TÝŽDEŇ 3 / WEEK 3

ÚVODNÉ OTÁZKY

A1 Aké podmienky musí spĺňať prvok simplexovej tabuľky, aby mohol byť pivotom?

A2 Ako zistíme, že simplexová tabuľka je optimálna?

A3 Ako zistíme, že úloha LP je neohraničená?

A4 Ako zistíme, že úloha LP je neprí pustná?

WARM-UP QUESTIONS

A1 Describe how to choose a pivot in a simplex tableau.

A2 How will one learn that a simplex table is optimal?

A3 How will one learn that a LP is unbounded?

A4 How will one learn that a LP is infeasible?

ZÁKLADNÉ ÚLOHY

Riešte simplexovou metódou.

B1

$$\begin{aligned} 3x_1 - 5x_2 &\to \min \\ -x_1 - x_2 &\leq 6 \\ x_1 + 2x_2 &\leq 14 \\ x_{1-2} &\geq 0 \end{aligned}$$

B2

$$\begin{aligned} 2x_1 - 3x_2 + x_3 &\to \min \\ x_1 + x_2 + x_4 &= 4 \\ x_2 + x_3 - x_4 &= 6 \\ x_{1-4} &\geq 0 \end{aligned}$$

Riešte simplexovou metódou s pomocnou úlohou.

B3

$$\begin{aligned} -3x_1 + 2x_2 + x_3 &\to \min \\ x_1 + x_2 + x_3 &= 6 \\ x_1 + 2x_2 - x_3 &= 3 \\ x_{1-3} &\geq 0 \end{aligned}$$

B4

$$\begin{aligned} x_1 - x_2 + 3x_3 - 2x_4 &\to \min \\ x_1 + x_2 + x_4 &= 1 \\ x_2 + x_3 - x_4 &= 2 \\ x_1 + 2x_2 + x_3 &= 2 \\ x_{1-4} &\geq 0 \end{aligned}$$

B5 Je daná takáto simplexová tabuľka

CORE EXERCISES

Solve the following LPs by Simplex Method.

Solve the following LPs by the 2-Phase Simplex Method (with auxiliary variables).

B5 Consider the following simplex tableau

B		x_1	x_2	x_3	x_4	x_5	x_6	x_7
	6	0	-3	0	0	2	0	0
x_1	0	1	1	0	0	1	0	1
x_6	4	0	-1	0	2	2	1	1
x_3	1	0	0	1	1	-1	0	1

a) Je bázické prípustné riešenie prislúchajúce k tejto tabuľke optimálne?

b) Nájdite všetky optimálne riešenia pre túto úlohu.

a) Is the basic feasible solution corresponding to this simplex tableau optimal?

b) Find all optimal solutions for this problem.

OTÁZKY NA ZAMYSLENIE

C1 Ukážte, že podmienka "$\theta_o = 0$ pre každého možného pivota v simplexovej tabuľke" nie je postačujúcou pre optimalitu zodpovedajúceho BPR.

C2 Môže sa v priebehu výpočtu nezáporná relatívna cena stĺpca zmeniť na zápornú? Môže takáto zmena nastať v priebehu výpočtu viackrát?

C3 Je možné, aby sa hneď v nasledujúcom pivotovaní vrátil do bázy vektor, ktorý ju práve opustil? Zdôvodnite!

C4 Dokážte, že ak je pôvodná úloha LP prípustná, tak v optimálnej simplexovej tabuľke pomocnej úlohy sú relatívne ceny pre pomocné premenné, ktoré nie sú v optimálnej báze, rovné 1 a nulové pre všetky ostatné premenné.

BRAIN TEASERS

C1 Show that condition "$\theta_o = 0$ for each possible pivot in a simplex tableau" is not sufficient for optimality of the corresponding BFS.

C2 Is it possible for a relative cost of a column to change from positive to negative in the course of simplex computations? Can such a change occur during the computations more than once?

C3 Consider a variable that has just been removed from the basis. Can it return to the basis in the immediately following pivot operation? Support your statement by a suitable argument.

C4 Suppose that the original LP is feasible and consider the optimal simplex tableau of Phase-1. Prove that the relative prices of auxiliary variables that are not in the optimal basis are equal to 1 and the relative prices of all other variables are equal to 0.

ĎALŠIE ÚLOHY / ADDITIONAL PRACTICE

D1

$$\begin{aligned} 5x_1 + 4x_2 + 3x_3 &\to \max \\ 2x_1 + 3x_2 + x_3 &\le 5 \\ 4x_1 + x_2 + 2x_3 &\le 11 \\ 3x_1 + 4x_2 + 2x_3 &\le 8 \\ x_{1-3} &\ge 0 \end{aligned}$$

D2

$$\begin{aligned} x_1 + x_2 + x_3 + x_4 + x_5 &\to \max \\ x_1 + x_2 + 4x_3 + 4x_4 + 3x_5 &= 3 \\ x_1 - x_2 - 2x_3 + 2x_4 + 5x_5 &= 1 \\ x_{1-5} &\ge 0 \end{aligned}$$

D3

$$\begin{aligned} -3x_1 + x_2 + 3x_3 - x_4 &\to \min \\ x_1 + 2x_2 - x_3 + x_4 &= 0 \\ 2x_1 - 2x_2 + 3x_3 + 3x_4 &= 9 \\ x_1 - x_2 + 2x_3 - x_4 &= 6 \\ x_{1-4} &\ge 0 \end{aligned}$$

D4

$$\begin{aligned} 4x_1 + 5x_2 + x_3 &\to \max \\ 3x_1 + 2x_2 &\leq 10 \\ x_1 + 4x_2 &\leq 11 \\ 3x_1 + 3x_2 + x_3 &\leq 13 \\ x_{1-3} &\geq 0 \end{aligned}$$

D5

$$\begin{aligned} x_1 + x_2 &\to \max \\ x_1 + x_2 &\geq 4 \\ x_1 - x_2 &\leq 8 \\ -x_1 + 2x_2 &\leq 8 \\ x_{1-2} &\geq 0 \end{aligned}$$

D6

$$\begin{aligned} x_1 + 2x_2 &\to \min \\ x_1 + x_2 &= 3 \\ x_1 - x_2 &= 4 \end{aligned}$$

D7

$$\begin{aligned} -2x_1 - 3x_2 - 3x_3 &\to \min \\ 3x_1 + 2x_2 + x_4 &= 60 \\ -x_1 + x_2 + 4x_3 + x_5 &= 10 \\ 2x_1 - 2x_2 + 5x_3 + x_6 &= 50 \\ x_{1-6} &\geq 0 \end{aligned}$$

D8

$$\begin{aligned} 2x_2 + x_3 &\to \max \\ x_1 + x_2 - 2x_3 &\leq 7 \\ -3x_1 + x_2 + 2x_3 &\leq 3 \\ x_{1-3} &\geq 0 \end{aligned}$$

D9

$$\begin{aligned} 2x_1 + 4x_2 - 4x_3 + 7x_4 &\to \min \\ 8x_1 - 2x_2 + x_3 - x_4 &\leq 50 \\ 3x_1 + 5x_2 + 2x_4 &\leq 150 \\ x_1 - x_2 + 2x_3 - 4x_4 &\leq 100 \\ x_{1-4} &\geq 0 \end{aligned}$$

D10

$$\begin{aligned} 9x_2 + 2x_3 - x_5 &\to \max \\ x_1 - 3x_2 - 4x_4 + 2x_6 &= 60 \\ 2x_2 - x_4 - x_5 + 4x_6 &= -20 \\ x_2 + x_3 + 3x_6 &= 10 \\ x_{1-6} &\geq 0 \end{aligned}$$

D11

$$\begin{aligned} 2x_1 - 3x_2 + x_3 + x_4 &\to \min \\ x_1 - 2x_2 - 3x_3 - 2x_4 &= 3 \\ x_1 - x_2 + 2x_3 + x_4 &= 11 \\ x_{1-4} &\geq 0 \end{aligned}$$

D12

$$\begin{aligned} x_1 + x_2 - x_4 &\to \min \\ 4x_1 + x_2 + x_3 + 4x_4 &= 8 \\ x_1 - 3x_2 + x_3 + 2x_4 &= 16 \\ x_{1-4} &\geq 0 \end{aligned}$$

D13

$$\begin{aligned} x_1 - 3x_3 &\to \min \\ x_1 + 2x_2 - x_3 &\leq 6 \\ x_1 - x_2 + 3x_3 &= 3 \\ x_{1-3} &\geq 0 \end{aligned}$$

D14

$$\begin{aligned} x_1 + x_2 + x_3 &\to \min \\ -x_1 + 2x_2 + x_3 &\leq 1 \\ -x_1 + 2x_3 &\geq 4 \\ x_1 - x_2 + 2x_3 &= 4 \\ x_{1-3} &\geq 0 \end{aligned}$$

D15

$$\begin{aligned} x_1 + 2x_2 + 3x_3 + 4x_4 &\to \max \\ x_1 + x_3 - 4x_4 &= 2 \\ x_2 - x_3 + 3x_4 &= 9 \\ x_1 + x_2 - 2x_3 - 3x_4 &= 21 \\ x_{1-4} &\geq 0 \end{aligned}$$

D16

$$\begin{aligned} x_1 + 4x_2 + 3x_3 + 2x_4 &\to \min \\ x_1 + 2x_2 + x_4 &= 20 \\ 2x_1 + x_2 + x_3 &= 10 \\ -x_1 + 4x_2 - 2x_3 + 3x_4 &= 40 \\ x_{1-4} &\geq 0 \end{aligned}$$

VÝSLEDKY / RESULTS

B1 $\mathbf{x}^{\text{opt}} = (0, 7)^T$ **B2** $\mathbf{x}^{\text{opt}} = (0, 4, 2, 0)^T$ **B3** $\mathbf{x}^{\text{opt}} = (\frac{9}{2}, 0, \frac{3}{2})^T$

B4 Infeasible.

B5 a) Yes.

b) $\mathbf{x}^1 = (0, 0, 1, 0, 0, 4, 0)^T, \mathbf{x}^2 = (0, 0, 0, 1, 0, 2, 0)^T$ and their convex combinations.

C1

B	0	0	0	0	-2	1
x_1	0	1	0	0	1	-1
x_2	1	0	1	0	-1	1
x_3	2	0	0	1	-1	2

C2 Yes. Yes.

C3 No.

D1 $\mathbf{x}^{\text{opt}} = (2, 0, 1)^T$

D2 $\mathbf{x}^{\text{opt}} = (2, 1, 0, 0, 0)^T$

D3 $\mathbf{x}^{\text{opt}} = (1, 1, 3, 0)^T$

D4 $\mathbf{x}^{\text{opt}} = (\frac{9}{5}, \frac{23}{10}, \frac{7}{10})^T$

D5 $\mathbf{x}^{\text{opt}} = (24, 16)^T$

D6 $\mathbf{x}^{\text{opt}} = (\frac{7}{2}, -\frac{1}{2})^T$

D7 $\mathbf{x}^{\text{opt}} = (8, 18, 0, 0, 0, 70)^T$

D8 Unbounded.

D9 $\mathbf{x}^{\text{opt}} = (0, 0, 50, 0)^T$

D10 $\mathbf{x}^{\text{opt}} = (250, 10, 0, 40, 0, 0)^T$

D11 $\mathbf{x}^{\text{opt}} = (19, 8, 0, 0)^T$

D12 Infeasible.

D13 $\mathbf{x}^{\text{opt}} = (0, \frac{21}{5}, \frac{12}{5})^T$

D14 Infeasible

D15 Infeasible.

D16 $\mathbf{x}^{\text{opt}} = (5, 0, 0, 15)^T$