Workshop Cycles and Colourings '93

Novy Smokovec, Slovakia, September 5-10, 1993

Problems

Some necessary notions and notations can be found in abstracts. For more information, please contact authors of problems.

Problem 1 (posed by H. Broersma). Does there exist a 2-tough graph on at least 3 vertices which is not Hamiltonian-connected?
Remark. By a construction due to Bauer, Broersma, van den Heuvel and Veldman, an affirmative answer to this question would imply that there exists a non-Hamiltonian graph with toughness greater than 2.

Problem 2 (posed by H. Broersma). Let G be a 2-connected claw-free graph on n vertices.
Conjecture: If every endvertex of each induced subgraph of G isomorphic to the graph of a 3-sided prism without edges of one triangle has degree at least $(n-2)/3$, then G is Hamiltonian.
Remarks. The truth of this conjecture would give a result that generalizes known results of Duffus, Gould and Jacobson, and of Matthew and Summer. The conjecture was also posed in Beijing (June '93).

Problem 3 (posed by J. Harant, harant@mathematik.tu-ilmenau.de). For an integer $i \geq 0$ consider consider the following Problem Bi: Does there exist a cubic 3-connected non-Hamiltonian bipartite graph of genus i?
(a) The answer is yes: $i \geq i_0$ for a certain i_0 (see M. N. Ellingham, J. D. Horton: Non-Hamiltonian 3-connected cubic bipartite graphs, J. Comb. Theory Ser. B 34 (1983), 350-353).
(b) Barnette's conjecture (B. Barnette, Conjecture 5: In Recent Progress in Combinatorics, W. T. Tutte, Ed., Academic Press, New York 1969) is true if and only if answer (B0) $=$ no.
What's up for $i = 1$?

Problem 4 (posed by J. Harant). Consider a 3-connected undirected graph G with vertex set V and edge set E on $n = |V|$ vertices. The integer $d(x, y)$ denotes the distance between $x, y \in V$. The integer $r = \min_{x \in V} \max_{y \in V} d(x, y)$ is called the radius of G. It is known that $r \leq \lfloor n/4 \rfloor + 8$ (J. Harant: An upper bound for the radius of a 3-connected graph, Discrete Math., to appear). There is a 3-connected graph on n vertices with radius $r = \lfloor n/4 \rfloor + 1$ (J. Harant and H. Walther: On the radius of graph, J. Comb. Theory Ser. B 30 (1981), 113-117). Does there exist a 3-connected graph with $r > \lfloor n/4 \rfloor + 1$?

Problem 5 (posed by M. Hornak). Achromatic index of a graph G is the maximum number of colours in a proper edge-colouring of G such that each pair of colours occurs on at least one pair of adjacent edges. Let $A(n)$ be the achromatic index of K_n. It is proved that for any odd integer $q \geq 3$ $A(q^2 + q + 1) = q(q^2 + q + 1)$ if and only if there exists a projective plane of order q (see A. Bouchet, Indice achromatique des graphes multiparti complets et regulier, Cahiers

Centre Etudes Rech. Oper. 20 (1978), 331-340). Besides $n = q^2 + q + 1$ where q is a power of an odd prime $A(n)$ is known only for $2 \leq n \leq 12$ and $n = 25$. Thus the first value to discover is $A(14)$.

Problem 6 (posed by S. Jendrol). Let G be a plane graph. An Eulerian trail T in G is defined to be a C-trial if no two consecutive edges of G share a common face. Characterize all plane graphs which have a C-trial.
Remark. The problem was also posed at the Czecho-Slovak conference in Janov nad Nisou (May '93).

Problem 7- uniqueness of irreducible decomposition of hereditary properties (posed by P. Mihok). Let I be the set of all nonisomorphic simple graphs. A property $P, \emptyset \neq P \subseteq I$ is said to be hereditary if $H \subseteq G \in P \to H \in P$. Let $P_1, P_2, ..., P_n$ be hereditary properties. A vertex $(P_1, P_2, ..., P_n)$-partition of a graph G is a partition $V(G) = V_1 \cup V_2 \cup ... \cup V_n$, $n \geq 2$, of vertices of G such that $\langle V_i \rangle \in Pi$ for each induced subgraph $\langle V_i \rangle$, $i = 1, 2, ..., n$. Let us define the property $P_1.P_2.\ldots.P_n$ as follows: $G \in P_1.P_2.\ldots.P_n$ iff G has a vertex $(P_1, P_2, ..., P_n)$-partition. A property P is said to be reducible if there exist properties P_1, P_2 such that $P = P_1.P_2$; otherwise P is called irreducible.
Problem: Is the decomposition of any reducible property into irreducible properties unique?
Remark. The problem was also posed at the Czecho-Slovak conference in Janov nad Nisou (May '93).

Problem 8 (posed by H. Walther). What is the smallest integer m such that there exist integers $q_i \equiv 0 \pmod 3$, $i = 1, 2, ..., m$, for which the class $G(5; q_1, q_2, ..., q_m)$ has non-Hamiltonian members (Theorem 1 - see page 6 - shows $m \leq 8$)?
Conjecture: $m \leq 3$.

Problem 9 (posed by H. Walther). The same question as in Problem 8 with shortness exponent of the class $G(5; q_1, q_2, ..., q_m)$ smaller than one.
Conjecture: $m \leq 3$.

Problem 10 (posed by H. Walther). What is the smallest integer m such that the class $G(5; q_1, q_2, ..., q_m)$ has non-Hamiltonian members?

Problem 11 (posed by H. Walther). The same question as in Problem 10 with shortness exponent of the class $G(5; q_1, q_2, ..., q_m)$ smaller than one.