Dear Participant,

welcome to the Twenty-fourth Workshop Cycles and Colourings. Except for the first workshop in the Slovak Paradise (Čingov 1992), the remaining twenty two workshops took place in the High Tatras (Nový Smokovec 1993, Stará Lesná 1994–2003, Tatranská Štrba 2004–2010, Nový Smokovec 2011–2014).

The series of C&C workshops is organised by combinatorial groups of Košice and Ilmenau. Apart of dozens of excellent invited lectures and hundreds of contributed talks, the scientific outcome of our meetings is represented also by special issues of journals Tatra Mountains Mathematical Publications and Discrete Mathematics (TMMP 1994, 1997, DM 1999, 2001, 2003, 2006, 2008, 2013).

The scientific programme of the workshop consists of 50 minute lectures of invited speakers and of 20 minute contributed talks. This booklet contains abstracts as were sent to us by the authors.

Invited speakers:

Daniel W. Cranston	Virginia Commonwealth University, Richmond, USA
Magnús M. Halldórsson	Reykjavik University, Reykjavik, Iceland
Roman Nedela	University of West Bohemia, Plzeň, Czech Republic
Kenta Ozeki	National Institute of Informatics, Tokyo, Japan JST, ERATO, Kawarabayashi Large Graph Project
Jakub Przybyło	AGH University of Science and Technology, Kraków, Poland
Éric Sopena	Université de Bordeaux, LaBRI, Bordeaux, France CNRS, LaBRI, Bordeaux, France

Have a pleasant and successful stay in Nový Smokovec.

Organising Committee:

Igor Fabrici
František Kardoš
Tomáš Madaras
Roman Soták

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(2-d)-kernels in graphs

Paweł Bednarz

(joint work with Andrzej Włoch and Iwona Włoch)

A subset of vertices is a (2-d)-kernel of a graph if it is 2-dominating and independent simultaneously. In the talk we describe classes of graphs with (2-d)-kernel and we shall show that the problem of the existence of (2-d)-kernels is NP-complete for a general graph.

Moreover we show an algorithm which determines a unique (2-d)-kernel in tree or shows that (2-d)-kernel does not exists.

References

[1] P. Bednarz, C. Hernández-Cruz, I. Włoch, On the existence and the number of (2-d)-kernels in graphs, Ars Combin. (to appear).

[2] P. Bednarz, I. Włoch, On the existence of (2-d)-kernels in trees, manuscript.

[3] A. Włoch, On 2-dominating kernels in graphs, Australas. J. Combin. 53 (2012), 273–284.

The number of $(A, 2B)$-edge colourings in trees

Urszula Bednarz

Let G be an undirected, connected, simple graph. Let $\mathcal{C} = \{A, B\}$ be the set of two colours. A graph G is $(A, 2B)$-*edge coloured* if for every maximal B-monochromatic subgraph H of G there is a partition of H into edge disjoint paths of the length 2. We have no restriction on the colour A. Let a graph G be $(A, 2B)$-edge coloured. Let $\mathcal{F}$ be a family of distinct $(A, 2B)$-edge colourings of a graph G and

$$\mathcal{F} = \{G^{(1)}, G^{(2)}, \ldots, G^{(r)}\}, r \geq 1.$$

By $\theta(G^{(p)})$ we denote the number of all partitions related to B-monochromatic subgraph of $G^{(p)}$ for $1 \leq p \leq r$. If $\sigma_{(A,2B)}(G)$ denotes the number of $(A, 2B)$-edge colourings of G, then

$$\sigma_{(A,2B)}(G) = \sum_{p=1}^{r} \theta(G^{(p)}).$$

In the talk we present some results obtained for the parameter $\sigma_{(A,2B)}(G)$ in trees.

On two generalizations of Lucas numbers and their interpretations in graphs

Dorota Bród

The well-known Lucas numbers L_n are defined by recurrence

$$L_n = L_{n-1} + L_{n-2} \text{ for } n \geq 2$$

with $L_0 = 2$, $L_1 = 1$. In the talk two generalizations of Lucas numbers in the distance sense ($(2,k)$-distance Lucas numbers) and some combinatorial and graph interpretations will be considered. Moreover, combinatorial formulas for $(2,k)$-distance Lucas numbers of two types will be presented.

References

[1] U. Bednarz, D. Bród, M. Wołowiec-Musiał, On two types of $(2,k)$-distance Lucas numbers, Ars Combin. 115 (2014), 467–479.

General upper bound on the game domination number of forests

Csilla Bujtás

In the domination game, introduced by Brešar, Klavžar, and Rall [1], Dominator and Staller alternately choose a vertex of a graph G and take it into a set D. The number of vertices dominated by the set D must increase with each move and the game ends when D becomes a dominating set of G. Dominator aims to minimize while Staller aims to maximize the number of turns (or equivalently, the size of the dominating set D obtained at the end). Assuming that Dominator starts and both players play optimally, the number of turns is the game domination number of G.

In the talk, we prove a new upper bound on the game domination number of forests and answer a question posed by Kinnersley, West, and Zamani [2] concerning the game domination number of caterpillars.

References

[1] B. Brešar, S. Klavžar, D.F. Rall, Domination game and an imagination strategy, SIAM J. Discrete Math. 24 (2010), 979–991.

[2] W.B. Kinnersley, D.B. West, R. Zamani, Extremal problems for game domination number, SIAM J. Discrete Math. 27 (2013), 2090–2107.

Graphs with high circular chromatic index

Barbora Candráková
(joint work with Edita Máčajová)

A *circular r-edge-coloring* of a graph G is an assignment of numbers from $[0, r)$ to the edges of G such that $1 \leq |c(e) - c(f)| \leq r - 1$ for any pair of adjacent edges $e, f \in E(G)$. The *circular chromatic index* of G, $\chi'_c(G)$, is the infimum of all rational numbers r, such that there exists a circular r-edge-coloring of G. It is known that the circular chromatic index of a graph G with maximum degree Δ lies in the interval $[\Delta, \Delta + 1]$. All rational numbers "sufficiently close" to Δ are attained as circular chromatic indices of graphs, where "sufficiently close" depends on Δ and whether the graph is a multigraph or simple graph. On the other hand, not much is known about the values closer to $\Delta + 1$. Afshani et al. proved a nonexistence of a graph G with $11/3 < \chi'_c(G) < 4$. Moreover, they conjectured that for any $k \geq 2$ there exists an $\varepsilon_k > 0$ such that there is no graph G with $k - \varepsilon_k < \chi'_c(G) < k$. We bound the values ε_k from below by showing that there exist multigraphs with $\chi'_c = k + 2/3$ and simple graphs with $\chi'_c = k + 1/2$ for any $k \geq 4$.

Fractional coloring of planar graphs and the plane

Daniel W. Cranston
(joint work with Landon Rabern)

The 4 Color Theorem is a landmark result. However, all known proofs rely heavily on computers for extensive case checking, and the search for a human checkable proof remains a major open problem in graph theory. In contrast, proving the 5 Color Theorem is easy. Here we present a $\frac{9}{2}$ Color Theorem, which we can prove by hand. More precisely, we give a short proof [1] that every planar graph G has a homomorphism to the Kneser graph $K_{9,2}$, which implies that G has fractional chromatic number at most $\frac{9}{2}$. This is the first proof (independent of the 4 Color Theorem) that there exists a constant $k < 5$ such that every planar G has fractional chromatic number at most k.

In the second part of the talk, we consider coloring the points of the plane so that every pair of points at distance 1 gets distinct colors. It is easy to show that the plane has chromatic number at least 4 and at most 7 (and these bounds are the best known). In the 1990s, Fisher and Ullman [3] first studied the fractional chromatic number of the plane; here [2] we improve the best lower bound.

References

[1] D.W. Cranston, L. Rabern, Planar graphs are 9/2-colorable and have independence ratio at least 3/13, arXiv:1410.7233.

[2] D.W. Cranston, L. Rabern, The fractional chromatic number of the plane, arXiv:1501.01647.

[3] D. Fisher, D. Ullman, The fractional chromatic number of the plane, Geombinatorics 2 (1992), 8–12.

Avoiding rainbow 2-connected subgraphs

Izolda Gorgol

While defining the anti-Ramsey number, Erdős, Simonovits and Sós [1] mentioned that the extremal colorings may not be unique. In the talk we discuss the uniqueness, generalize the idea of their construction and show how to use it to construct the colorings of the edges of complete split graphs avoiding rainbow 2-connected subgraphs. These colorings give the lower bounds for adequate anti-Ramsey numbers.

References

[1] P. Erdős, M. Simonovits, V. Sós, Anti–Ramsey theorems, in: A. Hajnal, R. Rado, V. Sós (eds.), Infinite and finite sets, Colloq. Math. Soc. J. Bolyai (1973), pp. 633–643.

Graph coloring as a model of wireless scheduling

Magnús M. Halldórsson

(joint work with Tigran Tonoyan)

As one of the most fundamental combinatorial problems, Graph Coloring has numerous applications. It is especially frequently used as a means to eliminate pairwise conflicts. Our interest here is in conflicts in the form of wireless interference.

Wireless scheduling has motivated numerous coloring-related work. In particular, it found an early role in the scheduling of ad-hoc networks, formulated as the coloring of graphs that are defined in terms of geometric ranges, such as *unit disc graphs*.

The role of disc graphs as natural models for wireless interference was discredited relatively early in the experimental literature (see, e.g., [3]). This has led in recent years to theoretical work for scheduling in more complex models, particularly the SINR or the *physical model* [4]. It is best viewed as a *directed edge-weighted coloring*, where the pairwise conflict relationship has been replaced by additive constraints (adding up the weights of edges from nodes of the same color) [1]. While we have seen some successes here, this still leaves two questions not fully addressed:

1. Is the true underlying problem truly well modeled by these more complicated models? If not, how can we overcome that?

2. Given that the design and analysis of algorithms in the physical model is significantly more difficult than in graphs, can we circumvent it by modeling the problem via a different, yet computationally tractable, class of graphs?

These are questions that we seldom need to ask ourselves, as the problem formulation is usually already there, conveniently supplied by knowledgeable "practitioners". I will only briefly outline possible answers to the first problem, but focus the talk on recent answers to the second problem [2].

References

[1] J. Bang-Jensen, M.M. Halldórsson, A note on vertex coloring edge-weighted digraphs, Inform. Process. Lett. 115 (2015), 791–796.

[2] M.M. Halldórsson, T. Tonoyan, How well can graphs represent wireless interference?, STOC, 2015.

[3] D. Kotz, C. Newport, R.S. Gray, J. Liu, Y. Yuan, C. Elliott, Experimental evaluation of wireless simulation assumptions, MSWiM, 2004.

[4] T. Moscibroda, R. Wattenhofer, The complexity of connectivity in wireless networks, INFOCOM, 2006.

Packing colouring of outerplanar graphs

Přemysl Holub

(joint work with Nicolas Gastineau and Olivier Togni)

A packing k-colouring of a graph G is a mapping from the vertex set $V(G)$ to the set $\{1, 2, \ldots, k\}$ (called colour set) such that any two vertices coloured with colour i are at distance at least $i+1$. Then the packing chromatic number $\chi_\rho(G)$ of G is the smallest integer l such that there is a packing l-colouring of G.

This concept was introduced by Goddard at el. in [2] under the name "broadcast colouring", but then the name was changed to "packing colouring" by Brešar et al. in [1]. Sloper in [3] showed that a complete binary tree of arbitrary height at least three is packing 7-colourable, while the infinite complete ternary tree is not packing colourable (i.e., we cannot colour vertices of the infinite complete ternary tree with a finite number of colours). In [2], it is shown that for paths and cycles, the packing chromatic number is at most 4. Hence is it a natural question which classes of graphs with maximum degree 3 can be packing colourable (i.e., for which classes of graphs with $\Delta = 3$, the packing chromatic number is finite). Since the problem is very difficult even for the class of planar graphs with $\Delta = 3$, we study the class of outerplanar graphs with $\Delta = 3$. In this talk we present some subclasses of outerplanar graphs with $\Delta = 3$, for which the packing chromatic number is finite.

References

[1] B. Brešar, S. Klavžar, D.F. Rall, On the packing chromatic number of Cartesian products, hexagonal lattice, and trees, Discrete Appl. Math. 155 (2007), 2303–2311.
[2] W. Goddard, S.M. Hedetniemi, S.T. Hedetniemi, J.M. Harris, D.F. Rall, Broadcast chromatic numbers of graphs, Ars Combin. 86 (2008), 33–49.
[3] C. Sloper, Broadcast-coloring of trees, Reports in Informatics 233 (2002), 1–11.

Facial colourings of plane graphs

Stanislav Jendroľ

(joint work with Igor Fabrici and Michaela Vrbjarová)

Let $G = (V, E, F)$ be a connected loopless and bridgeless plane graph, with vertex set V, edge set E, and face set F. Let $X \in \{V, E, F, V \cup E, V \cup F, E \cup F, V \cup E \cup F\}$. Two elements x and y of X are *facially adjacent* in G if they are incident, or they are adjacent vertices, or adjacent faces, or facially adjacent edges (i.e. edges that are consecutive on the boundary walk of a face of G).

A k-colouring is facial with respect to X if there is a k-colouring of elements of X such that facially adjacent elements of X receive different colours.

It is known that:
- G has a facial 4-colouring with respect to $X \in \{V, F\}$. The bound 4 is tight. (The Four Colour Theorem, Appel and Haken 1976, see [1]).
- G has a facial 6-colouring with respect to $X = V \cup F$. The bound 6 is tight. (The Six Colour Theorem, Borodin 1984, see [2]).

We prove that:
- G has a facial 4-colouring with respect to $X = E$. The bound 4 is tight.
- G has a facial 6-colouring with respect to $X \in \{V \cup E, E \cup F\}$. There are graphs requiring 5 colours in such a colouring.
- G has a facial 8-colouring with respect to $X = V \cup E \cup F$. There is a graph requiring 7 colours in such a colouring.

References

[1] K. Appel, W. Haken, Every planar graph map is four colourable, Bull. Amer. Math. Soc. 82 (1976), 711–712.
[2] O.V. Borodin, A new proof of the 6-colour theorem, J. Graph Theory 19 (1995), 507–521.

Dense on-line arbitrarily partitionable graphs

Rafał Kalinowski

A graph $G = (V, E)$ is called *arbitrarily partitionable* if for every partition $\tau = (\tau_1, \ldots, \tau_k)$ of the order n of G, there exists a partition $(V_1, \ldots, V_k)$ of V such that each V_i induces a connected subgraph of order n_i. This concept was introduced to model a problem in the design of computer networks by Barth, Baudon and Puech, and independently by Horňák and Woźniak.

The on-line version of this notion was defined by Horňák, Tuza and Woźniak. Suppose that the whole sequence $\tau = (\tau_1, \ldots, \tau_k)$ is initially not known, but its elements are requested on-line, i.e., one by one. In the i-th stage, where $i = 1, \ldots, k$, a positive integer n_i arrives and we have to choose a connected subgraph G_i of G of order n_i that is vertex-disjoint with all subgraphs $G_1, \ldots, G_{i-1}$ chosen in the previous stages (without a possibility of changing the choice in the future). If this procedure can be accomplished for any sequence of positive integers $\tau = (n_1, \ldots, n_k)$ adding up to the order n of G, then G is called *on-line arbitrarily partitionable*.

Clearly, each traceable graph is on-line arbitrarily partitionable. We try to replace some known sufficient conditions for traceability by weaker ones implying that the graphs satisfying them are on-line arbitrarily partitionable.

Sum list colorings of graphs

Arnfried Kemnitz
(joint work with Massimiliano Marangio and Margit Voigt)

Let $G = (V, E)$ be a simple graph and for every vertex $v \in V$ let $L(v)$ be a set (list) of available colors. The graph G is called *L-colorable* if there is a proper coloring φ of the vertices with $\varphi(v) \in L(v)$ for all $v \in V$. A function f from the vertex set V of G to the positive integers is called a *choice function* of G and G is said to be *f-list colorable* if G is L-colorable for every list assignment L with $|L(v)| = f(v)$ for all $v \in V$. Set $\text{size}(f) = \sum_{v \in V} f(v)$ and define the *sum choice number* $\chi_{sc}(G)$ as minimum of size(f) over all choice functions f of G.

It is easy to see that $\chi_{sc}(G) \leq |V| + |E|$ for every graph G and that there is a greedy coloring of G for the corresponding choice function f and every list assignment with $|L(v)| = f(v)$. A graph G is called *sc-greedy* if the sum choice number equals its upper bound, that is, $\chi_{sc}(G) = |V| + |E|$.

In this talk we investigate for different graph classes the question which graphs of the respective class are *sc*-greedy and which are not. Moreover, we determine the sum choice number for different non-*sc*-greedy graphs.

On the Erdős-Faber-Lovász Conjecture

Attila Kiss

In 1972 Erdős, Faber, and Lovász formulated the following conjecture: If k complete graphs, each having exactly k vertices, have the property that every pair of complete graphs has at most one shared vertex, then the union of the graphs can be colored with k colors. In this talk I prove another partial result. I define a special subclass of graphs and prove that the conjecture is true in this subclass. This question is important in real life applications too, these results can be used in a Wireless Sensor Network (WSN) to prolong the network lifetime for the benefit of various applications.

Wiener index and congruences

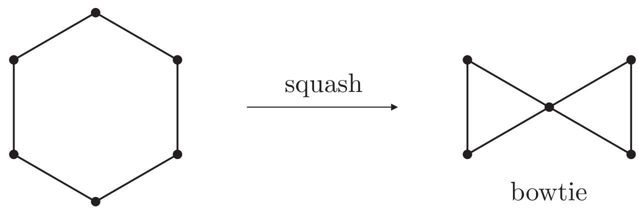
Martin Knor
(joint work with Katarína Hriňáková, Riste Škrekovski, and Aleksandra Tepeh)

The Wiener index, defined as the sum of all distances in a graph, is one of the most popular molecular descriptors. Congruence relations for the Wiener index for specific families of trees have been studied by several authors. In the talk we generalize a result of Lin, which itself generalizes a former result of Gutman and Rouvray, to the maximum possible extent. Our theorem covers large families of graphs with a tree-like structure.

How to squash a 6-cycle system into a Steiner triple system

Charles C. Lindner
(joint work with Mariusz Meszka and Alex Rosa)

The spectra for Steiner triple systems and 6-cycle systems agree when $n \equiv 1$ or 9 (mod 12). Let (X, C) be a 6-cycle system of order $n \equiv 1$ or 9 (mod 12). Let T be a collection of bowties obtained by squashing each 6-cycle of C into a bowtie.



If (X, T) is a Steiner triple system we say that the 6-cycle system (X, C) is squashed into the Steiner triple system (X, T). In this talk we construct, for every $n \equiv 1$ or 9 (mod 12), a 6-cycle system that can be squashed into a Steiner triple system.

Short cycle covers of weighted cubic graphs

Robert Lukoťka

Short cycle cover conjecture asserts that each bridgeless graph with m edges can be covered with circuits of total length at most $1.4m$ [1]. At the time when the conjecture was made, it was known that every bridgeless graph can be covered with circuit of total length at most $5/3 \cdot m$ [2, 1]. Despite a lot of effort this result still presents the best bound for general graphs. It is easy to see that it suffices to consider the problem of finding a short cycle cover of a graph on weighted cubic graphs. Despite the fact that for unweighted cubic graphs we have several methods that produce cycle covers of length smaller than $5/3 \cdot m$, none of these methods guarantees an improvement in the weighted case. In this talk we show several constructions of cycle covers shorter than $5/3 \cdot m$ under various cyclic connectivity and weight distribution assumptions.

References

[1] N. Alon, M. Tarsi, Covering Multigraphs by Simple Circuits, SIAM J. Algebraic Discrete Methods 6 (1985), 345–350.

[2] J.C. Bermond, B. Jackson, F. Jaeger, Shortest Coverings of Graphs with Cycles, J. Combin. Theory Ser. B 35 (1983), 297–308.

Permutation snarks

Edita Máčajová

(joint work with Martin Škoviera)

A permutation snark is a cubic graph with no 3-edge-colouring that contains a 2-factor consisting of two induced circuits. It is easy to see that a permutation snark on n vertices has $n \equiv 2 \pmod 4$. On the other hand, all known examples have order $n \equiv 2 \pmod 8$, leaving the existence of permutation snarks of order $6 \pmod 8$ open.

In his book 'Integer Flows and Cycle Covers of Graphs' C.-Q. Zhang made a conjecture that the only cyclically 5-edge-connected permutation snark is the Petersen graph. In 2013, Brinkmann et al. disproved this conjecture by exhibiting a cyclically 5-edge-connected permutation snark on 34 vertices found by an exhaustive computer search. This example has been recently extended by J. Hägglund and A. Hoffmann-Ostenhof to an infinite series of cyclically 5-edge-connected permutation snarks of order $24n + 10$ for each integer $n \geq 1$.

We study permutation snarks in a greater detail, focusing on the structure of edge-cuts of size 4 and 5. We prove that every permutation snark whose cyclic connectivity equals 4 is a dot product of two smaller permutation snarks. We also derive a necessary and sufficient condition for a dot product of two permutation snarks to be a permutation snark. As an application of our knowledge we provide rich families of cyclically 4- and 5-edge-connected permutation snarks of order $8n + 2$ for each integer $n \geq 2$ and $n \geq 4$, respectively. Whether there exist permutation snarks of order 6 (mod 8) remains unknown. The smallest such example, if it exists, must be cyclically 5-edge-connected.

Optimal unavoidable sets of types of 3-paths for planar graphs of given girth

Mária Maceková

(joint work with Stanislav Jendroľ, Mickaël Montassier, and Roman Soták)

In this talk we consider simple planar graphs with minimum degree at least two and a given girth. We describe the structure of the 3-paths in such graphs.

A 3-path of type (i, j, k) is a path uvw on three vertices u, v, and w such that the degree of u (resp. v, resp. w) is at most i (resp. j, resp. k).The elements i, j, k are called *parameters* of the type. The set S of types of paths is *unavoidable* for a family $\mathcal{F}$ of graphs if each graph G from $\mathcal{F}$ contains a path of the type from S. An unavoidable set S of types of paths is *optimal* for the family $\mathcal{F}$ if neither any type can be omitted from S, nor any parameter of any type from S can be decreased.

In the talk we present the unavoidable sets of types of 3-paths for the family of planar graphs having $\delta(G) \geq 2$ and $g(G) \geq g$. For some values of g we give two mutually incomparable optimal unavoidable sets of types of 3-paths for this family.

Lower bound on TSP in simple cubic graphs

Ján Mazák

(joint work with Robert Lukoťka et al.)

Of all the variants of the travelling salesman problem, we are concerned with *graphic* TSP on cubic graphs, where the goal is to find a closed trail as short as possible such that it contains all the vertices of a given unweighted cubic graph. It is known that every subcubic graph with n vertices has a TSP tour of length at most $(4n-2)/3$. This bound is attained for a certain infinite class of multigraphs containing parallel edges. Recently, it has been proved by Candráková, Lukoťka, and Ozeki that simple bridgeless cubic graphs on n vertices have a TSP tour of length at most $1.3n$, but this bound is probably not best possible. This talk is focused on the lower bound on the length of a TSP tour in simple bridgeless cubic graphs. We present several variants of a recursive construction which show that there are simple cubic graphs with no TSP tour shorter than roughly $1.25n$.

Bounded-diameter arboricity

Martin Merker

(joint work with Luke Postle)

The arboricity of a graph G is the smallest number of forests needed to cover the edges of G. Equivalently, it is the smallest number of colours needed to edge-colour G so that there are no monochromatic cycles. We introduce a new concept called the *diameter-d arboricity* of G, which is the smallest number of colours in an edge-colouring of G with no monochromatic cycles and no monochromatic paths of length $d+1$. In other words, we want the diameter of the trees in the forests to be at most d. If d is greater than the size of G, then the diameter-d arboricity of G is just the usual arboricity. For $d = 2$, the diameter-d arboricity of a graph is the same as the well-studied star arboricity.

We conjecture that for every natural number k there exists a number $f(k)$ such that every graph with arboricity k has diameter-$f(k)$ arboricity at most $k+1$. We verify this conjecture for $k \leq 3$ by giving an algorithm that shows $f(3) \leq 26$. As a corollary we get that every planar graph has diameter-26 arboricity at most 4, i.e. every planar graph can be decomposed into 4 forests in which each tree has diameter at most 26.

Bounds for k-Thue sequences

Martina Mockovčiaková

In this talk we consider a generalization of Thue sequence; that is, a sequence that does not contain a repetition of any length. A sequence S is called *k-Thue* if every subsequence of S, in which two consecutive terms are at indices of common differences from the set $\{1, 2, \dots, k\}$, is also Thue.

It was conjectured that $k+2$ symbols are enough to construct an arbitrarily long k-Thue sequence and shown that the conjecture holds for $k = 1, 2, 3$ and 5. We present a construction of 4-Thue sequences on 6 symbols, which confirms this conjecture. Moreover, we discuss known upper bounds on the number of symbols that suffice to construct such sequences.

The achromatic colourings of uniform hypergraphs

Paweł Naroski

(joint work with Paweł Rzążewski)

A *strong r-colouring* of a k-uniform hypergraph $H = (V, E)$ is any function $c : V \to [r]$ ($[r] := \{1, \dots, r\}$) such that $(\forall e \in E) |\{c(v) : v \in e\}| = k$ (i.e the vertices of every edge receive pairwise different colours).

An *achromatic r-colouring* of a k-uniform hypergraph $H = (V, E)$ is any of its strong colourings $c : V \to [r]$ satisfying

$$(\forall B \in \binom{[r]}{k})(\exists e \in E)\{c(v) : v \in e\} = B$$

(i.e. every set of k colours appears on some edge). In other words an achromatic r-colouring of a hypergraph $H = (V, E)$ is a partition of its vertex set into r sets $V = V_1 \cup \ldots \cup V_r$ such that the union of less than k sets of $V_1, \ldots, V_r$ is always an independent set (i.e. it does not contain any edge of H) and the union of every k sets is not an independent set.

Every graph G has an achromatic colouring (e.g. every proper $\chi(G)$-colouring is achromatic). However, in the case of $k > 2$ it is not true any longer. There are uniform hypergraphs which do not admit an achromatic colouring. Moreover in the case of many natural hypergraph classes it is a hard task to decide if a given hypergraph can be coloured in an achromatic way. In this talk we consider some classes of the hypergraphs which do not have any achromatic colouring, which do have such a colouring and for which the problem of deciding if a given hypergraph has an achromatic colouring is NP-complete.

Hamilton cycles in cubic maps

Roman Nedela

(joint work with Michal Kotrbčík and Martin Škoviera)

By Jordan theorem a hamilton cycle in a spherical map M separates the set F of faces of M into two disjoint connected subsets $F = F_1 \cup F_2$ such that the corresponding sets of vertices F_1^* and F_2^* induce trees in the dual map M^*. The other implication holds as well, namely, if the vertex-set of the dual map M^* decomposes into two disjoint induced trees, then M has a hamilton cycle. When one consisders hamilton cycles in polytopal (or circular) maps on closed surfaces (both orientable, or non-orientable) a hamilton cycle can be either contractible, or bounding (separating) but non-contractible, or it is not bounding (non-separating). It is worth to metion, that in the non-spherical case, the problem of existence of a Hamilton cycle of a given topological type is not the same as the problem of existence of a Hamilton cycle in the underlying graph. For instance, consider the hexagonal embedding of the 3-cube in the torus. While the graph itself is known to be hamiltonian, none of the Hamilton cycles is contractible in the prescribed embedding.

In the talk we first give a general characterisation of the hamiltonicity of maps of the first two types in terms of vertex decompositions of the vertex-sets of the dual graphs. We restrict our attention to a particular, but important case when the underlying graph of the map is cubic. Secondly, we show how the general methods can be applied to derive new results. Our research is motivated by a recent progress done by Glover, Marušič, Kutnar, and Malnič in a solution of particular case of the "Lovász problem" stating that the finite 2-generator Cayley graphs coming from presentations of the form $\langle x, y \mid y^2 = (xy)^3 = 1, \ldots \rangle$ are hamiltonian.

Further motivation comes from a recent result by Kardoš establishing that cubic spherical maps with faces of size at most are hamiltonian (Barnette conjecture). The crucial idea is based on the fact, that under some additional assumptions we can guarantee the existence of a "proper" vertex decomposition of the dual map, which proves the existence of a Hamilton cycle in the original map. An "effective" general solution of the problem of existence Hamilton cycles is hopeless, since it is known that even in the cubic planar case, the problem is NP-complete.

Hamiltonicity of graphs on surfaces

Kenta Ozeki

(joint work with Ken-ichi Kawarabayashi)

Tutte [2] showed that "every 4-connected planar graph is Hamiltonian", and Thomassen [1] extended it, showing that "every 4-connected planar graph is Hamiltonian-connected", i.e., there is a Hamiltonian path connecting any two prescribed vertices. From those results, several improvements have been shown: for example, properties stronger than Hamiltonian-connectedness, Hamiltonicity of graphs on non-spherical surfaces, and so on. In this talk, I will give a survey on recent results, and I also would like to give a basic strategy to prove them.

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Optimal pebbling number of grids

László F. Papp

(joint work with Ervin Győri and Gyula Y. Katona)

Graph pebbling is a one person mathematical game. Starting with a pebble distribution on the vertices of a simple connected graph, a *pebbling move* removes two pebbles from a vertex and adds one pebble at an adjacent vertex. A vertex is called *reachable* if a pebble can be moved to that vertex using pebbling moves.

$\pi_{\text{opt}}(G)$ is the *Optimal pebbling number* of graph G, which is the minimum number of pebbles that one can distribute on the vertices such that any vertex of G is reachable with a sequence of pebbling moves.

Optimal pebbling number of the square grid $P_n \Box P_m$ is investigated and the exact values of $\pi_{\text{opt}}(P_n \Box P_2)$ and $\pi_{\text{opt}}(P_n \Box P_3)$ are determined in [1] and [2]. In [2] the authors gave a construction showing that $\pi_{\text{opt}}(P_n \Box P_m) \leq \frac{4}{13}nm + O(n+m)$.

We make a better construction for $P_n \square P_m$ which gives $\frac{2}{7}nm + O(n+m)$ as an upper bound. We show for large grids that $\pi_{\text{opt}}(P_n \square P_m) \geq \frac{2}{13}nm + O(n+m)$. The method giving this result yields lower bounds for several other graphs too. Finally, we determine the optimal pebbling number of some vertex-induced subgraphs of the square grid.

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Locally irregular graph colourings and labellings

Jakub Przybyło

How to define an irregular graph? This very basic question was posed and exploited in 1988 as a title of a paper by Chartrand, Erdős and Oellermann. The confusion originates from the well known fact that no antonym of a regular graph, understood as a graph whose all vertices have pairwise distinct degrees, exists, except for the trivial 1-vertex case. This limitation does not concern multigraphs though. Consequently, the following extension of these research was developed as an attempt of designing a graph invariant measuring the level of 'irregularity' of a graph. Suppose that given a simple graph $G = (V, E)$ we are allowed to multiply some of its edges. How small can be the least k so that we are able to construct an *irregular multigraph* of G, i.e., a multigraph with pairwise distinct vertex degrees, using at most k copies of every edge? This value was named the *irregularity strength* of G, and denoted by $s(G)$. Alternatively, one may consider a (colouring) function $c : E \to \{1, 2, \ldots, k\}$, assigning every edge an integer corresponding to its multiplicity in a desired multigraph. The least k so that such colouring exists attributing every vertex of G a distinct sum of incident colours is then equal to $s(G)$.

This issue was a cornerstone of many other combinatorial questions and colouring problems including e.g. 1–2–3 Conjecture and Zhang's Conjecture, as well as some problems of a more structural flavour, like graph decompositions into locally irregular subgraphs, or complexity problems concerning these.

As appeared this field also constitutes a natural environment for nice applications of the probabilistic method, and provides some observations on random graphs themselves. A few of its consequential results reach far beyond this particular branch of graph theory.

A number of key questions of the field shall be presented during the talk, accompanied by representative results concerning these.

Perfect matchings of regular bipartite graphs

Edita Rollová

(joint work with Robert Lukoťka)

Let G be a graph and $X \subseteq E(G)$ be a chosen set of edges. We say that a set $Y \subseteq E(G)$ is equivalent to X if the symmetric difference of X and Y is an edge cut of G. The set X is minimal if there is no equivalent set with fewer elements.

In this talk we show that if G is regular and bipartite and X is minimal, then there exists a perfect matching of G containing no edge of X. Moreover, if X is not equivalent to $\emptyset$, then there exists a perfect matching containing one preselected edge of X. Furthermore, if G is cubic, then there exists a perfect matching of G containing exactly two preselected edges of X.

Characterization of graphs with exclusive sum labeling

Zdeněk Ryjáček

(joint work with Mirka Miller and Joe Ryan)

A *sum labeling* of a (simple undirected) graph G is a one-to-one mapping L of $V(G)$ onto a set of positive integers S such that, for $x, y \in V(G)$, $xy \in E(G)$ if and only if $L(x) + L(y) = L(z)$ for some $z \in V(G)$. It is easy to observe that a graph having a sum labeling must have at least one isolated vertex. A sum labeling is *exclusive* if the elements of S that are the sum of two other elements of S label a collection of isolated vertices associated with G. More formally, for $k \geq 1$, a graph G has a *k-exclusive sum labeling* (abbreviated k-ESL), if the graph $\overline{G} = G \cup \overline{K_k}$ has a sum labeling L such that, for any $x, y \in V(\overline{G})$, $xy \in E(\overline{G})$ if and only if $L(x) + L(y) = L(z)$ for some $z \in V(\overline{G}) \setminus V(G)$. Obviously, if G has a k-ESL, then necessarily $k \geq \Delta(G)$.

We first observe that the property of having a k-ESL is an induced hereditary property, i.e., if a graph G has a k-ESL for some $k \geq 1$, then so does every its induced subgraph. Using this fact, we provide a characterization of graphs having a k-ESL, for any $k \geq 1$, in terms of describing a universal graph for the property, i.e., a graph H_k such that a graph G has a k-ESL if and only if G is an induced subgraph od H_k.

Sequences with radius k for graphs

Paweł Rzążewski

(joint work with Michał Dębski and Zbigniew Lonc)

The notion of k-radius sequences was introduced by Jaromczyk and Lonc [1]. They were motivated by the following problem that appears in transmission of large data sets. A collection of n huge objects (such as medical images) is stored in a remote database. The objects have to be downloaded to a local memory to be compared pairwise. However, the size of the local memory is limited and it can store only at most $k+1$ objects at the same time. The problem is to desing a strategy minimizing the total number of download operations. Among other results, Jaromczyk and Lonc proved that the FIFO strategy for this problem is asymptotically optimal.

We say that a sequence of elements from a ground set A is a *k-radius sequence* (or, alternatively, has the *k-radius property*) if every two elements of A appear somewhere in the sequence at distance at most k. Note that the shortest possible k-radius sequence corresponds to an optimal FIFO strategy for the abovementioned data transmission problem.

In this talk we consider a generalization of k-radius sequences for graphs. We say that a sequence of vertices of a graph G has the k-radius property if every pair of vertices adjacent in G appears in this sequence in distance at most k. We are interested in finding the shortest sequence with radius k for a given graph.

We show that finding the length of the shortest k-radius sequence for a given graph is NP-hard for every $k \geq 2$. Moreover, we determine the length of asymptotically shortest k-radius sequence for a complete graph $K_{n,m}$ (provided that both n and m are sufficiently large). This required solving some interesting combinatorial problem concerning de Brujin graphs.

References

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Chromatic number of P_5-free graphs: χ-bounding functions

Ingo Schiermeyer

In this talk we study the chromatic number of P_5-free graphs. Gyárfas has shown the following theorem:

Theorem. Let G be a P_k-free graph for $k \geq 4$ with clique number $\omega(G) \geq 2$. Then $\chi(G) \leq (k-1)^{\omega(G)-1}$.

and has posed the following question:

Question. Is there a polynomial (χ-bounding) function f_k for $k \geq 5$ such that every P_k-free graph G satisfies $\chi(G) \leq f_k(\omega(G))$?

We will show that there is a polynomial χ-bounding function for several subclasses of P_5-free graphs. Our main result is the following.

Theorem. Let G be a P_5-free graph of order n and clique number $\omega(G)$. If G is
(i) $Claw$-free or
(ii) Paw-free or
(iii) $Diamond$-free or
(iv) $Dart$-free or
(v) $Cricket$-free or
(vi) Gem^+-free,
then $\chi(G) \leq \omega^2(G)$.

Here Gem^+ denotes the graph $(K_1 + (K_1 \cup P_4))$.

Light graphs in planar graphs of large girth

Pavol Široczki

A graph H is defined to be light in a graph family $\mathcal{G}$ if there exist finite numbers $\varphi(H, \mathcal{G})$ and $w(H, \mathcal{G})$ such that each $G \in \mathcal{G}$ which contains H as a subgraph, also contains its isomorphic copy K with

$$\Delta_G(K) \leq \varphi(H, \mathcal{G}) \quad \text{and} \quad \sum_{x \in V(K)} \deg_G(x) \leq w(H, \mathcal{G}).$$

In this contribution, we analyze light graphs in families of plane graphs of minimum degree 2 with prescribed girth and no adjacent 2-vertices, specifying several necessary conditions for their lightness and providing sharp bounds on φ and w for light $K_{1,3}$ and C_{10}.

Decomposition of flows on signed graphs into characteristic flows

Martin Škoviera
(joint work with Edita Máčajová)

We generalise to signed graphs a classical result of Tutte (1956) stating that every integer flow can be decomposed into a linear combination of characteristic flows of directed circuits. In our generalisation, the rôle of circuits is taken over by signed circuits of a signed graph which occur in two types – either balanced circuits or pairs of disjoint unbalanced circuits connected with a path intersecting them only at its ends. Furthermore, for the decomposition to be possible we need to allow 1/2 as a flow value, otherwise certain flows could not be decomposed. As an application of this result we show that a signed graph G admitting a nowhere-zero k-flow has a covering with signed circuits of total length at most $2(k-1)|E(G)|$.

Nonrepetitive vertex-colourings of products of graphs

Erika Škrabuľáková

In 1914 Felix Hausdorf defined the lexicographic product of graphs, that is beside the Cartesian and the strong product of graphs one of the best known binary operations on graphs. Recently some values of Thue vertex-colouring paramethers were determined also for these structures.

Let G be a simple graph and let φ be a proper colouring of its vertices, $\varphi : V(G) \to \{1, \dots, k\}$. We say that φ is *non-repetitive* if for any simple path on vertices $v_1, \dots, v_{2n}$ in G the associated sequence of colours $\varphi(v_1), \dots, \varphi(v_{2n})$ is not a repetition. The minimum number of colours in a non-repetitive colouring of a graph G is the *Thue chromatic number* $\pi(G)$. For the case of list-colourings let the *Thue choice number* $\pi_l(G)$ of a graph G denotes the smallest integer k such that for every list assignment $L : V(G) \to 2^{\mathbb{N}}$ with minimum list length at least k, there is a colouring of vertices of G from the assigned lists such that the sequence of vertex colours of no path in G forms a repetition.

Here we give some upper and lower bounds of the Thue chromatic number and the Thue choice number of products of graphs and discuss tightness of the bounds.

Incidence coloring of graphs

Éric Sopena

An *incidence* in a graph $G = (V, E)$ is a pair (u, uv) with $u \in V$ and $uv \in E$. Two distinct incidences (u, uv) and (w, wx) are *adjacent* if (i) $u = w$, (ii) $w = v$, or (iii) $u = x$. If we denote by G^s the graph obtained from G by subdividing once every edge of G, then incidences in G are in one-to-one correspondence with edges of G^s and two incidences in G are adjacent if and only if their corresponding edges in G^s are at distance at most 2.

An *incidence coloring* of G is a mapping that assigns colors to incidences in G in such a way that adjacent incidences get distinct colors. An incidence coloring of G thus corresponds to a strong edge coloring of G^s. The *incidence chromatic number* of G, denoted $\chi_i(G)$, is then defined as the smallest number of colors required for an incidence coloring of G.

Incidence colorings have been introduced by Brualdi and Massey in [1]. In this talk, we will survey the main results on incidence colorings (see e.g. [2] for an online survey) and propose some open problems.

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Bounds on the oriented diameter of graphs

Michel Surmacs
(joint work with Peter Dankelmann and Yubao Guo)

In 1939, Robbins [3] – inspired by an application in traffic control – gave the classical result that a graph permits a strong orientation, if and only if it is bridgeless/2-edge-connected. The practical application of his result, in particular, naturally gives rise to the problem of finding such a strong orientation of smallest diameter. While, in 1978, Chvátal and Thomassen [1] showed that the determination of the oriented diameter – i.e., the smallest diameter of a strong orientation – of a given bridgeless graph is NP-complete, over the last decades, the oriented diameter of several classes of graphs has been considered and bounds with respect to certain graph invariants were found. See [2] for a survey. In this talk, we mainly focus on some new bounds with respect to the maximum vertex and maximum edge degree, such as: Every bridgeless graph of order n and maximum vertex degree Δ has an orientation D with $\mathrm{diam}(D) \leq n - \Delta + 3$. We provide examples that show that our bounds are sharp and a polynomial time algorithm to find such an orientation.

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On cordial hypertrees

Michał Tuczyński
(joint work with Przemysław Wenus and Krzysztof Węsek)

Let $H = (V, E)$ be a hypergraph. A vertex labeling of H (with elements from $\mathbb{Z}_k$) is a function $f : V \to \mathbb{Z}_k$. A vertex labeling f induces an edge labeling (also denoted by f) $f : E \to \mathbb{Z}_k$ defined by $f(e) = \sum_{v \in e} f(v)$. A labeling is k-cordial if every element of $\mathbb{Z}_k$ is a label of exactly $\lfloor \frac{|V|}{k} \rfloor$ or $\lceil \frac{|V|}{k} \rceil$ vertices and exactly $\lfloor \frac{|E|}{k} \rfloor$ or $\lceil \frac{|E|}{k} \rceil$ edges. A hypergraph is called k-cordial if it admits a k-cordial labeling.

Cichacz, Görlich and Tuza [2] conjectured that all hypertrees (connected hypergraphs without cycles) are k-cordial for all k. We prove the conjecture for $k = 2, 3, 4$. These results generalize results on cordial labelins of graphs: Cahit's theorem [1] which states that every tree is 2-cordial and Hovey's theorem [3] which states that every tree is k-cordial for $k = 3, 4$. We also prove that every loose hypergraph (a hypergraph such that its every edge contains a vertex of degree 1) is 2-cordial.

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Tropical domination in graphs

Zsolt Tuza

(joint work with Jean-Alexandre Anglès d'Auriac, Csilla Bujtás, Hakim El Maftouhi, Marek Karpinski, Yannis Manoussakis, Leandro Montero, N. Narayanan, Laurent Rosaz, and Johan Thapper)

Let G be a vertex-colored graph. A *tropical dominating set* is a subset $D \subseteq V(G)$ such that every vertex in $V(G) \setminus D$ has at least one neighbor in D, moreover every color of the vertices appears at least once in D. We study tropical dominating sets of minimum cardinality, from structural and algorithmic points of view. These include upper bounds related to various parameters of the graph, NP-completeness, approximability and inapproximability results for general and restricted classes of graphs, and fixed-parameter tractability on interval graphs, parameterized by the number of colors.

On the number of hamiltonian cycles in triangulations with few separating triangles

Nico Van Cleemput

(joint work with Gunnar Brinkmann and Jasper Souffriau)

In 1931 Whitney proved that each triangulation containing no "separating triangles" – that is: no cycle of length 3 such that there are vertices inside as well as outside of the cycle – is hamiltonian [1]. For triangulations with at least 5 vertices this condition is equivalent to being 4-connected. One way how this classical result can be improved is to use the same prerequisites but prove a stronger lower bound for the number of cycles. The strongest result about the number of hamiltonian cycles so far was due to Hakimi, Schmeichel and Thomassen [2]. They prove that in a 4-connected triangulation with n vertices there are at least $n/(\log_2 n)$ different hamiltonian cycles.

We introduce a new abstract counting technique for hamiltonian cycles in general graphs. This technique is based on a set of subgraphs, their overlap with the hamiltonian cycles and a switching function. Using this technique for plane triangulations and the same subgraphs as Hakimi, Schmeichel and Thomassen used for their counting argument, we improved their bound to $\Omega(n)$. Using different types of subgraphs we were able to further improve the multiplicative and additive constants. We also show that in case of plane triangulations with one separating triangle there is still a linear number of hamiltonian cycles, and give computational results showing that their conjectured optimal value of $2n^2-12n+16$ holds up to $n = 25$.

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Rainbow triangles in 3-edge-colored graphs

Jan Volec

(joint work with Józef Balogh, Ping Hu, Bernard Lidický, Florian Pfender, and Michael Young)

We show that the maximum number of rainbow triangles in large 3-edge-colored graphs is attained by the following construction: take a blow-up of the properly 3-edge-colored complete graph on four vertices, where the sizes of every two blobs differ by at most 1, and inside every blob B place an extremal construction for $v(B)$ vertices. In particular, this implies that the maximum density of rainbow triangles in 3-edge-colored graphs is asymptotically equal to $\frac{2}{5}$. This question was originally raised by Erdős and Sós.

On edge-colorings of graphs with local constrains

Michaela Vrbjarová

(joint work with Stanislav Jendrol')

An *r-maximum k-edge-coloring* of graph G is an edge coloring of G using k colors such that for every vertex v of degree $d_G(v) = d$, $d \geq r$, the maximum color, that is present at vertex v, occurs at v exactly r times. The minimum number k of colors needed for an r-maximum k-edge-coloring of graph G is *r-maximum index*, denoted $\chi'_r(G)$. We show that $\chi'_r(G) \leq 3$ for any nontrivial connected graph G and $r = 1$ or 2. The bound 3 is tight. All connected graphs G with $\chi'_1(G) = i, i = 1, 2, 3$, are characterized. The precise value of the r-maximum index, $r \geq 1$, is determined for trees and complete graphs. For every $r \geq 3$, we show the existence of graphs for which no r-maximum edge coloring is defined.

On the Jacobsthal numbers in graphs

Iwona Włoch

(joint work with Anetta Szynal-Liana and Andrzej Włoch)

Let $n \geq 0$, $t \geq 1$ be integers. The nth generalized Jacobsthal number $J_{t,n}$ is defined recursively as follows

$$J_{t,n} = J_{t,n-1} + t \cdot J_{t,n-2}, \text{ for } n \geq 2$$

with initial conditions $J_{t,0} = 0$ and $J_{t,1} = 1$. It is interesting to note that $J_{t,n}$ generalizes the Fibonacci numbers and the Jacobsthal numbers, simultaneously. If $t = 1$ then $J_{1,n} = F_{n+1}$ and for $t = 2$ holds $J_{2,n} = J_n$. In the talk we give the graph interpretation of the Jacobsthal numbers and their generalization. This interpretation relates to the number of k-independent sets in the join of graphs.

Edge-shade-colouring of graphs

Małgorzata Wołowiec-Musiał

(joint work with Urszula Bednarz and Iwona Włoch)

In the talk we consider a new kind of edge-colouring of graphs that we call edge-shade-colouring and we present its connections with a recurrence relation of the form

$$a_n = b_1 a_{n-1} + b_2 a_{n-2} + \ldots + b_k a_{n-k},$$

where $b_1, b_2, \ldots, b_k$ are any non-negative integers and $a_0, a_1, \ldots, a_{k-1}$ are given integers. We also show that edge-shade-colouring of graphs allows to give a graph interpretation for almost all Fibonacci type numbers.

On non-hamiltonian graphs for which every vertex-deleted subgraph is traceable

Carol T. Zamfirescu

Let G be a non-hamiltonian graph such that for any vertex v the graph $G - v$ is traceable. We will call G a *platypus*. In 2012, Kenta Ozeki proposed the study of platypuses. Ozeki observed that every hypohamiltonian and every hypotraceable graph is a platypus, but there exist platypuses which are neither hypohamiltonian nor hypotraceable. In this talk we present properties of the platypus, investigate links to other families of graphs, and discuss connections to problems recently raised by Gábor Wiener concerning the minimum leaf number.

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