Dear Participant,

welcome to the Twenty-fifth Workshop Cycles and Colourings. Except for the first workshop in the Slovak Paradise (Čingov 1992), the remaining twenty three workshops took place in the High Tatras (Nový Smokovec 1993, Stará Lesná 1994–2003, Tatranská Štrba 2004–2010, Nový Smokovec 2011–2015).

The series of C&C workshops is organised by combinatorial groups of Košice and Ilmenau. Apart of dozens of excellent invited lectures and hundreds of contributed talks, the scientific outcome of our meetings is represented also by special issues of journals Tatra Mountains Mathematical Publications and Discrete Mathematics (TMMP 1994, 1997, DM 1999, 2001, 2003, 2006, 2008, 2013).

The scientific programme of the workshop consists of 50 minute lectures of invited speakers and of 20 minute contributed talks. This booklet contains abstracts as were sent to us by the authors.

Invited speakers:

Maria Axenovich	Karlsruhe Institute of Technology, Karlsruhe, Germany
Andreas Brandstädt	University of Rostock, Rostock, Germany
Hal Kierstead	Arizona State University, Tempe, Arizona, USA
Bernard Lidický	Iowa State University, Ames, Iowa, USA
Piotr Micek	Jagellonian University, Kraków, Poland
Benny Sudakov	ETH Zürich, Zürich, Switzerland
Bjarne Toft	University of Southern Denmark, Odense, Denmark

Have a pleasant and successful stay in Nový Smokovec.

Organising Committee:

Igor Fabrici
František Kardoš
Tomáš Madaras
Martina Mockovčiaková
Roman Soták

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A classification of the corona product of any tree with some graphs based on its f-chromatic index

Adiwijaya
(joint work with Bayu Erfianto and Maman Abdurohman)

Let $G(V, E)$ be a finite and simple graph and let f be a function from V to a positive integer set. An *f-coloring* of G is a generalized edge-coloring such that every vertex $v \in V$ has at most $f(v)$ edges colored with a same color. The minimum number of colors needed to define an f-coloring of G is called an *f-chromatic index* of G, denoted by $\chi'_f(G)$. Based on f-chromatic index, a graph G can be either in the $C_f 1$ or $C_f 2$. In this paper, we provide a classification of the corona product of any tree with some graphs based on its f-chromatic index.

References

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Cycles and colorings of hypergraphs and ordered graphs

Maria Axenovich
(joint work with Annette Karrer, Jonathan Rollin, and Torsten Ueckerdt)

A result of Erdős and Hajnal implies that forbidding a cycle of a fixed length does not necessarily bound the chromatic number of a hypergraph. On the other hand, forbidding a fixed forest in a graph forces the chromatic number of the graph to be bounded.

Here we show that the situation is drastically different for ordered graphs. While forbidding cycles still does not force the chromatic number to be bounded, we show that there are ordered forests, more precisely ordered paths, forbidding which also does not bound the chromatic number. On the other hand, we strengthen the Erdős-Hajnal theorem by finding hypergraphs of arbitrarily high girth that have either monochromatic or rainbow hyperedges in any vertex-coloring.

On extremal $(A, 2B)$-edge coloured trees

Urszula Bednarz

(joint work with Małgorzata Wołowiec - Musiał)

In the talk we consider a special kind of edge-shade colouring in graphs namely $(A, 2B)$-edge colouring. We present the succesive extremal graphs in the class of trees with respect to the number of all $(A, 2B)$-edge colourings.

Linear polyomino achievement

Jens-P. Bode

(joint work with Christian Löwenstein, Dirk Meierling,
Robert Scheidweiler, and Eberhard Triesch)

For a given set $P = \{p_1, \ldots, p_n\}$ of integers the following achievement game will be considered. Two players A (first move) and B alternatingly color the integers. Player A wins if he achieves a copy of P (that is $\{p_1 + k, \ldots, p_n + k\}$ or $\{k - p_n, \ldots, k - p_1\}$ for an integer k) in his color and B wins otherwise. The polyomino P is called a winner if there exists a winning strategy for A. Otherwise there exists a strategy for B to prevent A from winning and then P is called a loser.

Graph metrics and crossing numbers

Thomas Böhme

(joint work with Steffen Fischer)

A connected graph $G = (V, E)$ can be embedded into a metric space (M, d_M) with *distortion* $c \geq 1$ if there is an injection $f : V \to M$ such that

$$\frac{1}{c} \leq \frac{d_M(f(x), f(y))}{d_G(x, y)} \leq c$$

for all distinct vertices $x, y \in V$. (Here $d_G(x, y)$ denotes the graph theoretical distance of x and y in G, i.e. $d_G(x, y)$ is the number of edges of a shortest x-y-path in G.) Classes of graphs that can be embedded with bounded distortion into metric spaces (esp. the euclidean plane) were first considered in [1]. The topic of the talk is the relationship between bounded distortion embeddings of a graph and its crossing number.

References

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On interval colourings of some graphs

Marta Borowiecka-Olszewska

Many problems concerning arranging tasks and creating schedules which do not allow any pauses in work may be solved by the construction of a consecutive graph colouring. Such a colouring is defined as a proper edge colouring of a graph with natural numbers in which the colours of edges incident with each vertex form an interval of integers. The idea of this colouring called an *interval colouring* of a graph was first introduced in 1987 by Asratian and Kamalian [1]. The interval colouring of a graph was also investigated e.g. by Giaro and Kubale in [2] under the name of a *consecutive colouring*. There are many papers dealing with this topic but most of them concern bipartite graphs. In [3] Petrosyan considered the interval colourings of some product of graphs. In 1990 Sevastjanov showed that the problem to verify the existence of an interval colouring of a given graph is NP-complete even in a class of bipartite graphs.

In the talk we focus our attention on the interval colourings of some classes of graphs and some products of graphs in connection with 1-factorable graphs.

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On some graph classes based on cycle properties

Andreas Brandstädt

Many graph classes are based on cycle properties; typical examples are trees, bipartite graphs, perfect graphs, chordal graphs, split graphs, threshold graphs, weakly chordal graphs, Meyniel graphs, parity graphs, Gallai graphs, block graphs, distance-hereditary graphs, ptolemaic graphs, strongly chordal graphs and chordal bipartite graphs.

An important subclass of strongly chordal graphs are *leaf powers* introduced by Nishimura, Ragde and Thilikos; it has its background and motivation in computational biology and phylogeny: For an integer k, a tree T is a *k-leaf root* of a finite simple undirected graph $G = (V, E)$ if the set of leaves of T is the vertex set V of G and for any two vertices $x, y \in V$, $x \neq y$, $xy \in E$ if and only if $dist_T(x, y) \leq k$. Then graph G is a *k-leaf power* if it has a k-leaf root. G is a *leaf power* if it is a k-leaf power for some k. Two important examples are the classes of 3-leaf powers and of 4-leaf powers, which are characterized by cycle properties.

Finally we consider some cycle properties which lead to an efficient solution of the Maximum Independent Set (MIS) problem on some subclasses of odd-hole-free graphs. It is well known that MIS is NP-complete for triangle-free graphs and solvable in polynomial time for perfect graphs and for weakly chordal graphs but its complexity is open for hole-free graphs.

Minimum degree conditions for the proper connection number of graphs

Christoph Brause

(joint work with Trung Duy Doan and Ingo Schiermeyer)

An edge-coloured graph G is called *properly connected* if any two vertices are connected by a path whose edges are properly coloured. The *proper connection number* of a graph G, denoted by $pc(G)$, is the smallest number of colours that are needed in order to make G properly connected. In this paper we consider sufficient conditions in terms of the ratio between minimum degree and order of a 2-connected graph G implying that G has proper connection number 2.

Packing colorings of subcubic graphs

Boštjan Brešar

(joint work with Sandi Klavžar, Douglas Rall, and Kirsti Wash)

A *k-packing coloring* of a graph G is a function $c : V(G) \to \{1, \ldots, k\}$ such that if $c(u) = c(v) = i$, then $d(u, v) > i$, where $d(u, v)$ is the usual shortest-path distance between u and v (in other words, the vertices colored by color i form an i-packing in G). The *packing chromatic number* $\chi_\rho(G)$ of a graph G is the smallest integer k such that there exists a k-packing coloring of G. The invariant was introduced in [4] under a different name, and was later studied in a number of papers. In particular, it was shown in [2] that determining the packing chromatic number is NP-complete even when restricted to trees.

In the seminal paper [4] the following problem was posed: does there exist an absolute constant M, such that $\chi_\rho(G) \leq M$ holds for any subcubic graph G. (Recall that a graph is *subcubic*, if its largest degree is bounded by 3.) This problem led to a lot of research but remains unsolved at the present. For instance, it has been shown that the packing chromatic number of any subgraph of the hexagonal lattice is bounded by 7, and that the same bound holds for subcubic trees. Recently, Gastineau and Togni [3] found a cubic graph with packing chromatic number equal to 13 and posed an open problem whether there exists a cubic graph with packing chromatic number larger than 13.

In this talk, we consider the packing chromatic number in the class of subcubic graphs. It is shown that the packing chromatic number in the (subcubic) family of base-3 Sierpiński graphs is bounded from above by 9 [1]. On the other hand, we give a construction of a cubic graph on 78 vertices with packing chromatic number at least 14. A key technique in the related proof is edge subdivision. We hence give a closer look at this operation with respect to its effect on the packing chromatic number. We also present the effect on the packing chromatic number of some other standard local operations in graphs, such as vertex- and edge-deletion.

References

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Upper bound on the game total domination number

Csilla Bujtás

The total version of the domination game was introduced by Henning, Klavžar, and Rall [1]. This is a two-person competitive optimization game, where the players, Dominator and Staller, alternately select vertices of a graph G. Each vertex chosen must strictly increase the number of vertices totally dominated. This process eventually produces a total dominating set D of G. Dominator wishes to minimize the number of vertices chosen in the game, while Staller wishes to maximize it. The game total domination number of G, $\gamma_{\rm tg}(G)$, is the number of vertices chosen when Dominator starts the game and both players play optimally.

A general bound on the game total domination number was established in [2] where it is shown that if G is a graph on n vertices in which every component contains at least three vertices, then $\gamma_{\rm tg}(G) \le \frac{4}{5}n$. In the same paper [2], the authors posted the conjecture which states that the sharp upper bound is $\frac{3}{4}n$. Here, we take a step forward and prove that $\gamma_{\rm tg}(G) \le \frac{11}{14}n$ holds for every G which contains no isolated vertices or isolated edges.

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Colouring diamond-free graphs

Konrad Dąbrowski

(joint work with François Dross and Daniël Paulusma)

The Colouring problem is that of deciding, given a graph G and an integer k, whether G admits a (proper) k-colouring. The *diamond* is the graph obtained from a clique on four vertices by removing one edge. For a pair of graphs H_1, H_2, we say that a graph G is (H_1, H_2)-free if G does not contain an induced subgraph isomorphic to H_1 or H_2. For all graphs H up to five vertices, we classify the computational complexity of Colouring for $(\text{diamond}, H)$-free graphs.

Our proof is based on combining known results together with proving that the clique-width is bounded for $(\text{diamond}, P_1 + 2P_2)$-free graphs. Our technique for handling this case is to reduce the graph under consideration to a k-partite graph that has a very specific decomposition. As a by-product of this general technique we are also able to prove boundedness of clique-width for four other new classes of (H_1, H_2)-free graphs.

An extended abstract of the paper containing these results appeared in the proceedings of SWAT 2016 [1]. (This talk will not contain any algorithms and will be accessible to anyone who likes playing with graphs.)

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Strong chromatic index of unit distance graphs

Michał Dębski

The *strong chromatic index* of a graph G, denoted $s'(G)$, is the minimum possible number of colors in a coloring of edges of G such that each color class is an induced matching (or: if edges e and f have the same color, then both vertices of e are not adjacent to any vertex of f).

A graph G is a *unit distance graph in* $\mathbb{R}^n$ if vertices of G can be uniquely indentified with points in $\mathbb{R}^n$ so that uv is an edge of G if and only if the Euclidean distance between the points indentified with u and v is 1.

We try to estimate the largest possible value $s'(G)$, where G is a unit distance graph (in $\mathbb{R}^2$ or $\mathbb{R}^3$) of maximum degree Δ. It is related to the problem posed by Erdős and Nešetřil in 1985 (they conjectured that $s'(G) \leq \frac{5}{4}\Delta^2$ for every graph G, while it is easy to prove that $s'(G) \leq 2\Delta^2$).

We still do not know the correct order of magnitude. We show that $s'(G) \leq c\frac{\Delta^2}{\ln \Delta}$ (where G is a unit distance graph in $\mathbb{R}^3$ of maximum degree Δ). However, some considerations suggest that the correct answer may be much lower, maybe even linear in Δ.

Certified domination

Magda Dettlaff

(joint work with Magdalena Lemańska, Jerzy Topp,
Radosław Ziemann, and Paweł Żyliński)

Imagine that we are given a set D of officials and a set W of civils. For each civil $x \in W$, there must be an official $v \in D$ that can serve x, and whenever any such v is serving x, there must also be another civil $w \in W$ that observes v, that is, w may act as a kind of witness, to avoid any abuse from v. What is the minimum number of officials to guarantee such a service, assuming a given social network?

In this talk, we introduce the concept of certified domination that perfectly models the aforementioned problem. Specifically, a dominating set D of a graph $G = (V_G, E_G)$ is said to be certified if every vertex in D has either zero or at least two neighbours in $V_G \setminus D$. The cardinality of a minimum certified dominating set in G is called the certified domination number of G. We present the exact values of the certified domination number for some classes of graphs as well as provide some upper bounds on this parameter for arbitrary graphs. We then characterise a wide class of graphs with equal domination and certified domination numbers and characterise graphs with large values of certified domination numbers. Next, we examine the effects on the certified domination number when the graph is modified by deleting/adding an edge or a vertex. We also provide Nordhaus-Gaddum type inequalities for the certified domination number. Finally, we show that the (decision) certified domination problem is NP-complete. As a side result, we characterise a wider class of DD_2-graphs, thus generalizing a result of [1].

References

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Proper connection number 2, connectivity and forbidden subgraphs

Trung Duy Doan

(joint work with Christoph Brause and Ingo Schiermeyer)

An edge-coloured graph G is called *properly connected* if any two vertices are connected by a path whose edges are properly coloured. The *proper connection number* of a graph G, denoted by $pc(G)$, is the smallest number of colours that are needed in order to make G properly connected. In this talk we consider sufficient conditions in terms of connectivity and forbidden subgraphs, implying a graph to have proper connection number 2.

References

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$L(2,1)$-labelling of cacti

Anna Dresslerová
(joint work with Michal Forišek)

An $L(2,1)$-labelling is a labelling of the vertex set of graph with non-negative integers such that the labels of adjacent vertices differ by at least two and the labels of vertices at distance 2 are distinct. It is required to determine, for a given graph G, the smallest integer k such that G admits an $L(2,1)$-labelling with integers not exceeding k; this invariant is denoted by $\lambda(G)$. Determining $\lambda(G)$ is known to be a hard problem. To test whether $\lambda(G) \leq k$ is NP-complete even for series-parallel graphs [2]. On the other hand, there exist classes of graphs where this problem is polynomially solvable (e.g. trees or their mild generalisations [1] [3]). In this talk we derive tight upper and lower bounds for the λ-number of cacti. We also present a polynomial-time algorithm which computes the λ-number of an arbitrary cycle-tree (cactus with disjoint cycles).

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Acyclic-sum-list colourings of graphs

Agata Drzystek
(joint work with Ewa Drgas-Burchardt)

Let $\mathcal{D}_1$ be a class of acyclic graphs and G be a graph. A vertex colouring of a graph G is called *acyclic* if the subgraph induced by each colour class is a forest. Let f be a function on the vertex set of a graph G. The graph G is $(f, \mathcal{D}_1)$-*choosable* if for every collection of lists with sizes specified by values of f there is an acyclic colouring of G using colours from the lists. The $\mathcal{D}_1$-*sum-choice-number* of a graph G is the minimum of the sum of sizes in f over all f such that G is $(f, \mathcal{D}_1)$-choosable.

In the talk we shall present some results on $\mathcal{D}_1$-sum-choice-numbers of some classes of graphs, including generalized Petersen graphs and Cartesian products of paths and cycles.

Complete graph immersions and minimum degree

Zdeněk Dvořák
(joint work with Liana Yepremyan)

An *immersion* of a graph H in another graph G is a one-to-one mapping $\varphi : V(H) \to V(G)$ and a collection of edge-disjoint paths in G, one for each edge of H, such that the path P_{uv} corresponding to the edge uv has endpoints $\varphi(u)$ and $\varphi(v)$. The immersion is *strong* if the paths P_{uv} are internally disjoint from $\varphi(V(H))$. We prove that every simple graph of minimum degree at least $11t+7$ contains a strong immersion of the complete graph K_t. This improves on previously known bound of minimum degree at least $200t$ obtained by DeVos et al. [3]. Our result supports a conjecture of Lescure and Meyniel [1] (also independently proposed by Abu-Khzam and Langston [2]), which is the analogue of famous Hadwiger's conjecture for immersions and says that every graph without a K_t-immersion is $(t-1)$-colorable.

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On some of the Thue type graph colouring concepts

Erika Fecková Škrabuľáková

Axel Thue's name is well known in combinatoric. The programming language Thue invented by John Colagioia in early 2000 is named after him. Since 2002 Thue's name appears in graph theory quite often as well. Several graph colouring concepts and related graph colouring parameters came to be called with his name.

Here we give a list of Thue type graph colouring concepts (see e.g. [1] – [7]) and problems, show the differences between them, as well as an overview of some known results in the area. This will be supplemented by several open problems.

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On the H-irregularity strength of graphs

Andrea Feňovčíková

(joint work with Faraha Ashraf, Martin Bača, and Marcela Lascsáková)

An H-*covering* of G is a family of subgraphs $H_1, H_2, \ldots, H_t$, all isomorphic to a given graph H, such that each edge of $E(G)$ belongs to at least one of the subgraphs H_i, $i = 1, 2, \ldots, t$.

Let G be a graph admitting H-covering and let φ be a total k-labeling of G that assigns to vertices and edges of G the numbers from the set $\{1, 2, \ldots, k\}$.For the subgraph $H \subseteq G$ under the total k-labeling φ, we define the associated H-weight as

$$wt_{\varphi}(H) = \sum_{v \in V(H)} \varphi(v) + \sum_{e \in E(H)} \varphi(e).$$

A total k-labeling φ is called to be an H-*irregular total* k-*labeling* of the graph G if for every two different subgraphs H' and H'' isomorphic to H there is $wt_{\varphi}(H') \neq wt_{\varphi}(H'')$. The *total* H-*irregularity strength* of a graph G, denoted $\text{ths}(G, H)$, is the smallest integer k such that G has an H-irregular total k-labeling.

In the talk we will introduce this new graph characteristic. Some estimations on this parameter will be discussed and for some families of graphs we will present the precise values of this parameter.

Game in connected domination on graphs

Anna Fiedorowicz

(joint work with Mieczysław Borowiecki and Elżbieta Sidorowicz)

A new graph game is introduced, namely, a connected domination game on graphs. It is defined analogously to the well known domination game, first studied by Brešar, Klavžar and Rall in 2010 ([2]). For other results concerning the domination game, see for instance [1, 3, 4, 5] and [6].

The game is played by two players, Dominator and Staller, on a connected graph G. The players alternate taking turns choosing a vertex of G (Dominator starts). A move of a player by choosing a vertex v is legal, if the following two conditions are satisfied: the vertex v dominates at least one new vertex of G and the set of all chosen vertices induces a connected subgraph of G. The game ends when none of the players has a legal move (i.e., G is dominated). The aim of Dominator is to finish as soon as possible, the aim of Staller is opposite.

We present some preliminary results concerning this game, as well as bounds and exact values of the corresponding graph parameter — connected game domination number — for some classes of graphs, including outerplanar graphs and Cartesian products of graphs.

We also consider variations of the above described game. Namely, we let one of the players skip her/his move. We give some connections between the number of vertices played in these games and the connected game domination number.

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Graphs with a cycle of length s through an arbitrary chosen edge

Grzegorz Gancarzewicz

We consider only finite graphs without loops and multiple edges. Let $4 \leqslant s \leqslant n$ and let G be a graph of order n. J.A. Bondy and V. Chvátal introduced the notion of stability and they proved that for graph G the property of containing a cycle of length s is $(2n - s)$-stable and that the property of containing a hamiltonian cycle through an arbitrary chosen edge is $(n + 1)$-stable.

For $4 \leqslant s \leqslant n$, we prove that the property of containing a cycle of length s through an arbitrary chosen edge is $(2n - s + 1)$-stable and we prove that if G is a 3-connected graph on n vertices satisfying a Fan type condition:

$$\mathrm{d}\,(x\,,y) = 2 \;\Rightarrow\; \max\{\mathrm{d}\,(x)\,,\ \mathrm{d}\,(y)\} \geqslant \frac{2n - s + 1}{2}\,,$$

for each pair of vertices x and y in $\mathrm{V}\,(G)$, then for every edge $e \in \mathrm{E}\,(G)$, there is a cycle of length s containing e.

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Results on the zero forcing number

Michael Gentner
(joint work with Dieter Rautenbach)

A set Z of vertices of a graph G is a zero forcing set of G if iteratively adding to Z vertices from $V(G) \setminus Z$ that are the unique neighbor in $V(G) \setminus Z$ of some vertex in Z, results in the entire vertex set $V(G)$ of G. The zero forcing number $Z(G)$ of G is the minimum cardinality of a zero forcing set of G.

The zero forcing number was introduced by the AIM Minimum Rank - Special Graphs Work Group [1, 4] as an upper bound on the corank of matrices associated to a given graph. In [2], Davila and Kenter conjecture that for a graph G with girth $g \geq 3$ and minimum degree $\delta \geq 2$,

$$Z(G) \geq (g - 2)(\delta - 2) + 2.$$

They show that for $g > 6$ and sufficiently large δ their conjecture is true. In [3], we showed together with Penso and Souza that the conjecture is true for triangle-free graphs. We will show that it is also true for girth 5 and 6.

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t-hull number of graph products

Tanja Gologranc

(joint work with Polona Repolusk)

A walk W between two non-adjacent vertices in a graph G is called *tolled* if the first vertex of W is among vertices from W adjacent only to the second vertex of W, and the last vertex of W is among vertices from W adjacent only to the second-last vertex of W. In the resulting interval convexity, a set $S \subset V(G)$ is *toll convex* if for any two non-adjacent vertices $x, y \in S$ any vertex in a tolled walk between x and y is also in S. A toll convexity was introduced in [1] as a convexity for which exactly interval graphs are convex geometry. The toll closure $T_G[S]$ of a subset $S \subseteq V(G)$ is defined as the union of toll intervals between all pairs of vertices from S. If $T_G[S] = V(G)$, S is a *toll set* of a graph G. The order of a minimum toll set in G is called the *toll number* of G and is denoted by $tn(G)$. The *t-convex hull* of a set $S \subseteq V(G)$ is the intersection of all t-convex sets that contain S, and is denoted by $[S]_t$. A set S is a *t-hull set* of G if its t-convex hull $[S]_t$ coincides with $V(G)$. The *t-hull number* of G is the size of minimum t-hull set and is denoted by $th(G)$.

In this talk we consider the toll number and the t-hull number of the Cartesian and lexicographic product of two graphs. In both cases we establish formulas that express the exact toll number of $G \Box H$ and $G \circ H$, respectively. In the case of the Cartesian product $tn(G \Box H) = 2$ and consequently $th(G \Box H) = 2$. In the case of the lexicographic product we present some upper bounds for $tn(G \circ H)$

and prove the exact result using a new concept, a toll-dominating triple, the idea for which came from [2].

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Anti-Ramsey number for Hanoi graphs

Izolda Gorgol

(joint work with Anna Lechowska)

A graph is called rainbow if each of its edges has a different color. Anti-Ramsey number $ar(G, H)$ is the maximum number of colors such that we are able to color the edges of a graph G with this number of colors without creating any rainbow copy of H. It was defined in [1] and since then widely studied. The results for a variety of pairs of graphs can be found in [2]. Hanoi graphs H_p^n are the graph theoretical model of well-known Towers of Hanoi puzzle with p pegs and n discs. The vertices of the graph are permissible states of discs on pegs, coded by the appropriate integer sequences, and the two vertices are adjacent if and only if there is a legal move from one state to another. This model was proposed firstly in [4] for classical puzzle with three discs. It occurs that some of graph properties and parameters, such as hamiltonicity, planarity, chromatic number and index, are not difficult to establish for Hanoi graphs in general case. The survey of known results can be found in [3]. In the talk we will consider $ar(H_p^n, H_q^m)$ for various values of p, n, q, m. Among others we will show the exact value for $ar(H_p^n, H_p^m)$, $m \leq n$.

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Group connectivity: $\mathbb{Z}_4$ v. $\mathbb{Z}_2^2$

Radek Hušek

(joint work with Lucie Mohelníková and Robert Šámal)

A *flow* in a digraph $G = (V, E)$ is an assignment of values of some abelian group Γ to edges of G such that Kirchhoff's law is valid at every vertex. We say a flow is *nowhere-zero* if it does not use value 0 at any edge. One of nice properties of nowhere-zero flows discovered by Tutte is the following one:

Theorem 1. (Tutte [2]) *Let Γ be an abelian group with k-elements. Then for every digraph the existence of a nowhere-zero Γ-flow is equivalent with the existence of a nowhere-zero $\mathbb{Z}_k$-flow.*

Jaeger et al. [1] introduced a variant of nowhere-zero flows called *group connectivity*. A digraph $G = (V, E)$ is Γ*-connected* if for every mapping $h : E \to \Gamma$ there is a Γ-flow f on G that satisfies $f(e) \neq h(e)$ for every edge $e \in E$. As we may choose the "forbidden values" $h \equiv 0$, every Γ-connected digraph has a nowhere-zero Γ-flow; however, the converse is false.

Some results on nowhere-zero flows extend to group connectivity. The group connectivity analogy of Theorem 1 motivated following note in Section 3.1 of [1]: "… we do not know of any $\mathbb{Z}_4$-connected graph which is not $\mathbb{Z}_2 \times \mathbb{Z}_2$-connected, or vice versa. Neither can we prove that such graphs do not exist." Our main result is the resolution to this natural question:

Theorem 2. *There is a graph that is $\mathbb{Z}_2^2$-connected but not $\mathbb{Z}_4$-connected, and there is a graph that is $\mathbb{Z}_4$-connected but not $\mathbb{Z}_2^2$-connected.*

Our proof is computer aided. We find certifying graphs by trying subdivisions of random cubic graphs and checking whether they are Γ-connected. To make this fast enough, we devised an algorithm based on enumerating mappings of forbidden values satisfied a fixed flow.

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Exponential independence

Simon Jäger

(joint work with Dieter Rautenbach)

For a set S of vertices of a graph G, a vertex u in $V(G)\setminus S$, and a vertex v in S, let $\text{dist}_{(G,S)}(u,v)$ be the distance of u and v in the graph $G-(S\setminus\{v\})$. Dankelmann et al. [1] define S to be an exponential dominating set of G if $w_{(G,S)}(u)\geq 1$ for every vertex u in $V(G)\setminus S$, where $w_{(G,S)}(u)=\sum\limits_{v\in S}\left(\frac{1}{2}\right)^{\text{dist}_{(G,S)}(u,v)-1}$. Inspired by this notion, we define S to be an exponential independent set of G if $w_{(G,S\setminus\{u\})}(u)<1$ for every vertex u in S, and the exponential independence number $\alpha_e(G)$ of G as the maximum order of an exponential independent set of G.

Similarly as for exponential domination, the non-local nature of exponential independence leads to many interesting effects and challenges. In this talk we show tight bounds for the exponential independence number. Furthermore, we characterize all graphs G for which $\alpha_e(H)$ equals the independence number $\alpha(H)$ for every induced subgraph H of G.

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Counting cycles in graphs of order close to the Moore bound

Robert Jajcay

The excess of a k-regular graph G of girth g is defined to be the difference between the order of G and the well-known Moore bound, and k-regular graphs of girth g and minimal excess are called (k,g)-*cages*. Despite the fact that the Moore bound is widely believed to be a poor predictor of the order of cages, meaningful improvements are hard to come by.

We present a number of formulas for counting cycles of lengths close to the girth in k-regular graphs of girth g and small excess not exceeding 4. Based on these formulas, we are able to exclude the existence of graphs with small excess for infinite families of degree-girth pairs. In particular, we obtain lower bounds for families of even girth and excess 4; the case for which no such results have been previously known.

Overall, however, we observe that counting cycles does not exclude too many families, an observation made previously in the setting of strongly regular graphs by Vašek Chvátal.

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Secure sets in strong grid-like graphs

Marko Jakovac

(joint work with Ismael González Yero and Dorota Kuziak)

Given a graph $G = (V, E)$ and a set of vertices $S \subseteq V$ of G, the set S is a secure set if it can defend every attack of vertices outside of S, according to an appropriate definition of "attack" and "defence". The minimum cardinality of a secure set in G is the security number $s(G)$. In this talk the security number of strong grid-like graphs, which are the strong products of paths and cycles (grids, cylinders and toruses) is obtained.

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Homogeneous colourings of graphs

Mária Janicová

(joint work with Borut Lužar, Tomáš Madaras, and Roman Soták)

A k-homogeneous colouring of a graph G is a proper colouring of vertices of G such that the number of colours in the neighbourhood of any vertex equals k. We define palette of homogeneity of a graph G as a set of all positive integers k for which G admits a k-homogeneous colouring and range of k-homogeneity as a set of all positive integers n such that G admits a k-homogeneous n-colouring. We explore properties of such colourings in general as well as for regular and other particular graphs and completeness of the palette and the range of homogeneity of particular graph families.

Facial list colourings of plane graphs

Stanislav Jendroľ

(joint work with Igor Fabrici and Margit Voigt)

Let $G = (V, E, F)$ be a connected plane graph, with vertex set V, edge set E, and face set F. For $X \in \{V, E, F, V \cup E, V \cup F, E \cup F, V \cup E \cup F\}$, two distinct elements of X are facially adjacent in G if they are incident elements, adjacent vertices, adjacent faces, or facially adjacent edges (edges that are consecutive on the boundary walk of a face of G). A list k-colouring is facial with respect to X if there is a list k-colouring of elements of X such that facially adjacent elements of X receive different colours.

We prove that every plane graph $G = (V, E, F)$ has a facial list 4-colouring with respect to $X = E$, a facial list 6-colouring with respect to $X \in \{V \cup E, E \cup F\}$, and a facial list 8-colouring with respect to $X = V \cup E \cup F$. For plane triangulations, each of these results is improved by one and it is tight. These results complete the theorem of Thomassen that every plane graph has a (facial) list 5-colouring with respect to $X \in \{V, F\}$ and the theorem of Wang and Lih that every simple plane graph has a (facial) list 7-colouring with respect to $X = V \cup F$.

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$L(2,1)$-labeling of unit disk graphs

Konstanty Junosza-Szaniawski

(joint work with Paweł Rzążewski, Joanna Sokół, and Krzysztof Węsek)

$L(2,1)$-labeling is motivated by the problem of assigning frequencies to the transmitters in a radio networks. It asks for a vertex labeling with non-negative integers, such that adjacent vertices get labels that differ by at least two, and vertices at distance two get different labels. The *span* of an $L(2,1)$-labeling is the difference between the maximum and the minimum label used. The $L(2,1)$-*span* of a graph G, denoted by $\lambda(G)$, is the minimum span of an $L(2,1)$-labeling of G (note that the number of available labels is $\lambda(G) + 1$, but some may be not used).

Griggs and Yeh [2] proved that $\lambda(G) \leq \Delta(G)^2 + 2\Delta(G)$ and conjectured that $\lambda(G) \leq \Delta(G)^2$, where $\Delta(G)$ denotes the maximum degree of G. The conjecture initiated intensive research and is still not fully resolved. It is known to be true for many special graph classes and quite recently has been proved for graphs of large maximum degree [3]. Yet it is interesting to note that the Petersen and Hoffmann-Singleton graphs are the only two known graphs that satisfy equality in this bound (for maximum degree greater than 2).

Unit disk intersection graphs in a very natural way model radio networks. Shao et al. [4] showed $\lambda(G) \leq \frac{4}{5}\Delta(G)^2+2\Delta(G)$ if G is unit disk intersection graph. Actually, they gave an online algorithm that finds an $L(2,1)$-labeling of G with span at most $\frac{4}{5}\Delta(G)^2 + 2\Delta(G)$. We improve this bound to $\frac{3}{4}\Delta^2 + 3(\Delta - 1)$ in the offline case. Moreover, we show that the algorithm from [1] implies a linear bound $18\Delta + 18$, which is better for $\Delta \geq 22$. All these results are based on algorithms that require geometric representation as an input. In [1] there is also given robust algorithm for $L(2,1)$-labeling of unit disk graph, which does not use representation. We give more careful analysis of this algorithm reducing competitive ratio from 10.67 to 8.67.

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Breaking graph symmetries by proper colourings

Rafał Kalinowski

(joint work with Wilfried Imrich, Monika Pilśniak, and Mohammad Shekarriz)

We consider proper vertex-, edge-, and total colourings of graphs. In [1], the *distinguishing chromatic number* $\chi_D(G)$ of a graph G was defined as the least number of colours in a proper vertex-colouring that is preserved only by the trivial automorphism. Corresponding invariants, the *distinguishing chromatic index* $\chi'_D(G)$ for edge-colourings, and the *total distinguishing chromatic number* $\chi''_D(G)$ for total colourings, were introduced in [2] and [3], respectively. Upper bounds for $\chi_D(G)$, $\chi'_D(G)$ and $\chi''_D(G)$ in terms of maximum degree $\Delta(G)$ will be discussed, also for infinite graphs.

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Rainbow connection number two and clique number

Arnfried Kemnitz
(joint work with Philipp Krause and Ingo Schiermeyer)

An edge-colored connected graph G is called *rainbow connected* if each two vertices are connected by a path whose edges have different colors. Note that the edge coloring need not be proper. If such a coloring uses k colors then G is called *k-rainbow connected*. The rainbow connection number of G, denoted by $\mathrm{rc}(G)$, is the minimum k such that G is k-rainbow connected.

Some obvious properties of the rainbow connection number of connected graphs G of order n and diameter $\mathrm{diam}(G)$ are

1. $1 \leq \mathrm{rc}(G) \leq n-1$,
2. $\mathrm{rc}(G) \geq \mathrm{diam}(G)$,
3. $\mathrm{rc}(G) = 1$ if and only if G is complete,
4. $\mathrm{rc}(G) = n-1$ if and only if G is a tree.

In general, it is not an easy task to determine the rainbow connection number of a given graph. In fact, it is already NP-complete to decide whether $\mathrm{rc}(G) = 2$.

In this talk we determine all graphs G with rainbow connection number $\mathrm{rc}(G) = 2$ and clique number $n-4 \leq \omega(G) \leq n-1$.

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Disjoint cycles and equitable coloring

Hal Kierstead

I will discuss recent results with Kostochka, McConvey, Molla, and Yeager extending the Corrdi-Hajnal Theorem and the Hajnal-Szemerdi theorem. The goal is not only to strengthen and prove analogs of these theorems in various settings, but also to develop a theory strong enough to attack the Chen-Lih-Wu conjecture that if a t-colorable graph G with maximum degree at most t has no equitable t-coloring, then G contains $K_{t,t}$ and t is odd. Along the way we answer a question of Dirac and strengthen results of Dirac and Erdős from the sixties.

On the crossing numbers of products of wheels

Marián Klešč

The *crossing number* $cr(G)$ of a graph G is the minimum possible number of edge crossings in a drawing of G in the plane. Garey and Johnson have proved that the problem to determine the crossing number of a graph is NP-complete. The crossing numbers of some classes of graphs have been obtained. It was shown by D.J. Kleitman that the crossing number of the complete bipartite graph $K_{m,n}$ is $\lfloor \frac{m}{2} \rfloor \lfloor \frac{m-1}{2} \rfloor \lfloor \frac{n}{2} \rfloor \lfloor \frac{n-1}{2} \rfloor$ for all $m \leq 6$ and all n.

Let G and H be two disjoint graphs. The *join product* of G and H, denoted by $G + H$, is obtained from vertex–disjoint copies of G and H by adding all possible edges between $V(G)$ and $V(H)$. For $|V(G)| = m$ and $|V(H)| = n$, the edge set of $G + H$ is the union of disjoint edge sets of the graphs G, H, and the complete bipartite graph $K_{m,n}$. The Kleitman's result enables us to establish the crossing numbers of several join products.

In the talk, we show that the known values of crossing numbers of suitable join products can be used by estimating crossing numbers of Cartesian products of special graphs. We present the crossing numbers of Cartesian products of wheels and trees.

Graphs minimal with respect to Balaban index

Martin Knor

(joint work with Jaka Kranjc, Riste Škrekovski, and Aleksandra Tepeh)

We consider graphs of order n with the minimal value of Balaban index. This index is defined as

$$J(G) = \frac{m}{m-n+2} \sum_{uv \in E(G)} \frac{1}{\sqrt{w(u) \cdot w(v)}},$$

where the sum is taken over all edges of G and for $x \in V(G)$ we have $w(x) = \sum_{y \in V(G)} dist(x,y)$.

In this talk we show that $J(G) \geq 4/(n-1)$ and when n is large then $J(G) > 8/n + o(n^{-1})$. For small values of n, $n \leq 11$, we determine the extremal graphs. Finally, we show that balanced dumbbell graphs with clique size $\sqrt[4]{\pi/2}\sqrt{n}+o(\sqrt{n})$ have the value of Balaban index about $10.15/n$. Finally, we present a conjecture about the structure of extremal graphs.

On girth of minimal counterexample to 5-flow conjecture

Peter Korcsok
(joint work with Radek Hušek and Robert Šámal)

Given a graph $G = (V, E)$ with fixed (but arbitrary) orientation of the edges and a finite abelian group Γ, we define a *flow* as a mapping $f : E \to \Gamma$ satisfying Kirchhoff's law at each vertex, i.e. for each vertex total in-flow equals total out-flow. A *k-flow* is a flow using only integers strictly between $-k$ and k. Finally, a flow is *nowhere-zero* if no edge has value 0.

In 1954, Tutte [4] conjectured that there exists a nowhere-zero 5-flow for every bridgeless graph. Seymour [3] proved the existence of nowhere-zero 6-flow for every bridgeless graph.

Recently, Kochol [1, 2] studied a hypothetical minimal counterexample to Tutte's 5-Flow Conjecture. He introduced so-called *forbidden networks*, i.e. graphs that cannot be a subgraph of any such counterexample. He also used Tutte's contraction/deletion formula to count flows on a network using a given values on specific edges. Using these counts, he transformed the exclusion of a forbidden subgraph to the problem of equality of vector spaces and proved that such minimal counterexample cannot contain circuits of length at most 10.

We have verified the results of Kochol using an independent implementation of the computations and have improved the best known result.

Theorem. *Every minimal counterexample to the 5-Flow Conjecture has girth at least 12.*

We have also studied modifications of Kochol's approach, e.g. a replacement of the forbidden network by a smaller subgraphs. The aim of these modifications has been a reduction of the size of computed matrices. Furthermore, we have used this method also for larger circuits (lengths up to 15) but it seems not to work anymore.

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Coloring of graphs embedded in the plane

Daniel Kráľ

Problems concerning coloring graphs embedded in the plane have always been among the most intensively studied problems in graph theory. In the talk, we will survey results on classical and cyclic coloring of plane graphs, and we will present some recent results, which have been obtained with various groups of collaborators, on three classical problems in the area: Steinberg's Conjecture from 1976, the Cyclic Coloring Conjecture of Borodin from 1984, and the Cyclic Coloring Conjecture of Plummer and Toft from 1987.

Unique colorability and clique minors

Matthias Kriesell

For a graph G, let $h(G)$ denote the largest k such that G has k pairwise disjoint pairwise adjacent connected nonempty subgraphs, and let $s(G)$ denote the largest k such that G has k pairwise disjoint pairwise adjacent connected subgraphs of size 1 or 2. Hadwiger's conjecture states that $h(G) \leq \chi(G)$, where $\chi(G)$ is the chromatic number. Seymour conjectured $s(G) \geq |V(G)|/2$ for all graphs without antitriangles, i. e. three pairwise nonadjacent vertices. Here we concentrate on graphs with exactly one $\chi(G)$-coloring. We prove generalizations of

(i) if $\chi(G) \leq 6$ and G has exactly one $\chi(G)$-coloring then $h(G) \geq \chi(G)$, where the proof does *not* use the four-color-theorem, and

(ii) if G has no antitriangle and G has exactly one $\chi(G)$-coloring then $s(G) \geq |V(G)|/2$.

Convex dominating-geodetic partitions in graphs

Magdalena Lemańska

(joint work with Ismael Gonzalez Yero)

The distance $d(u, v)$ between two vertices u and v in a connected graph G is the length of a shortest $u-v$ path in G. A $u-v$ path of length $d(u, v)$ is called $u-v$ geodesic. A set X is convex in G if vertices from all $a-b$ geodesics belong to X for every two vertices $a, b \in X$. A set of vertices D is dominating in G if every vertex of $V-D$ has at least one neighbor in D. The convex domination number $\gamma_{con}(G)$ of a graph G equals the minimum cardinality of a convex dominating set in G. A set of vertices S of a graph G is a geodetic set of G if every vertex $v \notin S$ lies on a $x-y$ geodesic between two vertices x, y of S. The minimum cardinality of a geodetic set of G is the geodetic number of G and it is denoted by $g(G)$. Let D, S be a convex dominating set and a geodetic set in G, respectively. The two sets D and S form a convex dominating-geodetic partition of G if $|D| + |S| = |V(G)|$.

Moreover, a convex dominating-geodetic partition of G is called optimal if D is a $\gamma_{con}(G)$-set and S is a $g(G)$-set. We study the (optimal) convex dominating-geodetic partitions of graphs.

3-coloring triangle-free planar graphs

Bernard Lidický

(joint work with Ilkyoo Choi, Jan Ekstein, Zdeněk Dvořák, Přemek Holub, Alexandr Kostochka, and Matthew Yancey)

A well known theorem of Grötzsch states that every planar graph is 3-colorable. We will show a simple proof based on a recent result of Kostochka and Yancey on the number of edges in 4-critical graphs. Then we show strengthening of the Grötzsch's theorem in several different directions.

Shorter signed circuit covers of graphs

Robert Lukoťka

(joint work with Tomáš Kaiser, Edita Máčajová, and Edita Rollová)

A signed circuit is a minimal signed graph (with respect to inclusion) that admits a nowhere-zero flow. We show that each flow-admissible signed graph on m edges can be covered by signed circuits of total length at most $(3 + 2/3) \cdot m$, improving the recent result of Cheng et al. [1]. To obtain this improvement we prove several results on signed circuit covers of trees of Eulerian graphs, which are connected signed graphs such that removing all bridges results in a collection of Eulerian graphs.

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Incidence coloring: the $(\Delta + 2)$-conjecture

Borut Lužar

(joint work with Petr Gregor and Roman Soták)

An *incidence* in a graph G is a pair (v, e) where v is a vertex of G and e is an edge of G incident to v. Two incidences (v, e) and (u, f) are adjacent if at least one of the following holds: (1) $v = u$, (2) $e = f$, or (3) $vu \in \{e, f\}$. An *incidence coloring* of G is a coloring of its incidences assigning distinct colors to adjacent incidences. The originators [1] conjectured that every graph G admits an incidence coloring with at most $\Delta(G) + 2$ colors. The conjecture is false in general [4], but there are many classes of graphs for which it holds. We will present main results from the field and introduce some of our recent ones. Namely, we will focus on incidence coloring of Cartesian products of graphs [2] and subquartic graphs [3].

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Incidence coloring — cold cases

Mária Maceková

(joint work with František Kardoš, Martina Mockovčiaková, Éric Sopena, and Roman Soták)

An *incidence* of an undirected graph G is a pair (v, e) where v is a vertex of G and e an edge of G incident with v. Two incidences (v, e) and (u, f) are adjacent if one of the following holds: (i) $v = u$, (ii) $e = f$ or (iii) $vu = e$ or f. An *incidence coloring* of G is a coloring that assigns distinct colors to adjacent incidences. A corresponding chromatic number is called the *incidence chromatic number* of G, and is denoted by $\chi_i(G)$.

The general upper bound $\chi_i(G) \leq 2\Delta(G)$ was proved by Brualdi and Massey and improved by Guiduli to $\chi_i(G) \leq \Delta(G) + 20 \log \Delta(G) + 84$ for every graph G. Brualdi and Massey conjectured that for every graph G holds $\chi_i(G) \leq \Delta(G) + 2$, but this was disproved by Guiduli. However, this inequality seems to hold for many graph classes.

In this talk we present some results on graphs with prescribed maximum degree and maximum average degree. It is known that every planar graph G with $\Delta(G) \geq 5$ and mad(G) < 3 has the incidence chromatic number at most $\Delta(G)+2$. We obtain the same bound for such graphs with $\Delta(G) = 4$. Moreover, for graphs with $\Delta(G) \geq 8$ and mad(G) $< \frac{10}{3}$ we show that the incidence chromatic number is at most $\Delta(G) + 2$.

It was also proved that at most $\Delta(G) + 5$ colors are enough for an incidence coloring of any planar graph G except $\Delta(G) = 6$; in this case at most 12 colors are needed. We improve the bound for $\Delta(G) = 6$ to 10.

Global defensive structures in graphs

Michał Małafiejski

(joint work with Robert Lewoń, Anna Małafiejska, and Kacper Wereszko)

In the talk we give a survey of our recent results on the minimum global defensive structures: alliances [1, 5], edge alliances [3, 4] and defensive sets [2, 4, 5].

For a given graph G and a subset S of a vertex set of G we define for every subset X of S the predicate $SEC(X) = true$ iff $|N[X] \cap S| \geq |N[X] \setminus S|$ holds, where $N[X]$ is a closed neighbourhood of X in G.

Set S is an *alliance* iff for each vertex $v \in S$ we have $SEC(\{v\}) = true$. If S is also a dominating set of G (i.e., $N[S] = V(G)$), we say that S is a *global alliance*.

Set S is an *edge alliance* iff $G[S]$ has no isolated vertices and for each edge $e = \{v, u\} \in E(G[S])$ we have $SEC(\{v, u\}) = true$. Set S is a *global edge alliance* if it also dominates G.

Set S is a *defensive set* in G iff for each vertex $v \in S$ we have $SEC(\{v\}) = true$ or there exists a neighbour u of v such that $u \in S$ and $SEC(\{v, u\}) = true$. Similarly, if set S is also a dominating set of G, we say that S is a *global defensive set*.

Recently, in [4], the authors proved the upper bound for the edge alliance number for trees, i.e., $\gamma_{ea}(T) \leq 2n/3$, and characterized the class of trees reaching this upper bound. Moreover, in [4] the authors proved the upper bound for the defensive set number for trees, i.e., $\gamma_{ds}(T) \leq \lceil n/2 \rceil$.

In the paper [5] the authors showed the exact formulas for the global alliance number, the global edge alliance number and the global defensive set number for complete k-ary trees, and for complete k-partite graphs.

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Computational complexity of distance edge labeling

Tomáš Masařík
(joint work with Dušan Knop)

The problem of *Distance Edge Labeling* is a variant of *Distance Vertex Labeling* (also known as $L_{2,1}$ labeling) that has been studied for more than twenty years and has many applications, such as frequency assignment.

The *Distance Edge Labeling* problem asks whether the edges of a given graph can be labeled such that the labels of adjacent edges differ by at least two and the labels of edges at distance two differ by at least one. Labels are chosen from the set $\{0, 1, \ldots, \lambda\}$ for λ fixed.

We present a full classification of its computational complexity—a dichotomy between the polynomial-time solvable cases and the remaining cases which are NP-complete. We characterize graphs with $\lambda \leq 4$ which leads to a polynomial-time algorithm recognizing the class and we show NP-completeness for $\lambda \geq 5$ by several reductions from *Monotone Not All Equal 3-SAT*.

Moreover, there is an absolute constant $c > 0$ such that there is no 2^{cn}-time algorithm deciding the *Distance Edge Labeling* problem unless the exponential time hypothesis fails.

This result has been published at the conference IWOCA 2015 [1].

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Coloring and on-line coloring of geometric intersection graphs

Piotr Micek

Combinatorics of geometric intersection graphs is fascinating for both aesthethic and practical reasons. Within this talk we focus on coloring problems. We all know that the chromatic and clique numbers of a graph can be arbitrarily far apart. But what if we insist that our graph has a geometric representation? How large can the chromatic number of an intersection graph of segments in the plane be in terms of its clique number or in terms of the number of segments? What happens if instead of segments we have axis-aligned rectangles. We will discuss recent progress on these type of questions. We are also going to see when one can color effectively geometric objects incoming on-line.

Semistrong edge-coloring of subcubic graphs

Martina Mockovčiaková

A *strong edge-coloring* is a proper edge-coloring in which the edges of each color class form an induced matching. In this talk, we consider an edge-coloring of related type.

A matching M of a graph G is *semistrong* if every edge of M has an end-vertex of degree one in the induced subgraph $G[M]$. A proper edge-coloring of a graph G in which every color class induces a semistrong matching is called *semistrong edge-coloring*. This notion was introduced by Gyárfás and Hubenko in 2005.

We focus our attention to the results regarding subcubic graphs and derive a tight upper bound on the corresponding chromatic index.

Triangle-free planar graphs with the smallest independence number

Jan Musílek
(joint work with Zdeněk Dvořák, Tomáš Masařík, and Ondřej Pangrác)

Steinberg and Tovey [1] proved that every n-vertex planar triangle-free graph has an independent set of size at least $(n+1)/3$, and described an infinite class of tight examples. We show that all n-vertex planar triangle-free graphs except for this one infinite class have independent sets of size at least $(n+2)/3$.

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How “close” a graph to be a super edge-magic graph

Anak Agung G. Ngurah
(joint work with Rinovia Simanjuntak)

A graph G of order p and size q is called *super edge-magic* if there exists a bijection f from $V(G) \cup E(G)$ to $\{1, 2, 3, \cdots, p+q\}$ such that $f(V(G)) = \{1, 2, 3, \cdots, p\}$ and $f(u) + f(uv) + f(v)$ is a constant for every edge uv in $E(G)$. Furthermore, the *super edge-magic deficiency* of a graph G, is either the minimum nonnegative integer n such that $G \cup nK_1$ is super edge-magic or $+\infty$ if there exists no such integer n. This parameter, the super edge-magic deficiency, measure how "close" a graph to be a super edge-magic graph.

In this talk, we present the latest development in this area including the latest development of the super edge-magic deficiency of 2-regular graphs.

Locating-dominating sets in directed square grids

Ľudovít Niepel

(joint work with Mohammed Gebleh)

A set S of vertices of a digraph G is dominating if $\forall u \in V(G)$ $N^-[u] \cap S \neq \emptyset$. Set S is locating if for any two distinct vertices $u, v \in V(G) \setminus S$, $N^-[u] \cap S \neq N^-[v] \cap S$, where $N[u]$ and $N[v]$ are closed in-neighborhoods of vertices u and v. Set S is locating-dominating if it is both dominating and locating. We give a characterization of locating-dominating sets with minimal density in directed infinite n-dimensional square grids $\mathbb{Z}^n$.

A characterization of graphs with disjoint total dominating sets

Iztok Peterin

(joint work with Michael A. Henning)

A set S of vertices in a graph G is a total dominating set of G if every vertex is adjacent to a vertex in S. A fundamental problem in total domination theory in graphs is to determine which graphs have two disjoint total dominating sets. We provide a constructive characterization of the graphs that have two disjoint total dominating sets.

More detailed, every graph whose vertices can be partition into two total dominating sets can be obtain from one of four basic graphs with a finite sequence of 19 (simple) operations.

Computing vertex-disjoint paths using MAOs

Johanna E. Preißer

(joint work with Jens M. Schmidt)

Consider a simple graph G with minimum degree δ and a maximal adjacency ordering (MAO) $<$ of G. Let a subset S of vertices be *k-connected* if G contains k internally vertex-disjoint paths between every two vertices of S. Henzinger proved for every $1 \leq k \leq \delta$ that the last $\delta - k + 2$ vertices of $<$ are k-connected [1]. Nagamochi [2] improved this result to the more general vertex sets of trees derived from the *forest decomposition* of the MAO; one such tree actually contains the vertex set given above. This proof uses the machinery of mixed connectivity, which generalizes both edge- and vertex-connectivity.

However, no algorithm for computing k internally disjoint paths between two such given vertices is known so far that improves the traditional flow-based approaches. We give (the first) algorithm that computes these paths in linear time $O(n + m)$ by a sweep line algorithm.

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Distant irregularity strength of graphs

Jakub Przybyło

Consider a graph $G = (V, E)$ without isolated edges and with maximum degree Δ. Given a colouring $c : E \to \{1, 2, \ldots, k\}$, the weighted degree of a vertex $v \in V$ is the sum of its incident colours, i.e., $\sum_{e \ni v} c(e)$. For any integer $r \geq 2$, the least k admitting the existence of such c attributing distinct weighted degrees to any two different vertices at distance at most r in G is called the *r-distant irregularity strength* of G and denoted by $s_r(G)$. This graph invariant provides a natural link between the well known *1–2–3 Conjecture* and *irregularity strength* of graphs. We apply the probabilistic method in order to prove an upper bound $s_r(G) \leq (4 + o(1))\Delta^{r-1}$ for graphs with minimum degree $\delta \geq \ln^8 \Delta$, improving thus far best upper bound $s_r(G) \leq 6\Delta^{r-1}$. We also investigate a total variant of the same concept and discuss their relation with similar problems where vertices are distinguished by sets of their incident colours, not by sums.

Dynamic monopolies for degree proportional thresholds

Dieter Rautenbach

(joint work with Michael Gentner)

Let G be a graph, and let $\rho \in [0, 1]$. For a set D of vertices of G, let the set $H_\rho(D)$ arise by starting with the set D, and iteratively adding further vertices u to the current set if they have at least $\lceil \rho d_G(u) \rceil$ neighbors in it. If $H_\rho(D)$ contains all vertices of G, then D is known as an irreversible dynamic monopoly or a perfect target set associated with the threshold function $u \mapsto \lceil \rho d_G(u) \rceil$. Let $h_\rho(G)$ be the minimum cardinality of such an irreversible dynamic monopoly.

For a connected graph G of maximum degree at least $\frac{1}{\rho}$, Chang [1] showed $h_\rho(G) \leq 5.83\rho n(G)$, which was improved by Chang and Lyuu [2] to $h_\rho(G) \leq 4.92\rho n(G)$. We show that for every $\epsilon > 0$, there is some $\rho(\epsilon) > 0$ such that $h_\rho(G) \leq (2+\epsilon)\rho n(G)$ for every ρ in $(0, \rho(\epsilon))$, and every connected graph G that has maximum degree at least $\frac{1}{\rho}$ and girth at least 5. Furthermore, we show that $h_\rho(T) \leq \rho n(T)$ for every ρ in $(0,1]$, and every tree T that has order at least $\frac{1}{\rho}$.

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3-flows with large support

Edita Rollová

(joint work with Matt DeVos, Jessica McDonald, Irene Pivotto, and Robert Šámal)

A k-flow in an oriented graph G is a mapping $\phi : E(G) \to \{0, \pm 1, \ldots, \pm(k-1)\}$ such that at each vertex of G the sum of incoming values equals the sum of outgoing values. The support of ϕ is the set of all edges of G with $\phi(e) \neq 0$. Tutte's 3-flow conjecture states that every 4-edge-connected graph G has a 3-flow with support $E(G)$. This is not true for 3-edge-connected graphs. We prove that every 3-edge-connected graph G has a 3-flow with support that contains at least $\frac{5}{6}|E(G)|$ edges of G. The graph K_4 demonstrates that this is best possible.

Minimal unavoidable sets of cycles in planar graphs with restricted minimum degree

Martina Sabová

(joint work with Tomáš Madaras)

A set S of cycles is minimal unavoidable in a graph family $\mathcal{G}$ if each graph $G \in \mathcal{G}$ contains a cycle from S and, for each proper subset $S' \subset S$, there exists an infinite subfamily $\mathcal{G}' \subseteq \mathcal{G}$ such that no graph from $\mathcal{G}'$ contains a cycle from S'. We explore unavoidable sets of cycles in planar graphs with prescribed minimum vertex degree.

Two thirds of the Petersen conjecture

Robert Šámal

(joint work with Hana Bílková)

Petersen coloring (defined by Jaeger [2]) is a mapping from the edges of a cubic graph to the edges of the Petersen graph, so that three edges incident to a single vertex are mapped to three edges incident to a single vertex. Jaeger [2] conjectured the following:

Conjecture 1. *Every cubic bridgeless graph admits a Petersen coloring.*

This conjecture, if true, implies the cycle double cover conjecture and the Berge-Fulkerson conjecture.

We develop Jaeger's alternate formulation of Petersen coloring in terms of special 5-edge-colorings. Consider a proper 5-coloring of the edges of a cubic graph G. Let e be an edge of G, we look at the four edges adjacent to e. We call e a *poor edge* (in the given coloring) if this five-tuple of edges uses just three colors and a *rich edge*, if five colors are used. Finally, a proper coloring is called *normal*, if every edge is either rich or poor.

Theorem 2 ([1])**.** *Let G be a cubic bridgeless graph. Then G has a Petersen coloring if and only if G has a normal 5-edge-coloring.*

We suggest a weaker conjecture, and provide new techniques to solve it.

Conjecture 3. *Let G be a cubic bridgeless graph, M a perfect matching of G. Then there is a proper 5-edge-coloring of G so that every edge e not contained in M is either rich or poor.*

If we ask, on the other hand, that every edge e contained in M is either rich or poor, then this is easier to achieve; we prove the following:

Theorem 4. *Let G be a cubic bridgeless graph, M a perfect matching of G. Then there is a proper 5-edge-coloring of G so that every edge e in M is either rich or poor.*

Further, we prove a partial solution to Conjecture 3: Let G consist of two cycles of the same length and a perfect matching M between them. (Such graph is frequently called a generalized prism.) Then there is a proper 5-edge-coloring of G so that every edge e not contained in M is either rich or poor.

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On the chromatic number of $2K_2$-free graphs

Ingo Schiermeyer

(joint work with Christoph Brause, Bert Randerath, and Elkin Vumar)

A graph G is called *k-colourable*, if its vertices can be coloured with k colours so that adjacent vertices obtain distinct colours. The smallest k such that a given graph G is k-colourable is called its *chromatic number*, denoted by $\chi(G)$. It is well-known that $\omega(G) \leq \chi(G) \leq \Delta(G) + 1$ for any graph G, where $\omega(G)$ denotes its clique number and $\Delta(G)$ its maximum degree. A graph G is *perfect* if $\chi(H) = \omega(H)$ for every induced subgraph H of G. A *hole* in a graph is an induced cycle of length at least four, and an *antihole* is the complement of a hole.

A family $\mathcal{G}$ of graphs is called χ-bound with binding function f if $\chi(G') \leq f(\omega(G'))$ holds whenever $G \in \mathcal{G}$ and G' is an induced subgraph of G. For a fixed graph H let $\mathcal{G}(H)$ denote the family of graphs which are H-free.

Strong Perfect Graph Theorem. *A graph is perfect if and only if it contains neither an odd hole of length at least five nor its complement.*

In this paper we study the chromatic number of $2K_2$-free graphs. Our work was motivated by the following problem posed by Gyárfás [1].

Problem. *What is the order of magnitude of the smallest χ-binding function for $\mathcal{G}(2K_2)$?*

One of the earliest results is due to Wagon [3], who has considered graphs without induced matchings.

Theorem. *Let G be a $2K_2$-free graph with clique number $\omega(G)$. Then $\chi(G) \leq \binom{\omega(G)+1}{2}$.*

In this talk we will show linear binding functions for several subclasses of $(2K_2, H)$-free graphs, where $H \in \{C_4, Diamond, House, Gem, Paw\}$. We will also present binding functions for $(2K_2, Claw)$-free graphs. Finally, we will discuss extensions of our results to subclasses of P_5-free graphs [2].

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Weighted path vertex cover problem for cacti

Gabriel Semanišin
(joint work with Christoph Brause and Rastislav Krivoš-Belluš)

A subset S of vertices of a graph G is called a *k-path vertex cover* if every path of order k in G contains at least one vertex from S. The cardinality of a minimum k-path vertex cover is called the *k-path vertex cover number* of a graph G and it is denoted by $\psi_k(G)$.

In the weighted version of a k-Path Vertex Cover Problem (abbreviated by k-WPVCP) the vertices have assigned weights, and the problem is to find a minimum weight k-path vertex cover set in G. This problem was introduced in [2] and the first results were presented in [1].

In our talk we discuss recent progress in k-WPVCP and give a polynomial time algorithm for k-WPVCP for networks with a specific topology – cactus.

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Snarks that cannot be covered with four perfect matchings

Martin Škoviera
(joint work with Edita Máčajová)

The celebrated Berge-Fulkerson Conjecture suggests that every bridgeless cubic graph can have its edges covered with at most five perfect matchings. It is easy to see that three perfect matchings are enough precisely when the graph in question is 3-edge-colourable. Uncolourable cubic graphs with no bridges thus fall into two classes: those that can be covered with four perfect matchings, and those that require at least five. Cubic graphs that cannot be covered with four perfect matchings are extremely rare. Among the 64326024 snarks (uncolourable cyclically 4-edge-connected cubic graphs with girth at least five) on up to 36 vertices generated by Brinkmann et al. [2] there are only *two* that cannot be covered with four perfect matchings – the Petersen graph and a snark of order 34.

The first infinite family of snarks that require at least five perfect matchings to cover their edges was described by Esperet and Mazzuoccolo [3]. It combines three snarks of order n_1, n_2, and n_3, respectively, into a snark of order $n_1+n_2+n_3+4$. The first member of this family is obtained from three copies of the Petersen graph and coincides with the mentioned snark of order 34. Another construction has been recently proposed by Abreu et al. [1]. It takes a Halin graph (a cubic plane tree with a planar cycle through its leaves) and replaces each vertex of the Halin cycle with a copy of the Petersen graph in a similar manner as the former construction. Although the resulting snarks have a more general shape, the construction has two significant drawbacks. First, its building blocks are restricted to the Petersen graph, and second, the construction heavily depends on computer-aided arguments which employ lists of possible arrangements of four perfect matchings on the expanded Halin cycle. In this talk we describe a new construction which generalises the previous two: it also starts with a Halin graph, but its building blocks can be any snarks that cannot be covered with four perfect matchings. In addition, our proofs are completely computer-free.

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Online coloring and $L(2,1)$-labeling of unit disk graphs

Joanna Sokół

(joint work with Konstanty Junosza-Szaniawski, Paweł Rzążewski, and Krzysztof Węsek)

Graphs representing intersections of families of geometric objects are intensively studied for their practical applications and for their interesting theoretical properties. In particular, unit disk intersection graphs are interesting for applications in radio network modeling. We consider the problem of classical coloring, as well as the $L(2,1)$-labeling of such graphs.

Unit disk intersection graphs can be colored online with competitive ratio equal to 5. We improve this ratio using the j-fold coloring of the unit distance graph (see [2]).

Fiala, Fishkin and Fomin [1] presented an on-line algorithm for L(2, 1)-labeling of unit disk intersection graphs with competitive ratio 50/3. We improve this algorithm to the one with competitive ratio 40/3. Moreover, using the j-fold coloring, we manage to improve this ratio for unit disks intersection graphs with a large clique number.

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The crossing number of the products of special graph H with P_n

Michal Staš

(joint work with Sinha Gayathri, Rajan Bharati, and Jana Petrillová)

The *crossing number* $\mathrm{cr}(G)$ of a simple graph G with the vertex set V and the edge set E is defined as the minimal number of pairwise intersections of nonadjacent edges in any drawing of G in the plane. The investigation on the crossing numbers of graphs is a classical and moreover very difficult problem provided that an computing of the crossing number of a given graph in general is NP-complete problem. The exact values of the crossing numbers are known only for some graphs or some families of graphs.

In the talk we determine exact value of the crossing number of the products of special graph H on six vertices with the path P_n of length n.

2-factors and independent sets in edge-chromatic critical graphs

Eckhard Steffen

About 1965 Vizing stated two conjectures:

(1) Every edge-chromatic critical graph has a 2-factor.
(2) Every independent set of an edge-chromatic critical contains at most half of its vertices.

We briefly survey the main results that have been derived till date on these conjectures and then formulate some statements which are equivalent to them.

Cycles in graphs with forbidden subgraphs

Benny Sudakov

(joint with Jacques Verstraete and in part with Alexandr Kostochka)

The notion of cycle is one of the basic notions in Graph Theory and their study has long been fundamental. Many central questions in this area ask to show that graphs with certain properties have some particular range of cycle length. In this talk we discuss several such old problems, focusing on the graphs with forbidden subgraphs.

Interval edge colourings of bipartite graphs methods, results, unsolved problems

Bjarne Toft

An interval colouring of a graph G is a proper edge-colouring of the edges of G such that the colours incident to any vertex of G form an interval of integers. This concept was introduced (in Russian) by Asratian and Kamalian at Yerevan University in Armenia in 1987. In 1989 the first example (unpublished) of a bipartite graph without an interval colouring was provided by Mirumyan. The first such published example appeared in a paper (in Russian, in 1990) by Sevastjanov at Akademgorodok in Siberia. In fact Sevastjanov proved that it is an NP-complete problem to decide if a given bipartite graph has an interval colouring. In the early 1990ies the concept of interval colouring was independently thought of by Hansen and Toft at Odense University in Denmark, investigating a scheduling problem raised by Jesper Bang-Jensen, who was in charge of scheduling parent-teacher consultations at a high school in Odense.

The literature on interval colourings is now quite extensive with more than 25 published papers. But challenging unsolved problems remain.

In my talk I shall present a survey of methods used in the study of interval colourings, two such with basis in old results by Knig and by Petersen. For example Petersens 2-factor theorem from 1891 is equivalent to the fact that a $(2, b)$-biregular bipartite graph with b even has an interval b-colouring (a bipartite graph is (a, b)-biregular if all vertices on one side have degree a and all vertices on the other side degree b).

A main unsolved problem is

Conjecture 1 (Hansen and Toft 1992, in Hansens thesis in Danish, later featured in the book Unsolved Graph Coloring Problems (Wiley 1995) by Jensen and Toft**).** *A biregular bipartite graph always has an interval colouring.*

Even the cases of (3,4)- and (3,5)-biregular bipartite graphs are still unsolved. But recently (2015) Casselgren and Toft published a proof that any (3,6)-biregular bipartite graph has an interval 7-colouring (to decide if such a graph has an interval 6-colouring is NP-complete).

There are several variants of interval colourings such as near interval colourings, one sided interval colourings and cyclic interval colourings. In the latter case the largest colour and the smallest colour are also considered to be neighbouring colours. Cyclic interval colourings were first introduced and studied by de Werra and Solot in 1991 and later by Nadolski and Kubale (2005). If a graph has an interval colouring, then by taking colours modulo the maximum degree Δ one obtains a cyclic interval Δ-colouring. Thus the following conjecture is a weakening of Conjecture 1:

Conjecture 2 (Casselgren and Toft 2014**).** *An (a,b)-biregular bipartite graph always has a cyclic* $\max a, b$*-interval colouring.*

Cases (a,b) where cyclic interval colourings are known to exist, but interval colourings not known to exist, include (3,4), (3,5), (4,5), (4,6), (4,7) and (4,8). In the cases (3,5) and (4,7) the number of colours used are 6 and 8 (not 5 and 7 as Conjecture 2 suggests). These results are published by Asratian, Casselgren, Petrosyan and Toft, in particular Petrosyan first obtained the result for the case (3,5). Asratian, Casselgren and Petrosyan recently proved that any complete multipartite graph has a cyclic interval colouring, thus solving a conjecture of Petrosyan and Mkhitaryan.

I acknowledge a fruitful collaboration with Carl Johan Casselgren from Linkping University in Sweden, who is a leading expert within this field, and who provided much work (proofs), insight and information.

On hydra number of a graph

Michał Tuczyński

(joint work with Angelika Nicgorska)

We consider directed hypergraphs with hyperarcs of of size 3 od the form $\{u, v\} \to w$. $\{u, v\}$ is called the body and w is called the head of the hyperarc. Let $H = (V, F)$ be a hypergraph. We say that a vertex $w \in V$ is reachable from a set $S \subset V$ if the following process marks w: start by marking vertices in S, and as long as there is a hyperarc $\{a, b\} \to c$ such that a and b are both marked and c is unmarked, mark c as well.

Let $G = (V, E)$ be a graph. We say that a hypergraph $H = (V, F)$ represents G if for every pair $\{u, v\}$ the set of vertices reachable from $\{u, v\}$ in H is the whole vertex set V if $uv \in E$ and is $\{u, v\}$ otherwise. The minimum number of hyperarcs in a hypergraph representing G is called the hydra number $h(G)$ of G and every hypergraph with $h(G)$ hyperarcs representing G is called optimal for G. In other words, given a set of hypergraph bodies (the edge set of a given graph), we look for the minimal number of heads assigned to these bodies such that every vertex is reachable from every body.

The problem of finding the hydra number of a graph is related to the minimization problem for Horn formulas in propositional logic.

We show that for every graph there exist optimal hypergraphs satisfying some specific additional conditions. As an application we determine the hydra number of a disconnected graph.

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Cycles and colorings — game versions

Zsolt Tuza

We present a new 2-person game concerning colorings of cycles, and some of its variants.

Geometry of permutation limits

Máté Vizer

(joint work with Mustazee Rahman and Bálint Virág)

We investigate the limit theory of permutation valued stochastic processes with the goal of understanding geometric behaviour of large random sorting networks. The theory builds on the limit theory of permutations, called permutons. We use the limit theory to investigate paths of minimal Dirichlet energy between permutons. We prove that the conjectured limit of random sorting networks, the Archimedean path, uniquely minimizes the energy among all paths from the identity to the reverse permuton.

Grasshopper pattern avoidance

Krzysztof Węsek

(joint work with Michał Dębski and Urszula Pastwa)

Pattern avoidance is an important topic in the area of *combinatorics on words*, branch of mathematics partly inspired by the work of Axel Thue at the dawn of 20th century. We say that a pattern p (i.e. finite sequence over a set of variables E) *occurs* in a word w (over an alphabet A) if there exists a substitution f from E to the set of nonempty sequences over A such that $f(p)$ is a block of consecutive elements in w. The classic goal is to construct arbitrarily long words over a small alphabet without occurrence of a given pattern - for example, Thue proved that 3 symbols suffice for the pattern α^2.

In this talk we discuss a new variant of pattern avoidance, where jumping over a letter in the pattern occurrence is allowed. For a sequence w, a subsequence $w_{i_1}w_{i_2}...w_{i_n}$ is *almost consecutive* if $(i_{j+1} - i_j) \in \{1, 2\}$ for every j. We say that a pattern p *occurs with jumps* in a word w if p occurs in any almost consecutive subsequence of w. A pattern p is *grasshopper k-avoidable* if there exists an alphabet A of k elements, such that there exist arbitrarily long words over A in which p does not occur with jumps. The minimal such k is the *grasshopper avoidability index* of p. We almost completely determine the grasshopper avoidability index of patterns α^n. We use entropy compression method to obtain results on grasshopper avoidability of patterns in two classes: patterns without variables used exactly once, and patterns which are long in terms of their number of variables. Moreover, we state some open problems concerning this new notion and we describe connections with other problems in the field (especially, pattern-free colorings of the plane).

The Ramsey minimal graphs of matching versus graph containing C_3

Kristiana Wijaya

(joint work with Edy Tri Baskoro, Hilda Assiyatun, and Djoko Suprijanto)

For given graphs G and H, a (G, H)-*colouring* is a red-blue colouring of edges of F so that F contains neither a red G nor a blue H. The graph F (without isolated vertices) is called a *Ramsey* (G, H)*-minimal graph* if every red-blue colouring of edges of F contains a red copy of G or a blue copy of H but for each $e \in E(F)$ there exists a (G, H)-colouring of $F - e$. In this paper, we present a class graph belonging to $\mathcal{R}(mK_2, C_3)$. Furthermore, we generalize the result by replacing C_3 with an H, a connected graph containing a C_3.

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2-dominating kernels in graphs and their products

Iwona Włoch

(joint work with Paweł Bednarz and Andrzej Włoch)

A subset $J \subset V(G)$ is a 2-dominating kernel of a graph G if J is independent and every vertex not belonging to J has at least two adjacent vertices in J. Every graph does not always possess a 2-dominating kernel.

In the talk we give some conditions for the existence of 2-dominating kernels in graphs and their products.

On graph interpretations and generalization of telephone numbers

Małgorzata Wołowiec-Musiał
(joint work with Urszula Bednarz)

The telephone (involution) numbers are given by the recurrence relation

$$T_n = T_{n-1} + (n-1)T_{n-2}$$

with initial conditions $T_0 = T_1 = 1$. These numbers have interesting combinatorial and graph interpretations. In the talk we focus on connections between classical telephone numbers and a special kind of edge-colouring of a graph i.e. edge-shade colouring of a graph. We also give one-parameter generalization of these numbers.

Structure of cycles in c-partite tournaments

Rita E. Zuazua
(joint work with Ana Paulina Figueroa, Bernardo Llano, and Mika Olsen)

Let T be a c-partite tournament. We say that a vertex v is C_3-free if v does not lie on any directed triangle of T.

Zhou, Yao and Zhang (1998) proved that if T is a regular c-partite tournament with $4 \leq c$ then T does not have C_3-free vertices. On other hand, Tewes, Volkmann and Yeo (2002) showed that if T is an almost regular c-partite tournament with $5 \leq c$ then T does not have C_3-free vertices.

I this talk we study the set of C_3-free vertices in regular and almost regular 3-partite tournaments.

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