourable for every list assignment L with list size at least k, we call G *non-repetitively k-choosable*. The smallest number k such that G is non-repetitively k-choosable is called the *Thue choice number* of G and is denoted by $\pi_{ch}(G)$.

Czerwiński and Grytczuk [1] conjectured that $\pi(P_n) = \pi_{ch}(P_n) = 3$ for P_n being a path of length n. Grytczuk, Przybyło, and Zhu [2] proved that $\pi_{ch}(P_n) \leq 4$. Here we give to our knowledge the first example of an infinite family of graphs (Gummi-bear graphs) where $\pi(G) < \pi_{ch}(G)$. On the other hand we show that there exist examples of infinite families of graphs where for each graph of the family the Thue chromatic number and the Thue choice number are the same.

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Domination: min-count characterization via majorization versus max-count of minima

Zdzisław Skupień

A self-contained proof is presented of recent characterizations of n-vertex trees [1] and next of n-vertex graphs without isolated vertices [4] with minimum number of dominating sets in either case. In both cases the number is exponential and the same. Unions of disjoint stars as spanning subgraphs and majorization among related numerical partitions of vertices are objects and a tool of investigation. Cycle length among characterized graphs is bounded above by

$(n+2)/3$. The number of leaves among the graphs (including trees) is in the set $\{2n+r)/3 | r = -2, -1, 0, 1\}$. On the other hand, there is still a considerable gap between upper and lower bounds on the maximum number of minimal dominating sets among n-vertex graphs [2] as well as n-vertex trees [3], the number being not the same.

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Parity vertex colouring of regular and semiregular plane graphs

Peter Šugerek

A proper vertex colouring of a 2-connected plane graph G is a parity vertex colouring if for each face f and each colour c, either no vertex or an odd number of vertices incident with f is coloured with c. The minimum number of colours used in such a colouring of G is called parity chromatic number and denoted by $\chi_s(G)$. We determine the $\chi_s(G)$ of the Platonic solids graphs, the Archimedean solids graphs and some its extensions.

On discrepancy of neighborhood hypergraphs

Hovhannes Tananyan

Let $G = (V(G), E(G))$ is an undirected graph. The hypergraph $\mathcal{N}(G) = (V(G), \{\{w \in V(G) : (v, w) \in E(G)\} : v \in V(G)\})$ is called neighborhood hypergraph of the graph G and the graph $L(G) = (E(G), \{(e_1, e_2) : e_1, e_2 \in E(G), e_1$ and e_2 are incident$\})$ is called the line graph of the graph G. The arboricity of a graph G is the minimum number $arb(G)$ of forests into which its edges can be partitioned. The discrepancy of the hypergraph $\mathcal{H} = (V(\mathcal{H}), \mathcal{E}(\mathcal{H}))$ is defined by

$$disc(\mathcal{H}) = \min_{\chi:V(\mathcal{H})\to\{-1,1\}} \max_{E\in\mathcal{E}(\mathcal{H})} \left| \sum_{v\in E} \chi(v) \right|.$$

In this talk the discrepancy of the neighborhood hypergraphs is investigated. Some upper bounds and exact values are given. Particularly the following results are obtained.

- $disc(\mathcal{N}(G)) \leq 3arb(G) - 1$, and moreover if $arb(G) \geq 3$ then $disc(\mathcal{N}(G)) \leq 3arb(G) - 3$ (for instance if G is planar graph then $disc(\mathcal{N}(G)) \leq 6$).
- For every $k \geq 2$ there exist simple graph Ch_k such that $disc(\mathcal{N}(Ch_k)) = arb(Ch_k) + 1 = k + 1$ and multigraph GC_k such that $disc(\mathcal{N}(GC_k)) = 2arb(GC_k) - 1 = 2k - 1$.
- For $n \geq 5$ the quantity $disc(\mathcal{N}(L(K_n)))$ is equal to 2 if and only if n is of the form $4k + 1$, and equal to 4 otherwise.
- $disc(\mathcal{N}(L(K_{n,m})))$ is equal to 4 if both $n \geq 7$ and $m \geq 7$ are odd, equal to 3 if $n \geq 6$ is even and $m \geq 3$ is odd, and equal to 4 if both $n \geq 4$ and $m \geq 4$ are even.

HISTs of k-holed triangulations

Shoichi Tsuchiya
(joint work with Atsuhiro Nakamoto)

A spanning tree with no vertices of degree two of a graph is called a *homeomorphically irreducible spanning tree* (or *HIST*) of the graph. A *k-holed triangulation* is a 2-connected graph on the plane with k distinguished faces bounded by pairwise disjoint cycles $C_1, \ldots, C_k$ such that the length of C_i is at least three for $i = 1, \ldots, k$ and that all other faces are triangular. Each face of a k-holed triangulation G bounded by a cycle C_i, where $1 \leq i \leq k$, is called a *hole* of G. Albertson, Berman, Hutchinson, and Thomassen have proved that every 1-holed triangulation with at least four vertices has a HIST [1]. Moreover, Davidow, Hutchinson, and Huneke have proved that every 2-holed triangulation has a HIST [2]. Following their results, we consider whether a k-holed triangulation has a HIST when $k \geq 3$.

Let G be a k-holed triangulation and let $C_1, \ldots, C_k$ be k boundary cycles of G. Let C be a cycle of G. Note that C separates the plane into two regions since G is a graph on the plane. If both regions separated by C contain at least one hole, then C is called an *essential cycle* of G. In my talk, we prove that every k-holed triangulation G has a HIST if G has no essential cycle whose length is less than 7.

By our main result, we can prove that every triangulation G on closed surfaces has a HIST if the representativity of G is sufficiently large.

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Cycles and Colorings twenty years after

Zsolt Tuza

There are many beautiful open problems which are more than twenty years old. Wouldn't you like to solve some of them? ;-)

Fractional total $(a, b)_{\mathcal{P},\mathcal{Q}}$-list coloring

Margit Voigt

(joint work with Arnfried Kemnitz and Peter Mihók)

Let $\mathcal{P}$ and $\mathcal{Q}$ be hereditary properties.

A *total stable* $(\mathcal{P}, \mathcal{Q})$*-set* $T = V_T \cup E_T \subseteq V \cup E$ is the union of a set V_T of vertices and a set E_T of edges of G such that

$$G[V_T] \in \mathcal{P},\ G[E_T] \in \mathcal{Q},\ G[V_T] \text{ and } G[E_T] \text{ are disjoint.}$$

Let $\mathcal{T}_{\mathcal{P},\mathcal{Q}}$ be the set of all total stable $(\mathcal{P}, \mathcal{Q})$-sets of G.

A fractional total $(\mathcal{P}, \mathcal{Q})$*-coloring* of G is a mapping $\varphi : \mathcal{T}_{\mathcal{P},\mathcal{Q}} \to [0, 1]$ such that

$$\forall x \in V \cup E : \sum_{T \in \mathcal{T}_{\mathcal{P},\mathcal{Q}}; x \in T} \varphi(T) \geq 1 \qquad (*)$$

The *fractional total* $(\mathcal{P}, \mathcal{Q})$*-chromatic number* $\chi''_{f,\mathcal{P},\mathcal{Q}}(G)$ is the solution of the linear program $(*)$ with objective function

$$\min \sum_{T \in \mathcal{T}_{\mathcal{P},\mathcal{Q}}} \varphi(T).$$

A graph G is *total* $(a, b)_{\mathcal{P},\mathcal{Q}}$*-list colorable* if for every list assignment L with $|L(x)| = a\ \forall x \in V \cup E$ we can choose color sets $C(x) \subseteq L(x)$ with $|C(x)| = b$ $\forall x \in V \cup E$ such that we have for every color i: $T_i \in \mathcal{T}_{\mathcal{P},\mathcal{Q}}$ where $T_i := \{x \in V \cup E; i \in C(x)\}$.

$$chr_{\mathcal{P},\mathcal{Q}}(G) := \inf \Big\{ \frac{a}{b}; G \text{ is total } (a, b)_{\mathcal{P},\mathcal{Q}} - \text{list colorable} \Big\}$$

The above defined parameters are generalizations of the fractional chromatic number $\chi_f(G)$ and the choice ratio $chr(G)$. Some properties and relations of these parameters will be discussed in the talk.

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Extending fractional precolorings

Jan Volec

(joint work with Jan van den Heuvel, Daniel Kráľ, Martin Kupec, and Jean-Sébastien Sereni)

Let G be a graph with fractional chromatic number k and let P be an independent set in G such that any two vertices from P are at distance at least $d \geq 4$. For given k and d, we seek the minimum ϵ such that it is possible to extend every fractional $(k+\epsilon)$-precoloring of P to a proper fractional $(k+\epsilon)$-coloring of the whole graph G.

For the case $k = 2$ and $k \geq 3$ the question was completely solved in [1]. However, the case $2 < k < 3$ seems to be more difficult and interesting. In this talk, we will solve the problem for $k \in (2, 3)$ and $d = 4$. We will also discuss some progress for larger values of d.

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On the Merrifield-Simmons index in graphs with two elementary cycles

Iwona Włoch

(joint work with Andrzej Włoch)

A subset $S \subseteq V(G)$ is independent if no two vertices of S are adjacent in G. The number of independent sets in G is denoted $NI(G)$. This parameter appears in the mathematical literature in a paper of Prodinger and Tichy [2] and this paper gave impetus to the counting of independent sets in graphs. Independently Merrifield and Simmons introduced the number of independent sets to the chemical literature. The parameter $NI(G)$ of a graph G is called the Merrifield-Simmons index in mathematical chemistry and now there have been many papers studying the Merrifield-Simmons index, see last survey [1]. We will present the extremal values of the Merrifield-Simmons index in graphs with two elementary cycles.

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Edge colourings and the number of palettes

Mariusz Woźniak

(joint work with Mirko Horňák and Rafał Kalinowski)

A proper edge colouring $f : E \longrightarrow \{1, 2, \ldots, k\}$ of a graph $G = (V, E)$ defines, for each vertex $x \in V$, the set $\{f(e) : x \in e\}$ called *colour-set (palette)* (at x). We are interested in the minimum number of palettes (taken over all possible proper colourings of G).

Degree sum conditions concerning the order, the connectivity and the independence number for the circumference

Tomoki Yamashita

(joint work with Shuya Chiba and Masao Tsugaki)

In this talk, we research degree sum conditions for cycles. Let σ_k be the minimum degree sum of an independent set of k vertices in a graph. Let n, κ and α be the order, the connectivity and the independence number of a graph, respectively. Fraisse and Jung (1989) proved that a graph is hamiltonian if $\sigma_2 \geq n+\kappa-\alpha+1$. On the other hand, Bauer, Broersma, Li, and Veldman (1989) showed that a graph is hamiltonian if $\sigma_3 \geq n + \kappa$. Recently, we showed that a graph is hamiltonian if $\sigma_4 \geq n + \kappa + \alpha - 1$, and obtained similar degree sum conditions for cyclability. Motivated these results, we proposed a problem; a graph is hamiltonian if $\sigma_{k+1} \geq n+\kappa+(k-2)(\alpha-1)$. Concerned this problem, we consider degree sum conditions for the circumference.

Planar hypohamiltonian graphs

Carol T. Zamfirescu

A graph is called *hypohamiltonian* if it is not hamiltonian but, when omitting an arbitrary vertex, it becomes hamiltonian. The smallest hypohamiltonian graph is the famous Petersen graph (found by Kempe in 1886) on 10 vertices. In 1963, Sousselier posed a "recreational" problem, and thus began the study of hypohamiltonian graphs. Many authors followed, in particular Thomassen with a series of very interesting papers written in the Seventies and Eighties.

Among the work concerning hypohamiltonian graphs, Chvátal [1] asked in 1973 if there existed hypohamiltonian graphs with the additional requirement of planarity, while Grünbaum conjectured that there are no such graphs. An infinite family of such graphs was subsequently found by Thomassen [4], the smallest among them having 105 vertices. In 1979, Hatzel [3] improved this lower bound to 57 vertices. Many years later, in 2007, Zamfirescu and Zamfirescu [7] found a planar hypohamiltonian graph on 48 vertices, and only very recently Araya and Wiener [6] constructed the currently smallest known example, which has 42 vertices. All of these graphs were constructed by applying Grinberg's hamiltonicity criterion for planar graphs [2]. This leads to the natural question whether one might construct even smaller planar hypohamiltonian graphs using Grinberg's criterion.

In this talk we shall investigate the pivotal role of Grinberg's criterion in the context of planar hypohamiltonian graphs, and present a result answering (partially) the above question in the negative. We will also discuss the recent constructions in [6], which by applying a method of Thomassen [5] and results from [3] and [7] settle the open question whether there exists an N such that there is a planar hypohamiltonian graph of every order $n \geq N$.

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Programme of the Conference

Sunday	
16:00 - 22:00	Registration
18:00 - 21:00	Dinner

Monday			
07:00 - 09:00		Breakfast	
09:00 - 09:50	A	Hell P.	Variants of interval graphs
09:55 - 10:15	A	Kráľ D.	Points covered by many simplices
	B	Niepel Ľ.	Locating and identifying codes in circulant networks
10:20 - 10:40	A	Borowiecki M.	Local ranking of graphs
	B	Chiba S.	Circumferences of 2-factors in claw-free graphs
10:40 - 11:10		Coffee break	
11:10 - 11:30	A	Kemnitz A.	Grundy numbers of strong products of graphs
	B	Rollová E.	Covering signed graphs with cycles
11:35 - 11:55	A	Bacsó G.	Progress in mixed hypergraph coloring for finite planes
	B	Adiwijaya	Some sufficient conditions of a graph to be in C_f2
12:00 - 12:20	A	Holub P.	The packing chromatic number of distance graphs
	B	Kyppö J.	Odd and even cycles in digraphs of small strategy games
12:30 - 14:00		Lunch	
15:30 - 16:20	A	Kriesell M.	Disjoint cycles and dicycles in digraphs
16:20 - 16:50		Coffee break	
16:50 - 17:10	A	Schreyer J.	The facial Thue choice index of plane graphs
	B	Noguchi K.	The empire problem in even embeddings on closed surfaces
17:15 - 17:35	A	Škrabuľáková E.	On the Thue choice number of graphs
	B	Ozeki K.	Recent results on the hamiltonicity of graphs on surfaces
17:45 - 18:05	A	Bode J.-P.	Minimum size of rainbow k-connected graphs of given order
	B	Bezegová Ľ.	A construction of balanced degree-magic graphs
18:10 - 18:30	A	Knor M.	Triangular embeddings of K_n in non-orientable surfaces
	B	Ali G.	On super edge magic and edge antimagic labeling of $C_n(n-2)$ and C_n^{n-3} graphs
18:30 - 20:00		Dinner	
20:00 -		Welcome party	

Tuesday			
07:00 - 09:00		Breakfast	
09:00 - 09:50	A	Nešetřil J.	Existence & counting
09:55 - 10:15	A	Ryjáček Z.	Closures for strong hamiltonian properties
	B	Kupec M.	Complexity of $\lambda - L(p,q)$-labelling
10:20 - 10:40	A	Škoviera M.	Cycle bases, matchings, and the maximum genus of a graph
	B	Scheidweiler R.	Gap k-colorings of trees and cycles
10:40 - 11:10		Coffee break	
11:10 - 11:30	A	Woźniak M.	Edge colourings and the number of palettes
	B	Tananyan H.	On discrepancy of neighborhood hypergraphs
11:35 - 11:55	A	Bujtás Cs.	Induced cycles in triangle graphs
	B	Lukoťka R.	Possible values of the circular chromatic index of a snark
12:00 - 12:20	A	Rosa A.	Triple metamorphosis of twofold triple systems
	B	Włoch I.	On the Merrifield-Simmons index in graphs with two elementary cycles
12:30 - 14:00		Lunch	
15:30 - 16:20	A	Li H.	Degrees and neighborhoods of vertices and Hamiltonicity of graphs
16:20 - 16:50		Coffee break	
16:50 - 17:10	A	Madaras T.	On doubly light triangles in plane graphs
	B	Tsuchiya S.	HISTs of k-holed triangulations
17:15 - 17:35	A	Hudák D.	1-planarity of complete multipartite graphs
	B	Dąbrowski K.K.	The complexity of risk-free marriage
17:45 - 18:05	A	Fiedorowicz A.	Acyclic chromatic indices of graphs
	B	Nikodem M.	On vertex stability of complete k-partite graphs
18:10 - 18:30	A	Dvořák Z.	5-choosability of near-planar graphs
	B	Polláková T.	On magic joins of graphs
18:30 - 20:00		Dinner	
20:00 - 21:00		Videopresentation C&C 2010	

Wednesday	
06:30 - 08:00	Breakfast
08:00 - 15:00	Trip
13:00 - 16:00	Lunch
18:30 - 20:00	Dinner

Thursday			
07:00 - 09:00		Breakfast	
09:00 - 09:50	A	Tuza Zs.	Cycles and Colorings twenty years after
09:55 - 10:15	A	Schiermeyer I.	Greedy is good to approximate minimum rainbow subgraphs
	B	Zamfirescu C.T.	Planar hypohamiltonian graphs
10:20 - 10:40	A	Kardoš F.	On computing the minimum 3-path vertex cover and dissociation number of graphs
	B	Esfandiari H.	$\{1,2,3\}$-weight colorability of claw-free graphs
10:40 - 11:10		Coffee break	
11:10 - 11:30	A	Nedela R.	Six-decompositions of snarks
	B	Klešč M.	On the crossing numbers of join products with stars
11:35 - 11:55	A	Voigt M.	Fractional total $(a,b)_{\mathcal{P},\mathcal{Q}}$-list coloring
	B	Kravecová D.	On the crossing numbers of products of some special graphs
12:00 - 12:20	A	Bielak H.	Ramsey numbers for graphs versus disjoint union of some small graphs
	B	Draženská E.	On the crossing numbers of Cartesian products
12:30 - 14:00		Lunch	
15:30 - 16:20	A	Ota K.	Minimum degree condition for forests
16:20 - 16:50		Coffee break	
16:50 - 17:10	A	Yamashita T.	Degree sum conditions concerning the order, the connectivity and the independence number for the circumference
	B	Petříčková Š.	Online Ramsey games on planar graphs
17:15 - 17:35	A	Göring F.	Week odd 3-colorings of planar graphs
	B	Mockovčiaková M.	Vertex-distinguishing edge colorings of circulant graphs
17:45 - 18:05	A	Hałuszczak M.	On Ramsey $(K_{1,m},\mathcal{G})$-minimal graphs
	B	Rucký O.	On the vertex suppression in 3-connected graphs
18:10 - 18:30	A	Volec J.	Extending fractional precolorings
	B	Hurajová J.	On selfcentric graphs
19:00 -		Farewell party	

Friday			
07:00 - 09:00		Breakfast	
09:00 - 09:50	A	Skupień Z.	Domination: min-count characterization via majorization versus max-count of minima
09:55 - 10:15	A	Lichiardopol N.	New Ore's type results on hamiltonicity and existence of paths of given length in graphs
10:20 - 10:40	A	Kopperová M.	Enforced hamiltonian cycles in two classes of graphs
10:40 - 11:10		Coffee break	
11:10 - 11:30	A	Simanjuntak R.	Graphs with relatively constant metric dimensions
11:35 - 11:55	A	Petrillová J.	Join products with crossing number one
12:00 - 12:20	A	Šugerek P.	Parity vertex colouring of regular and semiregular plane graphs
12:30 - 14:00		Lunch	