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C&C
High Tatras, Slovakia

# Workshop

# Cycles and Colourings 2013

**8th – 13th September 2013 Nový Smokovec**

**Abstracts of the 22nd Workshop Cycles and Colourings**
*8–13 September 2013, Nový Smokovec, High Tatras, Slovakia*

Editors: Igor Fabrici, František Kardoš

Dear Participant,

welcome to the Twenty-second Workshop Cycles and Colourings. Except for the first workshop in the Slovak Paradise (Čingov 1992), the remaining twenty workshops took place in the High Tatras (Nový Smokovec 1993, Stará Lesná 1994–2003, Tatranská Štrba 2004–2010, Nový Smokovec 2011–2012).

The series of C&C workshops is organised by combinatorial groups of Košice and Ilmenau. Apart of dozens of excellent invited lectures and hundreds of contributed talks the scientific outcome of our meetings is represented also by special issues of journals Tatra Mountains Mathematical Publications and Discrete Mathematics (TMMP 1994, 1997, DM 1999, 2001, 2003, 2006, 2008, 2013).

The scientific programme of the workshop consists of 50 minute lectures of invited speakers and of 20 minute contributed talks. This booklet contains abstracts as were sent to us by the authors.

**Invited speakers:**

| | |
|---|---|
| Boštjan Brešar, | University of Maribor, Slovenia |
| Jiří Fiala, | Charles University, Prague, Czech Republic |
| Anna Fiedorowicz, | University of Zielona Góra, Poland |
| Florian Pfender, | University of Colorado Denver, USA |
| Oleg Pikhurko, | University of Warwick, United Kingdom |
| Stéphan Thomassé, | École Normale Supérieure de Lyon, France |

Have a pleasant and successful stay in Nový Smokovec.

**Organising Committee:**

Igor Fabrici
František Kardoš
Tomáš Madaras
Roman Soták

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# The Workshop Programme

| Sunday | |
|---|---|
| 16:00 - 22:00 | Registration |
| 18:00 - 21:00 | Dinner |

| Monday | | | |
|---|---|---|---|
| 07:00 - 09:00 | | Breakfast | |
| 09:00 - 09:50 | A | **Thomassé S.** | Forbidding long cycles and anticycles |
| 09:55 - 10:15 | A | **Bujtás Cs.** | On the $(n, n/2, n/4)$-conjecture |
| 10:20 - 10:40 | A | **Tuza Zs.** | Identifying chromatic number of graphs |
| 10:45 - 11:15 | | Coffee break | |
| 11:15 - 11:35 | A | **Ryjáček Z.** | 1-Hamilton-connected (claw,hourglass)-free graphs |
| 11:40 - 12:00 | A | **Teska J.** | Shortness exponent and trestles |
| 12:05 - 12:25 | A | **Problem session 1** | |
| 12:30 - 14:00 | | Lunch | |
| 15:50 - 16:40 | A | **Fiedorowicz A.** | Acyclic colourings of graphs |
| 16:45 - 17:15 | | Coffee break | |
| 17:15 - 17:35 | A | **Scheidweiler R.** | New estimates for the gap chromatic number |
| | B | **Milz S.** | Connectivity of local tournaments |
| 17:40 - 18:00 | A | **Mockovčiaková M.** | Strong chromatic index of subcubic bipartite graphs |
| | B | **Włoch I.** | On classes of graphs with 2-dominating kernels |
| 18:05 - 18:25 | A | **Jakovac M.** | The b-chromatic index of regular graphs |
| | B | **Wołowiec-Musiał M.** | Distance Fibonacci numbers |
| 18:30 - 20:00 | | Dinner | |
| 20:00 - | | Welcome party | |

| Tuesday | | | |
|---|---|---|---|
| 07:00 - 09:00 | | Breakfast | |
| 09:00 - 09:50 | A | **Pikhurko O.** | Minimising the number of cliques |
| 09:55 - 10:15 | A | **Bielak H.** | Remarks on Turán and Ramsey numbers for some families of semi-topological graphs |
| 10:20 - 10:40 | A | **Dąbrowski K.** | Colouring of graphs with Ramsey-type forbidden subgraphs |
| 10:45 - 11:15 | | Coffee break | |
| 11:15 - 11:35 | A | **Kemnitz A.** | $(\mathcal{P}, \mathcal{Q})$-total $(r, s)$-colorings of graphs |
| 11:40 - 12:30 | A | **Problem session 2** | |
| 12:30 - 14:00 | | Lunch | |
| 15:50 - 16:40 | A | **Fiala J.** | Computational complexity of distance constrained labelings |
| 16:45 - 17:15 | | Coffee break | |
| 17:15 - 17:35 | A | **Kalinowski R.** | Symmetry breaking by edge-colourings |
| | B | **Knor M.** | Deterministic models of self-similar networks |
| 17:40 - 18:00 | A | **Pilśniak M.** | Symmetry breaking in infinite graphs |
| | B | **Coroničová Hurajová J.** | On decay centrality |
| 18:05 - 18:25 | A | **Hudák P.** | On subgraphs of 4-critical planar graphs |
| | B | **Wolska K.** | On the adjacent eccentric distance sum |
| 18:30 - 20:00 | | Dinner | |

| Wednesday | |
|---|---|
| 06:30 - 08:00 | Breakfast |
| 08:00 - 15:00 | Trip |
| 13:00 - 16:00 | Lunch |
| 18:30 - 20:00 | Dinner |

| Thursday | | | |
|---|---|---|---|
| 07:00 - 09:00 | | Breakfast | |
| 09:00 - 09:50 | A | **Pfender F.** | The 1-2-3 Conjecture and its relatives on graphs and hypergraphs |
| 09:55 - 10:15 | A | **Dvořák Z.** | List coloring in minor-closed families |
| 10:20 - 10:40 | A | **Barát J.** | List colouring, minimum degree, minors and girth |
| 10:45 - 11:15 | | Coffee break | |
| 11:15 - 11:35 | A | **Schiermeyer I.** | Rainbow connection and size of graphs |
| 11:40 - 12:00 | A | **Holub P.** | Rainbow connection and forbidden induced subgraphs |
| 12:05 - 12:25 | A | **Woźniak M.** | Local irregularity and decompositions |
| 12:30 - 14:00 | | Lunch | |
| 15:50 - 16:40 | A | **Brešar B.** | Bucolic graphs |
| 16:45 - 17:15 | | Coffee break | |
| 17:15 - 17:35 | A | **Diner Ö.Y.** | Monochromatic connected cover and partition of 2-coloured graphs |
| | B | **Meszka M.** | From squashed 6-cycles to triple systems |
| 17:40 - 18:00 | A | **Broere I.** | Hom-properties of graphs and the Hedetniemi Conjecture |
| | B | **Pavlič P.** | Retracts of strong graph bundles |
| 18:05 - 18:25 | A | **Škrabuľáková E.** | A new Thue-type colouring invariant of graphs |
| | B | **Junosza-Szaniawski K.** | Beyond homothetic polygons - recognition and maximum clique |
| 19:00 - | | Farewell party | |

| Friday | | | |
|---|---|---|---|
| 07:00 - 09:00 | | Breakfast | |
| 09:00 - 09:20 | A | **Taranenko A.** | On the Hausdorff graphs |
| 09:25 - 09:45 | A | **Gologranc T.** | Cover-incomparability graphs |
| 09:50 - 10:10 | A | **Kardoš F.** | Saturation number of fullerene graphs |
| 10:15 - 10:35 | A | **Bode J.-P.** | Achievement games for polyominoes on Catalan tessellations |
| 10:40 - 11:10 | | Coffee break | |
| 11:10 - 11:30 | A | **Široczki P.** | Densities of chromatically-critical unit-distance graphs |
| 11:35 - 11:55 | A | **Vrbjarová M.** | Unique-maximum edge-colourings of plane pseudographs |
| 12:00 - 13:00 | | Lunch | |

# List colouring, minimum degree, minors and girth

**János Barát**

(joint work with David R. Wood)

Assume every vertex of a given graph has a list of colours of size $k$. We have to colour the vertices properly from the given lists. Consider a class $\mathcal{C}$ of graphs, where every graph $G$ in the class satisfies $\delta(G) \leq k-1$. Now every graph in the class can be $k$-list coloured by the greedy algorithm, always colouring a vertex of minimum degree. Therefore, one would be happy to prove things like the following:
Every $K_5$-minor free graph with girth at least 5 has a vertex of degree at most 2.

We survey some results and conjectures of this nature. We also consider the question to determine the maximum number of edges in a $K_5$-minor-free graph with $n$ vertices and girth $g$. We determine the answer completely when $g \equiv 0 \pmod 4$.

**Theorem 1** For every integer $k \geq 1$ and $n \geq 4$, the maximum number of edges in a $K_5$-minor-free graph with $n$ vertices and girth $4k$ equals $\frac{3k}{3k-2}(n-3)$.

For other values of $g$ the question is more challenging. We obtain the following exact answer for $g = 5$.

**Theorem 2** Every $K_5$-minor-free graph with $n \geq 4$ vertices and girth at least 5 has at most $\frac{9n-21}{5}$ edges, except for $C_5$ and the Petersen graph with one edge deleted, which has 10 vertices and $14 = \frac{9n-20}{5}$ edges. Moreover, for infinitely many values of $n$, there is a $K_5$-minor-free graph with $n$ vertices and girth 5 and exactly $\frac{9n-21}{5}$ edges.

# Remarks on Turán and Ramsey numbers for some families of semi-topological graphs

**Halina Bielak**

Let $\mathcal{G}$ be a family of simple graphs. Let $ex(n, \mathcal{G})$ be the Turán number for the family $\mathcal{G}$, i.e., the maximum number of edges in a graph on $n$ vertices which does not contain $G \in \mathcal{G}$ as a subgraph. We give the Turán numbers for some families of semi-topological graphs extending the results of Jiang [4], Horev [3], Jiang and Seiver [5], and Bielak [1] and generalizing some other results presented in [2]. We consider extremal graphs for the problem. Moreover, we count Ramsey numbers for some semi-topological graphs versus a simple graph.

### References


[1] H. Bielak, Turán and Ramsey numbers for some semi-topological graphs, submitted.

[2] B. Bollobás, Extremal graph theory, Academic Press, 1978.

[3] E. Horev, Extremal graphs without a semi-topological wheel, J. Graph Theory 306 (2011), 326–339.

[4] T. Jiang, A note on a conjecture about cycle with many incident chords, J. Graph Theory 306 (2004), 180–182.

[5] T. Jiang, R. Seiver, Turán numbers of subdivided graphs, SIAM J. Discrete Math. 26:3 (2012), 1238–1255.


# Achievement games for polyominoes on Catalan tessellations

**Jens-P. Bode**
(joint work with Heiko Harborth)

Catalan tessellations are the duals of the eight Archimedean tessellations of the plane. A polyomino is a simply edge-connected set of polygons of a tessellation, that is, the set and its complement are edge-connected. Two sets of polygons are considered to be the same polyomino if there is a mapping (generated by translations, rotations, and reflections) of the tessellation onto itself which also maps one of the sets of polygons onto the other one.

For a given polyomino $P$ the following achievement game will be considered. Two players $A$ (first move) and $B$ alternatingly color the polygons (cells) of the corresponding tessellation. Player $A$ wins if he achieves a copy of $P$ in his color and $B$ wins otherwise. The polyomino $P$ is called a winner if there exists a winning strategy for $A$. Otherwise there exists a strategy for $B$ to prevent $A$ from winning and then $P$ is called a loser.

# Bucolic graphs

**Boštjan Brešar**
(joint work with Jeremie Chalopin, Victor Chepoi,
Tanja Gologranc, and Damian Osajda)

In this talk we present recently introduced class of the so-called (strongly) bucolic graphs [1], which are a common generalization of median graphs and bridged graphs, two central classes of graphs in metric graph theory.

Bridged graphs are known as the graphs in which there are no isometric cycles of length greater than 3; that is, for any cycle $C$ of length at least 4 in a bridged graph there exist two vertices $a, b$ from $C$ such that any shortest path from $a$ to $b$ passes also a vertex not in $C$. Median graphs arise in different guises and applications, relate to several other mathematical structures, and enjoy many different characterizations. For instance, they are precisely the retracts of hypercubes, the graphs with windex 2, and the $K_{2,3}$-free bipartite graphs in which the so-called quadrangle property holds. Several natural non-bipartite generalizations that capture various properties of median graphs have also been studied; in particular quasi-median graphs, weakly median graphs and fiber-complemented graphs. These classes of graphs are closed for operations of Cartesian products and gated amalgamations, and admit characterizations of similar flavor as median graphs. As it turns out, (strongly) bucolic graphs also fit into this frame.

We prove that (strongly) bucolic graphs are precisely the retracts of Cartesian products of (weakly) bridged graphs; in turn they are exactly the weakly modular graphs (i.e., the graphs in which the quadrangle and the triangle property hold), not containing induced $K_{2,3}$, 4-wheels, and 4-wheels without a spoke (and 5-wheels, respectively). Moreover, finite bucolic graphs are the graphs obtainable by a sequence of gated amalgamations from products of (weakly) bridged graphs. One of the motivations for their study comes from geometric group theory, where the notion of bucolic complexes becomes interesting. Bucolic complexes can be defined as simply connected prism complexes satisfying some local combinatorial conditions, and as it turns out, their 1-skeletons are precisely the bucolic graphs. Among other results that will be presented, let us mention fixed point theorem for finite group actions on such complexes (and graphs).

## References

[1] B. Brešar, J. Chalopin, V. Chepoi, T. Gologranc, D. Osajda, Bucolic complexes, Adv. Math. 243 (2013), 127–167.

# Hom-properties of graphs and the Hedetniemi Conjecture

**Izak Broere**

(joint work with Johannes Heidema and Moroli D.V. Matsoha)

A *graph property* is a set of countable graphs. A *homomorphism* from a graph $G$ to a graph $H$ is an edge-preserving map from the vertex set of $G$ into the vertex set of $H$. If such a map exists, we write $G \to H$. Given any graph $H$, the *hom-property* $\to H$ is the set of *$H$-colourable graphs*, i.e., the set of all (countable) graphs $G$ satisfying $G \to H$. A graph property $\mathcal{P}$ is *of finite character* if, whenever we have that $H \in \mathcal{P}$ for every finite induced subgraph $H$ of a graph $G$, then we have that $G \in \mathcal{P}$ too.

The well-known conjecture of Hedetniemi states that, for all (finite) graphs $G$ and $H$, we have that $\chi(G \times H) = \min\{\chi(G), \chi(H)\}$.

We study the (distributive) lattice of hom-properties of finite character and discuss a number of equivalent formulations of this conjecture of Hedetniemi which can be made in terms of (amongst others) the meet irreducibility of the property $\to K_n$ of $n$-colourable graphs in this lattice.

# On the $(n, n/2, n/4)$-conjecture

**Csilla Bujtás**

The transversal of a hypergraph $\mathcal{H}$ is a set $T \subseteq V(\mathcal{H})$ of vertices which meets each edge $E \in E(\mathcal{H})$. The transversal number $\tau(\mathcal{H})$ is the minimum cardinality of a transversal in $\mathcal{H}$. In the talk, we consider 6-uniform hypergraphs and prove an upper bound on the transversal number in terms of $n = |V(\mathcal{H})|$ and $m = |E(\mathcal{H})|$. This bound improves the existing ones for a range of $m/n$.

The topic is inspired by a conjecture of Tuza and Vestergaard [1] stating that for every 6-uniform hypergraph with $n$ vertices and with at most $n/2$ edges, the minimum size of a transversal is at most $n/4$.

## References

[1] Zs. Tuza, P.D. Vestergaard, Domination in partitioned graphs, Discuss. Math. Graph Theory 22 (2002), 199–210.

# On decay centrality

**Jana Coroničová Hurajová**
(joint work with Silvia Gago and Tomáš Madaras)

The centrality indices represent a core concept for the analysis of social networks since they help to quantify the role that a given object plays in the network. Decay centrality introduced in [1], [2] is a centrality measure based on the proximity between a choosen vertex and every other vertex weighted by the decay. More precisely, decay centrality of a given vertex $x$ of a graph $G$ is define as the sum $\sum_{y \in V(G)} \delta^{d(x,y)}$ where $d(x, y)$ denotes the distance between $x$ and $y$ and $\delta \in (0, 1)$ is a parameter.

We study the general properties of decay centrality, the stability of vertex ranking depending on the choise of parameter $\delta$ and we look for the graphs whose vertices do not change their mutual position according to this measure.

### References

[1] Ch. Dangalchev, Residual closeness and generalized closeness, Internat. J. Found. Comput. Sci. 22 (2011), 1939–1948.

[2] M.O. Jackson, A. Wolinsky, A strategic model of social and economic networks, J. Econom. Theory 71 (1996), 44–74.

# Colouring of graphs with Ramsey-type forbidden subgraphs

**Konrad Dąbrowski**
(joint work with Petr Golovach and Daniël Paulusma)

A colouring is an assignment of colours to the vertices of a graph such that no two vertices of the same colour are adjacent. A $k$-colouring is a colouring that uses at most $k$ colours. The Colouring problem is that of testing whether a given graph has a $k$-colouring for some given integer $k$.

The Colouring problem is difficult to solve in general (it is NP-hard). However, this is not the end of the story. Sometimes we only want to solve the problem on certain types of graphs. This gives us some extra knowledge about the structure of the graph, which we can use to our advantage.

For example, for fixed integers $s$ and $t$, if we insist that our input graph $G$ has no independent set on $s$ vertices and no clique on $t$ vertices, Ramsey's Theorem tells us that the graph is bounded in size. In this case, Colouring can be solved in polynomial time. We generalize this idea to larger classes of graphs, in which the number of vertices is no longer bounded. The talk will focus on graph-theoretic proofs and will be very light on algorithmic details.

# Monochromatic connected cover and partition of 2-coloured graphs

**Öznur Yaşar Diner**

(joint work with Shinya Fujita)

We consider the following graph invariants: Monochromatic connected cover and monochromatic connected partition of 2-coloured graphs. These invariants correspond to the minimum number of classes in a connected cover (resp. connected partition) of a 2-coloured graph. This problem was first introduced in general by Erdös, Gyárfás and Pyber [1] with the name *tree partition number* of an $r$-coloured graph for a given positive integer $r$. In this talk we compare these two parameters for some graph families, such as $k$-partite graphs. We give upper bounds restricting these invariants for some instances.

References

[1] P. Erdös, A. Gyárfás, L. Pyber, Vertex coverings by monochromatic cycles and trees, J. Combin. Theory Ser. B 51 (1991), 90–95.

# List coloring in minor-closed families

**Zdeněk Dvořák**

(joint work with Robin Thomas)

A graph $H$ is $t$-*apex* if $H - X$ is planar for some set $X \subset V(H)$ of size $t$. For any fixed $t$-apex graph $H$, we give a polynomial-time algorithm to decide whether a $(t+3)$-connected $H$-minor-free graph is colorable from a given assignment of lists of size $t + 4$. The connectivity requirement is the best possible in the sense that for every $t \geq 1$, there exists a $t$-apex graph $H$ such that testing $(t+4)$-colorability of $(t + 2)$-connected $H$-minor-free graphs is NP-complete. Similarly, the size of the lists cannot be decreased, since for every $t \geq 1$, testing $(t+3)$-list-colorability of $(t + 3)$-connected $K_{t+4}$-minor-free graphs is NP-complete.

# Computational complexity of distance constrained labelings

**Jiří Fiala**

(joint work with many good friends)

An $L(p, q)$-labeling is an assignment of integers to vertices of a given graph, s.t. labels of adjacent vertices differ by at least $p$, and labels of vertices with a common neighbor differ by at least $q$. The span of a labeling is the difference between the largest and the smallest label used.

We present a selection of algorithmic and hardness results for the existence of such an $L(p, q)$-labeling. Among others:

- NP-hardness of $L(2, 1)$-labeling of given span [4],
- NP-hardness of $L(2, 1)$-labeling of span four [3],
- a polynomial-time algorithm for $L(2, 1)$-labelings for trees [1],
- NP-hardnes for $L(p, q)$-labeling of given span for trees when $q$ is not a divisor of $p$ [2],
- and also more recent results for restricted graph classes.

### References


[1] G.J. Chang, D. Kuo, The $L(2, 1)$-labeling problem on graphs, SIAM J. Discrete Math. 9:2 (1996), 309–316.

[2] J. Fiala, P.A. Golovach, J. Kratochvíl, Computational complexity of the distance constrained labeling problem for trees (extended abstract), in: L. Aceto et al. (eds.), Automata, languages and programming, ICALP 2008, LNCS 5125, 294–305.

[3] J. Fiala, J. Kratochvíl, T. Kloks, Fixed-parameter complexity of $\lambda$-labelings, Discrete Appl. Math. 113:1 (2001), 59–72.

[4] J.R. Griggs, R.K. Yeh, Labelling graphs with a condition at distance 2, SIAM J. Discrete Math. 5:4 (1992), 586–595.


# Acyclic colourings of graphs


**Anna Fiedorowicz**

(joint work with Mariusz Hałuszczak and Elżbieta Sidorowicz)


In this talk we consider three aspects of acyclic colourings of graphs. Let us recall that a colouring (not necessarily proper) of vertices or edges of a graph $G$ is called *acyclic*, if there is no 2-coloured alternating cycle in $G$.

We start with the problem of acyclic vertex colourings of graphs with bounded maximum degree and with small maximum average degree. We also consider acyclic colourings in which each colour class induces a subgraph with bounded degree or acyclic, for graphs with maximum degree at most $d$ ($d = 4, 5$).

Next we move to proper acyclic edge colourings of graphs. The well-known Acyclic Edge Colouring Conjecture, stated by Fiamčík in 1978 and later restated by Alon, Sudakov and Zaks in 2001, says that any graph $G$ admits a proper acyclic edge colouring with at most $\Delta(G) + 2$ colours. This conjecture has been verified by now only for some special classes of graphs. We survey recent results.

Finally, we use the notion of acyclic edge colouring to introduce a new type of Ramsey numbers. Namely, let $G, H$ be graphs with at least one edge each. The *acyclic Ramsey number* $R^a(G, H)$ is defined as the smallest integer $t$ such that in any acyclic (red,blue)-colouring of the edges of $K_t$ there is a red copy of $G$ or a blue copy of $H$. We determine the values of $R^a(G, H)$ for several families of graphs, among others, for complete and complete bipartite graphs.

# Cover-incomparability graphs

**Tanja Gologranc**

Cover-incomparability graphs (C-I graphs, for short) are the graphs whose edge-set is the union of edge-sets of the incomparability and the cover graph of a poset [2]. As the problem of recognizing cover-incomparability graphs of posets is NP-complete in general [3], we investigate the classes of graphs in which the recognition complexity of the C-I graphs is polynomial. We concentrate on chordal graphs and cographs and characterize the posets whose cover-incomparability graph is a block graph, a split graph and a cograph, respectively [1]. We also characterize the cover-incomparability graphs among block graphs, split graphs and cographs, respectively [1]. Furthermore, we present polynomial algorithms for recognition of block graphs, split graphs and cographs, respectively, that are cover-incomparability graphs [1]. In addition, some properties of the cover-incomparability chordal graphs are presented.

### References


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# Rainbow connection and forbidden induced subgraphs

**Přemek Holub**

(joint work with Zdeněk Ryjáček and Ingo Schiermeyer)

A connected edge-coloured graph $G$ is rainbow-connected if any two distinct vertices of $G$ are connected by a path whose edges have pairwise distinct colours; the rainbow connection number $\mathrm{rc}(G)$ of $G$ is the minimum number of colours such that $G$ is rainbow-connected. The concept of rainbow connection in graphs was introduced by Chartrand et al. in [1] and has various applications. One interesting example is the secure transfer of classified information between agencies (see, e. g., [2]).

Let $\mathcal{F}$ be a family of connected graphs. We say that a graph $G$ is *$\mathcal{F}$-free* if $G$ does not contain an induced subgraph isomorphic to a graph from $\mathcal{F}$. Specifically, for $\mathcal{F} = \{X\}$ we say that $G$ is *$X$-free*, and for $\mathcal{F} = \{X, Y\}$ we say that $G$ is *$(X, Y)$-free*. The members of $\mathcal{F}$ will be referred to in this context as *forbidden induced subgraphs*.

Graphs characterized in terms of forbidden induced subgraphs are known to have many interesting properties. In the first part of this talk we give a brief list of results for some hamiltonian properties of graphs. For the rainbow connection, we consider families $\mathcal{F}$ of connected graphs for which there is a constant $k_{\mathcal{F}}$ such that, for every connected $\mathcal{F}$-free graph $G$, $\mathrm{rc}(G) \leq \mathrm{diam}(G) + k_{\mathcal{F}}$, where $\mathrm{diam}(G)$ is the diameter of $G$. Then we give a complete answer for $|\mathcal{F}| \in \{1, 2\}$.

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## On subgraphs of 4-critical planar graphs

**Peter Hudák**

A graph is $k$-critical if its chromatic number is $k$ and each its proper subgraph is $(k-1)$-colorable. In 1974 Greenwell and Lovász [1] characterized the class of all proper subgraphs of $k$-critical graphs. A $(k-1)$ colorable graph is proper subgraph of some $k$-critical graph if and only if by contracting arbitrary edge $e$ graph $G/e$ is $(k-1)$-colorable. It is easy to verify that this theorem is not valid if we consider class of 4-critical planar graphs. We will present results about properties of some subgraphs of 4-critical planar graphs. We will also show some examples of forbidden subgraphs of 4-critical planar graphs that fulfill Greenwell, Lovász condition.

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# The b-chromatic index of regular graphs

**Marko Jakovac**

(joint work with Iztok Peterin)

In the talk regular graphs are considered with focus on cubic graphs. It is proved that with four exceptions, the b-chromatic index of cubic graphs is 5. The exceptions are $K_4$, $K_{3,3}$, the prism over $K_3$, and the cube $Q_3$.

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# Beyond homothetic polygons - recognition and maximum clique

**Konstanty Junosza-Szaniawski**

(joint work with Jan Kratochvíl, Martin Pergel, and Paweł Rzążewski)

We study the Clique problem in classes of intersection graphs of convex sets in the plane. The problem is known to be NP-complete in convex-sets intersection graphs and straight-line-segments intersection graphs, but solvable in polynomial time in intersection graphs of homothetic triangles. We extend the latter result by showing that for every convex polygon $P$ with $k$ sides, every $n$-vertex graph which is an intersection graph of homothetic copies of $P$ contains at most $n^{2k}$ inclusion-wise maximal cliques. We actually prove this result for a more general class of graphs, so called $k_{DIR}$-CONV, which are intersection graphs of convex polygons whose all sides are parallel to at most $k$ directions. We further provide lower bounds on the numbers of maximal cliques, discuss the complexity of recognizing these classes of graphs and present relationship with other classes of convex-sets intersection graphs.

# Symmetry breaking by edge-colourings

**Rafał Kalinowski**
(joint work with Monika Pilśniak)

The *distinguishing index* $D'(G)$ of a connected graph $G$ of order $n \geq 3$ is the minimum number of colours in an edge-colouring of $G$ such that the identity is the only automorphism of $G$ preserving this colouring. We prove that $D'(G) \leq \Delta(G)$ unless $G$ is a small cycle $C_3$, $C_4$ or $C_5$.

If we restrict ourselves to proper edge-colourings, then we speak about the *distinguishing chromatic index* $\chi'_D(G)$ of a graph $G$. It occurs that $\chi'_D(G) \leq \Delta(G) + 1$ except for four graphs $C_4, K_4, C_6$ and $K_{3,3}$. In consequence, every connected Class 2 graph $G$ admits a proper edge-colouring with $\chi'(G)$ colours that is not preserved by any nontrivial automorphism.

We also investigate a correlation of $\chi'_D(G)$ with the *distinguishing index by colour walks* introduced in [1], and with the *distinguishing index by colour paths with palettes*.

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# Saturation number of fullerene graphs

**František Kardoš**
(joint work with Vesna Andova and Riste Škrekovski)

The saturation number of a graph $G$ is the cardinality of any smallest maximal matching of $G$, and it is denoted by $s(G)$. Fullerene graphs are cubic planar graphs with exactly twelve 5-faces; all the other faces are hexagons. They are used to capture the structure of carbon molecules. We show that the saturation number for fullerenes on $n$ vertices is essentially $n/3$.

# $(\mathcal{P},\mathcal{Q})$-total $(r,s)$-colorings of graphs

**Arnfried Kemnitz**
(joint work with Massimiliano Marangio, Anja Pruchnewski, and Margit Voigt)

Let $r, s \in \mathbb{N}$, $r \geq s$, and $\mathcal{P} \supseteq \mathcal{O}$ and $\mathcal{Q} \supseteq \mathcal{O}_1$ be two additive and hereditary graph properties. A $(\mathcal{P}, \mathcal{Q})$*-total* $(r,s)$*-coloring* of a graph $G$ is a coloring of the vertices and edges of $G$ by $s$-element subsets of $\mathbb{Z}_r$ such that for each color $i$,

$0 \leq i \leq r-1$, the vertices colored by subsets containing $i$ induce a subgraph of $G$ with property $\mathcal{P}$, the edges colored by subsets containing $i$ induce a subgraph of $G$ with property $\mathcal{Q}$, and color sets of incident vertices and edges are disjoint. The *fractional* $(\mathcal{P}, \mathcal{Q})$*-total chromatic number* $\chi''_{f,\mathcal{P},\mathcal{Q}}(G)$ of $G$ is defined as the infimum of all ratios $r/s$ such that $G$ has a $(\mathcal{P}, \mathcal{Q})$-total $(r, s)$-coloring.

We present lower and upper bounds and also some exact values for $\chi''_{f,\mathcal{P},\mathcal{Q}}(G)$ for specific properties and classes of graphs.

# Deterministic models of self-similar networks

**Martin Knor**

(joint work with Riste Škrekovski)

A real-life network typically satisfies the following properties:

- The number of edges is in $O(n \ln n)$, where $n$ is the number of vertices.
- The diameter is in $O(\ln n)$.
- The clustering coefficient is at least $c$ for some $c > 0$.
- The proportion of vertices of degree at least $k$ is approximately equal to $k^{1-\gamma}$, where $2 \leq \gamma \leq 3$.
- The network is self-similar.

We propose a new model of complex network which satisfies the above properties and generalizes many previous models. We also calculate the main network parameters of our model. Finally, we propose several new models (special cases of our general one) with prescribed clustering coefficient.

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# From squashed 6-cycles to triple systems

**Mariusz Meszka**

(joint work with Elizabeth Billington, Curtis Lindner, and Alexander Rosa)

A *Steiner triple system* is a pair $(V, \mathcal{B})$ where $V$ is a finite set and $\mathcal{B}$ is a collection of 3-element subsets of $V$ called *triples* such that every 2-subset of $V$ is contained in exactly one triple in $\mathcal{B}$. Similarly, a 6*-cycle system* of order $v$ is a pair $(V, \mathcal{C})$ where $V$ is a finite set and $\mathcal{C}$ is a collection of 6-cycles with vertices in $V$ such that every edge of the complete graph on the set $V$ is contained in exactly one 6-cycle in $\mathcal{C}$.

One of possible ways to convert a 6-cycle into two triangles is to *squash* the 6-cycle by identifying its two opposite vertices, and renaming one of them with the other. More precisely, given the 6-cycle $(a, b, c, d, e, f)$, we may identify $a$ and $d$ to get the *bowtie* $\{\{a, b, c\}, \{a, e, f\}\}$ or $\{\{b, c, d\}, \{d, e, f\}\}$.

A complete answer to the question about the existence spectrum for 6-cycle systems having property that its 6-cycles can be squashed to produce triples of a Steiner triple system will be presented. Moreover, maximum packings and minimum coverings of complete graphs with 6-cycles that can be squashed to some partial triple systems will be discussed.

# Connectivity of local tournaments

**Sebastian Milz**
(joint work with Yubao Guo and Andreas Holtkamp)

For a local tournament $D$ with minimum out-degree $\delta^+$, minimum in-degree $\delta^-$ and irregularity $i_g(D)$ we give a lower bound on the connectivity of $D$, namely

$$\kappa(D) \geq \left\lceil \frac{2 \cdot \max\{\delta^+, \delta^-\} + 1 - i_g(D)}{3} \right\rceil$$

if there exists a minimum separating set $S$ such that $D - S$ is a tournament, and

$$\kappa(D) \geq \left\lceil \frac{2 \cdot \max\{\delta^+, \delta^-\} + 2|\delta^+ - \delta^-| + 1 - 2i_g(D)}{3} \right\rceil$$

otherwise. This generalizes a result on tournaments presented by C. Thomassen [1]. An example shows the sharpness of this result.

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# Strong chromatic index of subcubic bipartite graphs

**Martina Mockovčiaková**

A *strong edge coloring* of a graph $G$ is a proper edge coloring in which each color class is an induced matching of $G$; that is, there is no bichromatic path of length three in $G$. The minimum number of colors for which a strong edge coloring of $G$ exists is the *strong chromatic index of* $G$, denoted by $\chi'_s(G)$.

In 1993 Brualdi and Quinn conjectured that every bipartite graph with bipartition $X$ and $Y$ without any cycle of length four such that the maximum degree of any vertex in $X$ is two and the maximum degree of any vertex in $Y$ is $\Delta$ can be strongly colored with $\Delta + 2$ colors. We partially solve this problem for such graphs with $\Delta = 3$. We also confirm one of the conjectures for subcubic bipartite graphs proposed by Faudree et al. in 1990.

# Retracts of strong graph bundles

**Polona Pavlič**

(joint work with Blaž Zmazek and Janez Žerovnik)

Graph bundles generalize the notion of covering graphs and graph products. They can be defined with respect to an arbitrary graph product [2]. We present how the Cartesian, strong and direct graph bundles are defined and give some of their basic properties. As many problems on graph bundles were studied in the literature, motivated by the results of Imrich and Klavžar [1] on retracts of strong products graphs, we present retracts of strong graph bundles.

Let $B$ and $F$ be connected graphs and let $B \boxtimes_\varphi F$ be the strong graph bundle over base $B$ with fibre $F$, where $\varphi$ is a mapping, which assigns an automorphism of the fibre graph $F$ to every arc of the base graph $B$. We show that every retract $R$ of $B \boxtimes_\varphi F$ is of the form $R = B' \boxtimes_{\varphi'} F'$, where $B'$ and $F'$ are subgraphs of $B$ and $F$, respectively, and $\varphi'$ is a restriction of $\varphi$ on $B'$. For triangle-free graphs $B$ and $F$ both $B'$ and $F'$ are retracts of $B$ and $F$.

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# The 1-2-3 Conjecture and its relatives on graphs and hypergraphs

**Florian Pfender**

(joint work with Maciej Kalkowski and Michal Karoński)

The 1-2-3 Conjecture from 2004 by Karoński, Łuczak and Thomason states that you can weigh the edges of any connected graph on at least 3 vertices with weights from the set $\{1, 2, 3\}$ such that the weighted vertex degrees induce a proper vertex coloring.

This conjecture has spurred a lot of activity in recent years, but it looks like we are still ways away from solving it completely. In this talk I will survey some results on this and related conjectures. Further, I will present some new results on the equivalent question for hypergraphs. Surprisingly, we can give sharp bounds for large classes of hypergraphs.

# Minimising the number of cliques

**Oleg Pikhurko**

(joint work with Emil R. Vaughan)

Motivated by Ramsey's theorem, Erdős asked in 1962 about the value of $f(n, k, l)$, the minimum number of $k$-cliques in a graph with order $n$ and independence number less than $l$. The case $(k, l) = (3, 3)$ was solved by Lorden. By applying flag algebras, we solve the problem (for all large $n$) for $(3, l)$ with $4 \leq l \leq 7$ and $(k, 3)$ with $4 \leq k \leq 7$. Independently, Das, Huang, Ma, Naves, and Sudakov resolved the cases $(k, l) = (3, 4)$ and $(4, 3)$.

# Symmetry breaking in infinite graphs

**Monika Pilśniak**

The *distinguishing index* $D'(G)$ of a graph $G$ is the least cardinal $d$ such that $G$ has an edge colouring with $d$ colours that is only preserved by the trivial automorphism. This concept is similar to the notion of the *distinguishing number* $D(G)$ of a graph $G$, which is defined for vertex colourings.

In this talk, we present some results for infinite graphs. In particular, we show that if $G$ is a connected, infinite graph such that the degree of any vertex of $G$ is not greater than a cardinal $\mathfrak{n}$, then $D'(G) \leq \mathfrak{n}$. However, $D'(G) \leq 2$ for infinite trees with at most one leaf, and for infinite tree-likes graphs. Moreover, an infinite random graph almost surely has the distinguishing index not greater than two.

We also investigate the concept of a motion of edges and its interesting relationship with the Infinite Motion Lemma.

# 1-Hamilton-connected (claw,hourglass)-free graphs

**Zdeněk Ryjáček**
(joint work with Tomáš Kaiser and Petr Vrána)

A graph $G$ is *Hamilton-connected* if $G$ has a hamiltonian $(u, v)$-path for any pair of vertices $u, v \in V(G)$, and, for an integer $k \geq 0$, a graph $G$ is *k-Hamilton-connected* if the graph $G - X$ is Hamilton-connected for every set $X \subset V(G)$ with $|X| = k$. It is easy to observe that a Hamilton-connected graph is 3-connected, and a $k$-Hamilton-connected graph is $(k + 3)$-connected. Specifically, $G$ is 1-Hamilton-connected if the graph $G - x$ is Hamilton-connected for every vertex $x \in V(G)$, and every 1-Hamilton-connected graph is 4-connected. The *hourglass* is the unique graph with degree sequence $4, 2, 2, 2, 2$.

Using a recently introduced closure concept for 1-Hamilton-connectedness, we show that every 4-connected (claw,hourglass)-free graph is 1-Hamilton-connected. The result gives a partial affirmative answer to a conjecture by Thomassen (every 4-connected line graph is hamiltonian), which is known to be equivalent to a (seemingly stronger) statement that every 4-connected claw-free graph is 1-Hamilton-connected. Some computational complexity consequences will be also discussed.

# New estimates for the gap chromatic number

**Robert Scheidweiler**
(joint work with Eberhard Triesch)

We investigate the gap chromatic number, a graph coloring parameter introduced 2012 by M. A. Tahraoui, E. Duchêne, and H. Kheddouci. From an edge labeling $f : E \to \{1, \cdots, k\}$ of a graph $G = (V, E)$ on $n$ vertices, $G$ gets an induced coloring $l$ of its vertex set. Vertices of degree greater than one are colored with the difference between their maximum and their minimum incident edge label, i.e., with their so-called gap, and vertices of degree one get their incident edge label as color. The gap chromatic number gap$(G)$ of $G$ is the minimum $k$, for which a labeling $f$ of $G$ exists that distinguishes the vertices of $G$, meaning that $l(v) \neq l(w)$ for $v, w \in V$ with $v \neq w$.

We show that the gap chromatic number of connected graphs is bounded by $n$, if the graph contains a vertex of degree one, and by $n + 1$ otherwise. Furthermore, we show how to prove the general upper bound $n + 9$ for this graph parameter and investigate a connection to Skolem sequences.

# Rainbow connection and size of graphs

**Ingo Schiermeyer**

An edge-coloured connected graph $G$ is called *rainbow-connected* if each pair of distinct vertices of $G$ is connected by a path whose edges have distinct colours. The *rainbow connection number* of $G$, denoted by $rc(G)$, is the minimum number of colours such that $G$ is rainbow-connected. In [2] the following problem was introduced.

**Problem** For all integers $n$ and $k$ with $1 \leq k \leq n-1$ compute and minimize the function $f(n,k)$ with the following porperty: If $|V(G)| = n$ and $|E(G)| \geq f(n,k)$ then $rc(G) \leq k$.

In [2] the following lower bound for $f(n,k)$ has been shown.

**Proposition** For $n$ and $k$ with $1 \leq k \leq n-1$ it holds that $f(n,k) \geq \binom{n-k+1}{2} + k - 1$.

This lower bound is tight.

**Problem** Determine all values of $n$ and $k$ such that

$$f(n,k) = \binom{n-k+1}{2} + k - 1.$$

It has been shown that $f(n,k) = \binom{n-k+1}{2} + k - 1$

- for $k = 1, 2, n-2$, and $n-1$ in [2],
- for $k = 3$ and 4 in [3],
- and for $n-6 \leq k \leq n-3$ in [1].

In this talk we will report about these results and show some further recent progress obtained for this problem.

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# Densities
# of chromatically-critical unit-distance graphs

**Pavol Široczki**
(joint work with Tomáš Madaras)

Given a graph $G$, let $u(G)$ be the maximal number of edges of an unit-distance subgraph of $G$; for a graph family $\mathcal{G}$, we also define the numbers $u(n, \mathcal{G}) = \max\{u(G) : G \in \mathcal{G}, |V(G)| = n\}$ and $u(\rho, \mathcal{G}) = \max\{u(G) : G \in \mathcal{G}, \frac{|E(G)|}{|V(G)|} = \rho\}$. In the connection with the Hadwiger-Nelson problem, a "common belief" is that if $G$ is a $k$-chromatic graph with $k \geq 5$ then $\frac{u(G)}{|E(G)|} < 1$. On the other hand, for $k = 4$, there exist infinitely many graphs for which this ratio equals 1. We study the estimates of limit value $\lim\limits_{n\to\infty} \frac{u(G)}{|E(G)|}$ for $n$-vertex $k$-critical graphs, $k \geq 5$; in addition, we show that, for each rational number $\rho$ between $\frac{5}{3}$ and $\frac{9}{5}$, there exists a 4-critical planar unit-distance graph with density $\rho$.

# A new Thue-type colouring invariant of graphs

**Erika Škrabuľáková**
(joint work with Jens Schreyer)

There are several ways how to relax the requirements in the original Thue colouring problem (introduced by Alon at al. in 2002) and hence several kinds of so called Thue's types of problems dealing with non-repetitive colourings. Here we deal with total Thue colourings of graphs and investigate the strong and the weak total Thue number of graphs.

A finite sequence $R = r_1 r_2 \ldots r_{2n}$ of symbols is called a *repetition* if $r_i = r_{n+i}$ for all $i = 1, 2, \ldots, n$. A sequence $S$ is called *repetitive* if it contains a subsequence of consecutive terms that is a repetition. Otherwise $S$ is called *non-repetitive*.

Let $\varphi$ be a colouring of the vertices of a graph $G$. We say that $\varphi$ is a *non-repetitive vertex-colouring* of $G$ if for any simple path on vertices $v_1, v_2, \ldots, v_{2n}$ in $G$ the associated sequence of colours $\varphi(v_1)\ \varphi(v_2) \ldots \varphi(v_{2n})$ is not a repetition. A *non-repetitive edge-colouring* of $G$ is defined analogously.

A total colouring of a graph is a colouring of its vertices and edges such that no two adjacent vertices or edges have the same colour and moreover no edge coloured $c$ has its endvertex coloured $c$ too. By a *weak total Thue colouring* of a graph $G$ we mean a colouring $\phi$ of both vertices and edges of $G$ such that every path on edges (without end-vertices) and vertices of $G$ is coloured nonrepetitively. Moreover if both the induced vertex-colouring and the induced edge-colouring of $G$ are non-repetitive under $\phi$, we speak about a *strong total Thue colouring* of $G$. The minimum number of colours required in every weak total Thue colouring is called the *weak total Thue number* and it is denoted $\pi^{T_w}(G)$. The minimum number of colours required in every strong total Thue colouring is called the *strong total Thue number* and it is denoted $\pi^{T_s}(G)$.

We show that the strong total Thue number is lesser then $18 \cdot \Delta^2$, where $\Delta \geq 3$ is the maximum degree of the graph. For the weak total Thue number we show $\pi^{T_w}(G) \leq |E(G)| - |V(G)| + 5$, that for planar graphs with $k$ faces gives $\pi^{T_w}(G) \leq 3 + k$. We also give some upper and lower bounds for these parameters considering special classes of graphs.

# On the Hausdorff graphs

**Andrej Taranenko**

(joint work with Iztok Banič)

In topology, one way to measure the closeness of two objects is to use the so called Hausdorff metric. Suppose $X$ is a nonempty compact metric space. The family of all nonempty closed subsets of $X$ is usually denoted by $2^X$. For each $r > 0$, denote by

$$N(A,r) = \bigcup_{x \in A} K(x,r)$$

the union of open $r$-balls $K(x,r)$ in $X$ with centers $x \in A$. Think of $N(A,r)$ as $A$ being 'inflated' by factor $r$. Then the *Hausdorff metric* $H_d$ is defined on $2^X$ by

$$H_d(A,B) = \inf\{\varepsilon > 0 \mid A \subseteq N(B,\varepsilon), B \subseteq N(A,\varepsilon)\},$$

for any $A, B \in 2^X$, to measure closeness between $A$ and $B$.

We apply this idea of closeness into the language of graphs. We realize this idea by introducing the new concept of so-called Hausdorff graph $2^G$ of a graph $G$, where the vertices of $2^G$ are all nonempty subgraphs of $G$. As the main result we show that $G$ is connected if and only if $2^G$ is connected. Moreover, we present the new concepts of H-distance, which helps examining the closeness of any two graphs.

# Shortness exponent and trestles

**Jakub Teska**

(joint work with Adam Kabela)

For any integer $r > 1$, an *$r$-trestle* is a 2-connected graph $F$ with maximum degree $\Delta(F) \leq r$. We say that a graph $G$ has an $r$-trestle if $G$ contains a spanning subgraph which is an $r$-trestle. This notion is a generalization of a hamiltonian cycle, since 2-trestle is exactly a hamiltonian cycle in a graph. We show new results concerning the existence of a 3-trestle in the square of a tree and a general graph. We also present new results concerning the shortness exponent for the class of 1-tough chordal planar graphs.

# Forbidding long cycles and anticycles

**Stéphan Thomassé**

(joint work with Marthe Bonamy, Nicolas Bousquet, and Aurélie Lagoutte)

A class $\mathcal{C}$ of graphs closed under induced subgraphs is said to satisfy the *Erdős-Hajnal property* if there exists some $c > 0$ such that every graph on $n$ vertices of $\mathcal{C}$ contains a clique or a stable set of size $n^c$. The Erdős-Hajnal conjecture [5] asserts that every strict class of graphs satisfies the Erdős-Hajnal property, see [3] for a survey. One of the best general result up to date about this conjecture is by Fox and Sudakov [6], where they show the existence of a polynomial size clique or empty bipartite graph.

This fascinating question is even open for graphs not inducing a cycle of length five. When excluding a single graph $H$, Alon, Pach and Solymosi showed in [1] that it suffices to consider prime $H$ (i.e. without nontrivial modules). A natural approach is then to study classes of graphs with intermediate difficulty, hoping to get a proof scheme which could be extended. A natural prime candidate to forbid is certainly the path. Chudnovsky and Zwols studied the class $\mathcal{C}_k$ of graphs not inducing the path $P_k$ on $k$ vertices nor $\overline{P_k}$. They proved the Erdős-Hajnal property for $P_5$ and $\overline{P_6}$-free graphs [2]. This was extended for $P_5$ and $\overline{P_7}$-free graphs by Chudnovsky and Seymour [4].

I will show in this talk that for every fixed $k$, the class $\mathcal{C}_k$ satisfies the Erdős-Hajnal property. A slight extension of the argument also gives EH-property for the class of graphs excluding induced cycles and complement of cycles of length at least $k$.

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# Identifying chromatic number of graphs

**Zsolt Tuza**
(joint work with Csilla Bujtás)

Let $G = (V, E)$ be an undirected graph. As defined in [2], an *identifying coloring* of $G$ with respect to a set $S \subseteq V$ is a mapping from the vertex set into the power set of $S$, $f : V \to \mathcal{P}(S) \setminus \{\emptyset\}$, such that each vertex is mapped to a non-empty subset of its closed neighborhood inside $S$ and no two vertices have the same image under $f$. The notion is closely related to, but different from, the one termed *identifying code* introduced in [1]. We study the *identifying chromatic number* of graphs, which is the minimum cardinality of a set $S$ admitting an identifying coloring $f$.

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# Unique-maximum edge-colourings of plane pseudographs

**Michaela Vrbjarová**
(joint work with Igor Fabrici and Stanislav Jendroľ)

A unique-maximum $k$-edge-colouring with respect to faces of a 2-edge-connected plane pseudograph $G$ is an edge-colouring with colours from the set $\{1, 2 \ldots k\}$ such that for each face $f$ of $G$ the maximum colour occurs exactly once on the edges of $f$.

We will prove that any 2-edge-connected plane pseudograph has such a colouring with 3 colours in general and with 6 colours if we require the colouring to be facially-proper. We will also discuss relations of these colourings to other types of colourings of plane pseudographs.

# On classes of graphs with 2-dominating kernels

**Iwona Włoch**
(joint work with Andrzej Włoch)

A subset $S \subseteq V(G)$ is an independent set of $G$ if no two vertices of $S$ are adjacent in $G$. A subset $Q \subseteq V(G)$ is a 2-dominating set of $G$ if each vertex from $V(G) \setminus Q$ has at least two neighbours in $Q$.

Using existing concepts of an independent set and a 2-dominating set, we define in the natural way the concept of 2-dominating kernels in graphs.

A subset $J \subset V(G)$ is a 2-dominating kernel of $G$ if $J$ is independent and 2-dominating. Clearly a 2-dominating kernel of $G$ is a kernel of $G$.

Every graph does not always posses a 2-dominating kernel. For example the graph $P_4$ is a graph without 2-dominating kernel.

In the talk we give a characterization of some classes of graphs with 2-dominating kernels.

# Distance Fibonacci numbers

**Małgorzata Wołowiec-Musiał**
(joint work with Urszula Bednarz)

In the talk we present generalizations of Fibonacci numbers in the distance sense. We give combinatorial representations, graph representations and matrix generators of these numbers. We study different properties of these numbers, in Pascal's triangle too.

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# On the adjacent eccentric distance sum

**Katarzyna Wolska**
(joint work with Halina Bielak)

In this paper we are going to show bounds for the adjacent eccentric distance sum in terms of some graph invariants or topological indices such as total eccentricity, Wiener index, maximum degree and minimum degree. We extend some earlier results by Hua and Yu [3] and show the extremal graphs that satisfy the bounds we present.

The adjacent eccentric distance sum index of the graph $G$ is

$$\xi^{sv}(G) = \sum_{v \in V(G)} \frac{\varepsilon(v) D(v)}{deg(v)},$$

where $\varepsilon(v)$ is the eccentricity of the vertex $v$, $deg(v)$ is the degree of the vertex $v$ and $D(v) = \sum_{u \in V(G)} d(u, v)$ is the sum of all distances from the vertex $v$.

References

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# Local irregularity and decompositions

**Mariusz Woźniak**
(joint work with Olivier Baudon, Julien Bensmail, and Jakub Przybyło)

A *locally irregular graph* is a graph whose adjacent vertices have distinct degrees. We say that a graph $G$ *can be decomposed into $k$ locally irregular subgraphs* if its edge set may be partitioned into $k$ subsets each of which induces a locally irregular subgraph in $G$. We characterize all connected graphs which cannot be decomposed into locally irregular subgraphs.

Moreover we conjecture that apart from these exceptions all other connected graphs can be decomposed into 3 locally irregular subgraphs. In particular, we prove this statement to hold for all regular graphs of degree at least $10^7$.

We also investigate a total version of this problem, where in some sense also the vertices are being prescribed to particular subgraphs of a decomposition.

The both concepts are closely related to the known 1-2-3 Conjecture and 1-2 Conjecture, respectively, and other similar problems concerning edge colourings. In particular, we improve the result given in [1] in the case of regular graphs.

More details can be found in [2].

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