

**Jednota slovenských matematikov a fyzikov
Pobočka Košice**

**Prírodovedecká fakulta UPJŠ
Ústav matematických vied**

**Fakulta elektrotechniky a informatiky TU
Katedra matematiky**

10. Konferencia košických matematikov

Názov: 10. Konferencia košických matematikov

Editori: Ján Buša, Stanislav Jendroľ, Štefan Schrötter

Vydala: Technická univerzita, Fakulta elektrotechniky a informatiky

Vydanie: prvé

Počet strán: 24

Náklad: 50 ks

Vydané v Košiciach, 2009

Elektronická sadzba programom pdfT_EX

ISBN 978-80-553-0187-7

**Herľany
22. – 25. apríla 2009**

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Predhovor

Milí priatelia,

vítame Vás na jubilejnej 10. Konferencii košických matematikov, ktorá sa koná v rámci osláv 50. výročia založenia Univerzity Pavla Jozefa Šafárika v Košiciach a 40. výročia založenia Fakulty elektrotechniky a informatiky TU v Košiciach.

Konferenciu organizuje Jednota slovenských matematikov a fyzikov, pobočka Košice, v spolupráci s Ústavom matematických vied Prírodovedeckej fakulty UPJŠ, katedrami matematiky Technickej univerzity a pobočkou Slovenskej spoločnosti aplikovanej kybernetiky a informatiky pri KRVP BF TU v Košiciach. Konferencia sa koná, tak ako aj jej predchádzajúce ročníky, v útulnom prostredí Učebno-výcvikového zariadenia TU Košice v Herľanoch.

Cieľom konferencie je zintenzívniť stavovský život všetkých, ktorí sa v Košiciach a okolí profesionálne zaoberajú matematikou (t. j. učiteľov všetkých typov škôl, pracovníkov na poli matematických a informatických vied a aplikácií matematiky v priemysle, technike, bankovníctve a inde) a formulovať základné oblasti ich stavovských záujmov. Odborný program konferencie tradične pozostáva z pozvaných prednášok, prihlásených referátov a diskusií o stavovských problémoch. Prvé dva dni sú venované prezentácii výsledkov mladých (nielen košických) matematikov a doktorandov. Piťatkový a sobotňajší program naplňajú prednášky pozvaných prednášateľov. V programe konferencie je vytvorený priestor aj pre diskusiu o našich problémoch.

Takáto štruktúra programu sa vyprofilovala z poznatkov minulých ročníkov konferencie. Doktorandom a mladším matematikom je poskytnutý priestor na získanie skúseností pri prezentácii svojich výsledkov. Je potešujúce vidieť, ako sa každým rokom zlepšujú ich vystúpenia. Organizačný výbor konferencie sa snaží pozývať významné osobnosti matematiky, ktoré v rámci svojich prednášok ukážu miesto matematiky v spoločenskom živote a súčasné trendy jej rozvoja. V živej pamäti máme prednášky profesorov V. Bálinta, P. Brunovského, L. Bukovského, K. Cechlárovej, M. Fiedlera, A. Chechkina, M. Komana, J. Kopku, R. Mesiara, A. Plockého, I. Podlubného, A. Recskiho, I. Schiermeyera, O. Šedivého, K. Zimmermanna a radu ďalších uznávaných matematikov. Nejedna z pozvaných prednášok mala taký pozitívny ohlas, že ich autori boli pozvaní predniesť ich na iných konferenciách.

Program 10. Konferencie košických matematikov**Streda 22. 4. 2009**

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- 14⁵⁰ – Július Czap (ÚMV PF UPJŠ) *Looseness of plane graphs*
- 15¹⁵ – Martin Tajboš (ÚMV PF UPJŠ) *Váha hrán grafov pre vybrané triedy grafov s dedičnými vlastnosťami*
- 15⁴⁰ – Michal Staš (ÚMV PF UPJŠ) *Classification of Π_1^1 -complete sets*
- 16⁰⁵ – Martin Boďa (KM FPV UMB BB) *The framework for empirical verification of the Sharpe-Lintner version of the CAPM*
- 16³⁰ – Kávová prestávka
- 17⁰⁰ – Emília Draženská (KM FEI TU) *On the crossing numbers of Cartesian product of stars with the graph $K_{2,2,2}$*
- 17²⁵ – Daniela Kravecová (KM FEI TU) *The crossing number of $P_5^2 \times C_n$*
- 17⁵⁰ – Erika Škrabuľáková (ÚMV PF UPJŠ) *Facial non-repetitive edge-colouring of plane graphs*

18³⁰ – Večera**Štvrtok 23. 4. 2009**

- 9⁰⁰ – 12³⁰ Workshop: *Voľná diskusia – výmena skúseností*
- 12³⁰ – Obed
- 14⁵⁰ – Sergej Ševce (ÚMV PF UPJŠ) *Noninscribability and hard inscribability*
- 15¹⁵ – Kotorová Renáta (KM FEI TU) *Vlastnosti riešení diferenciálnych rovníc tretieho rádu s posunutým argumentom*
- 15⁴⁰ – Tomáš Škovránek (ÚRIVP BERG TU) *Identifikácia sústav ľubovoľného reálneho rádu: Nová metóda založená na sústavách diferenciálnych rovníc neceločíselného rádu a ortogonálnej regresie*
- 16⁰⁵ – Eva Pillárová (ÚMV PF UPJŠ) *Problém spravodlivého delenia*
- 16³⁰ – Kávová prestávka
- 17⁰⁰ – Viera Borbeľová (ÚMV PF UPJŠ) *Matematické metódy vo výmenách obličiek*
- 17²⁵ – Mária Kolková (ÚMV PF UPJŠ) *Problémy žiakov s matematickou reflexiou pri riešení stochastického problému*

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Identification of systems of arbitrary real order: a new method based on solution of systems of fractional order differential equations and orthogonal distance fitting

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A new method for identification of systems of arbitrary real order based on numerical solution of systems of fractional order differential equations and orthogonal distance fitting is presented. The main idea is to fit experimental or measured data using a solution of a system of fractional differential equations, where parameters of these equations, including the orders of derivatives, are subject to optimization process. The criterion of optimization is the minimal sum of orthogonal distances of the data points from the fitting line. The examples are presented in 2-dimensional and 3-dimensional problems.

On weight of graphs for peculiar graphs family – hereditary properties

Mgr. Martin Tajboš

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For a graph $G = (V, E)$ and $e = \{x, y\} \in E(G)$ the weight of e is defined as $w(e) = \deg(x) + \deg(y)$. Erdős asked the question what is the minimum weight of an edge e of a graph G having n vertices and m edges? Let $\mathcal{G}(n, m)$ be the family of all graphs having n vertices and m edges. Motivated by Erdős's question Ivančo, Jendroľ [1] and Jendroľ, Schiermeyer [2] solved the problem

$$W(n, m) = \max_{G \in \mathcal{G}(n, m)} \left\{ \min_{e \in E(G)} w(e) \right\}.$$

Achromatic index of a complete graph versus finite projective planes

Mirko Horňák

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Let G be a simple finite undirected graph. An edge colouring $\varphi : E(G) \rightarrow C$ is regular if $\varphi(e_1) \neq \varphi(e_2)$ whenever e_1 is adjacent to e_2 ; it is complete provided for any two distinct colours $c_1, c_2 \in C$ there is a pair of adjacent edges $e_1, e_2 \in E(G)$ such that $\varphi(e_i) = c_i$, $i = 1, 2$. The achromatic index of the graph G is the maximum number of colours in a complete regular edge colouring of G . The lecture will be devoted to $A(n)$, the achromatic index of the complete n -vertex graph K_n . A special emphasis will be put on a remarkable result by Bouchet who proved that for an odd integer $q \geq 3$ the following statements are equivalent:

- (1) $A(q^2 + q + 1) = q(q^2 + q + 1)$.
- (2) There is a projective plane of order q .

State Complexity of Cyclic Shift

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Cyclic shift is a unary operation on formal languages defined as $\text{SHIFT}(L) = \{vu \mid uv \in L\}$. The operation preserves both regularity and context-freeness. Its state complexity has been addressed in Maslov's paper on the state complexity of regular language operations (1970, Soviet Math. Dokl. 11, 1373–1375), where a high lower bound for partial deterministic finite automata using a growing alphabet was given.

We improve this result by using a fixed four-letter alphabet and obtaining a lower bound $2^{n^2 + n \log n - O(n)}$ on the state complexity of this operation.

An algorithm for fair division

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Fair division, also known as the cake cutting problem, is the problem of dividing a divisible good among n players in such a way that each player believes that he has received a fair amount. Cake is represented by interval $\langle 0, 1 \rangle$ and players value various pieces of the cake differently, i.e. it is assumed that player i has a utility function $u_i : \langle 0, 1 \rangle \rightarrow R^+$. Fairness can be conceptualized in different ways, like proportionality (each player receives a piece with utility at least $\frac{1}{n}$ of the cake from his point of view), envy-freeness (players do not envy each other) or equitability (utilities of pieces assigned to all players are equal).

We propose a new Bisection procedure for 2 players that leads to an ε -equitable division of the cake (the difference between utilities of pieces assigned to players is not higher than the predetermined value ε). Players do not need to reveal their utility functions. The obtained cake division is also proportional and envy-free.

Qualitative evaluation of the probability of events (Galton desk at the mathematics lesson)

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This contribution deals with qualitative evaluation of the probability of events, using statistical data. We present a proposal of the teaching lesson, where the Galton desk is used as a random data generator. We also give our remarks and comments from such lesson, which was realized in the fourth grade of the eight-year grammar school.

probability space. Since then several other models were presented. We will focus our attention mainly on two of them – on quantum logical spaces and on MV-algebras (many-valued algebras). We say something about why they are important, what have these models in common and in what aspects they are different.

Acknowledgement

The work on this paper has been supported by Science and Technology Assistance Agency under the contract No. APVV-0375-06, and by the VEGA grant agency, grant number 1/0373/08.

Exhibits of the strategies of problem solving

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In a problem-solving situation, we have a goal to achieve but may not have the means to achieve it immediately. The strategies are tools that might be useful in discovering or constructing means to achieve a goal. Important strategies are for examples: Experimentation, Analogy, Examine a simpler case, Identify a subgoal, Examine a related problem, Work backward, Geometric way, Algebraic way, Use indirect reasoning. But we will talk about *Strategy of generalization* and *Strategy of infinite descent*. The first one can be used also outside the mathematics, but the second one is characteristic only for mathematics.

Strategy of infinite descent: It is assumed that a given problem has a solution S . Starting with S , a never-ending sequence of solutions is constructed, although the nature of the problem indicates that any sequence of solutions must have a final term. This contradiction proves that the problem has no solution.

Facial non-repetitive edge-colouring of plane graphs

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and Erika Škrabuľáková²

¹Projet Mascotte, I3S (CNRS and University of Nice-Sophia Antipolis) and INRIA, 2004 Route des Lucioles, BP 93, 06902 Sophia-Antipolis Cedex, France.

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A sequence r_1, r_2, \dots, r_{2n} such that $r_i = r_{n+i}$ for all $1 \leq i \leq n$, is called a *repetition*. A sequence S is called *non-repetitive* if no *block* (i.e. subsequence of consecutive terms of S) is a repetition. Let G be a graph whose edges are coloured. A trail is called *non-repetitive* if the sequence of colours of its edges is non-repetitive. If G is a plane graph, a *facial non-repetitive edge-colouring* of G is an edge-colouring such that any *facial trail* (i.e. trail of consecutive edges on the boundary walk of a face) is non-repetitive. We denote $\pi'_f(G)$ the minimum number of colours of a facial non-repetitive edge-colouring of G . In this paper, we show that $\pi'_f(G) \leq 8$ for any plane graph G . We also get better upper bounds for $\pi'_f(G)$ in the cases when G is a tree, a plane triangulation, a simple 3-connected plane graph, a hamiltonian plane graph, an outerplanar graph or a Halin graph. The bound 4 for trees is tight.

Characteristics of competencies in the reflection cluster and strategies of their developing

Mária Kolková

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In the presentation the characteristics of competencies in the reflection cluster will be identified. These characteristics have arisen from analyse of pupils' solutions of one probabilistic problem and have inspired strategies of developing competencies in the reflection cluster. Designed strategies will be demonstrated on the same problem.

Discrete mathematics in the class

Ingrid Semanišínová

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We shall discuss the questions of what discrete mathematics belongs in the middle school level and how it should be incorporated, from the perspective of the curriculum. We shall present examples of learning environments as a setting for a mathematical investigation that leads students to creating representations and engaging them in a mathematical reasoning, abstraction and generalization. We shall show that the use of properly selected and presented discrete mathematics problems can foster independent thinking, encourage flexibility in approaches and representations, and encourage a focus on problem mathematical structure.

Conference contributions

The framework for empirical verification of the Sharpe-Lintner version of the CAPM

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One of the most illustrious outcomes of portfolio theory is the Capital Asset Pricing Model (CAPM) which describes, in terms of linear relationships, the determination of the price of risky assets in the presence of riskless assets. The CAPM results directly from the hypothesis of efficiency of financial markets and its empirical validity is directly linked to the trueness of the hypothesis of efficiency. It is therefore vital to financial risk management to validate the CAPM empirically. The contribution under this notion submits an appropriate statistical framework for estimating the Sharpe-Lintner version of the CAPM and for testing its validity.

Mathematical methods in kidney exchange

Viera Borbeľová

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The most effective currently known treatment for a kidney failure is a transplantation. Because of the lack of deceased donors, in recent years the number of living donors (genetically or emotionally relative of a patient) has increased. However, immunological reasons often make the transplantation impossible and the willing living donor is lost. Therefore organized kidney exchange programs have been initiated in several countries.

In our talk, we model a kidney exchange as a combinatorial problem and present several mathematical methods used to find a solution optimal in some sense.

Looseness of plane graphs

J. Czap, S. Jendroľ, F. Kardoš, and J. Miškuf

Institute of Mathematics, P. J. Šafárik University,
Jesenná 5, 040 01 Košice, Slovakia

All considered graphs are finite, loops and multiple edges are allowed. Let $G = (V, E, F)$ be a connected plane graph with the vertex set V , the edge set E and the face set F .

A k -colouring of a graph G is a mapping $\varphi : V \rightarrow \{1, \dots, k\}$.

For a face $f \in F$ we define $\varphi(f)$ to be the set of colours used on the vertices incident with the face f . A face f is called *loose* if $|\varphi(f)| \geq 3$.

Question: What is the minimum number of colours $ls(G)$ that any surjective vertex colouring of a connected plane graph G with $ls(G)$ colours enforces a loose face?

The invariant $ls(G)$ of a plane graph G is called the *looseness* of G .

We prove that the looseness of a connected plane graph G equals 2 plus the maximum number of vertex disjoint cycles in the dual graph G^* .

We also show upper bounds on the looseness of graphs based on the edge connectivity, the girth of the dual graphs and other basic graph invariants. Moreover, we present infinite classes of graphs where these equalities are attained.

References

- [1] J. Czap, S. Jendroľ, F. Kardoš, and J. Miškuf, *Looseness of plane graphs*, (<http://umv.science.upjs.sk/preprints/dokumenty/A3-2009.pdf>)

On the crossing numbers of Cartesian product of stars with the graph $K_{2,2,2}$

Emília Draženská, Marián Klešč

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The *crossing number*, $cr(G)$, of a graph G is the minimum number of pairwise intersections of nonadjacent edges in any drawing of G in the plane. Finding the exact value of crossing number of a given graph is in general an elusive problem. There are known several exact results on the crossing numbers of the Cartesian products of stars and graphs of order four or five.

We extend these results and we prove that the crossing number of the Cartesian product of the complete tripartite graph $K_{2,2,2}$ with the star S_n is $6 \lfloor \frac{n}{2} \rfloor \lfloor \frac{n-1}{2} \rfloor + 6n$.

Numerical modelling of complex flow

Mária Lukáčová-Medvid'ová

Institute of Numerical Simulation, Hamburg University
of Technology, Hamburg, Germany

In this talk we will present our recent results obtained for numerical simulation of complex flows. In particular, we will concentrate on the so-called finite volume evolution Galerkin scheme that will be used in order to model the shallow water equations. This is a model having wide applications in meteorology, oceanography or river flow engineering. The second part of the talk is devoted to the non-Newtonian fluids and their applications in hemodynamics.

Matrix approach to discretization of ordinary and partial differential equations of arbitrary real order

I. Podlubný, A. Chechkin, T. Škovránek, Y. Q. Chen,
B. M. Vinagre Jara

Technical University in Košice, Faculty BERG, Slovakia

A method that enables easy and convenient discretization of ordinary and partial differential equations with derivatives of arbitrary real order (so-called fractional derivatives) and delays is presented and illustrated on numerical solution of various types of fractional differential equations.

The method was developed by Podlubny (Fractional Calculus and Applied Analysis, vol. 3, no. 4, 2000, 359–386) for ordinary fractional differential equations and extended recently by this team of authors (I. Podlubny, A. Chechkin, T. Skovranek, YQ. Chen, B. M. Vinagre Jara, *Matrix approach to discrete fractional calculus II: partial fractional differential equations*, Journal of Computational Physics, vol. 228, no. 8, 1 May 2009, 3137–3153).

A set of MATLAB routines for the implementation of the method as well as sample code used to solve the examples have been developed. The toolbox is available at Matlab Central File Exchange (<http://www.mathworks.com/matlabcentral/fileexchange/22071>).

Properties of the solutions of the third order differential equations with delayed argument

Renáta Kotorová

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We investigate criteria for asymptotic and oscillatory properties of the third order trinomial differential equation with delay argument

$$y'''(t) - p(t)y'(t) + g(t)y(\tau(t)) = 0,$$

by transforming this equation to binomial second/third order differential equation.

The crossing number of $P_5^2 \times C_n$

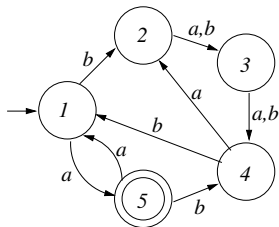
Daniela Kravecová, Marián Klešč

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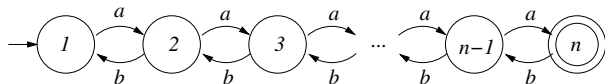
Patil and Krishnamurthy established family of graphs for which power graphs have crossing number one. This is the only result concerning crossing numbers of power of some graphs. Let P_m^2 denote the power of the path P_m . We start to determine crossing numbers of a new infinite family of graphs, concretely for the Cartesian products $P_m^2 \times C_n$ where $m \geq 2$ and $n \geq 3$. The main result is that the crossing number of the graph $P_5^2 \times C_n$ is $4n$ for all $n \geq 3$.

In a binary case, we get $2^{\Theta(n^2)}$ state complexity. This is extremely hard operation: while the star of a 5-state language requires at most 24 states, the cyclic shift, in the worst case, requires 56 million!

We also investigate the nondeterministic state complexity of this operation. In contrast to the hard deterministic case, the nondeterministic state complexity of cyclic is as low as $2n^2 + 1$. The worst case example is defined over a binary alphabet.



Obr. 1: A 5-state language that requires 56 817 428 states for its cyclic shift.



Obr. 2: A language that requires $2n^2 + 1$ nondeterministic states for its cyclic shift.

Some generalisations of conditioning and joint distributions

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The classical model of a probability space was introduced by Kolmogorov in 1933. Shortly after Kolmogorov's construction was published it was clear that there are situations which cannot be modeled using the Kolmogorov

Classification of Π_1^1 -complete sets

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The Hierarchy of sets gives us many possibilities of characterizations of Π_1^1 (coanalytic) resp. Σ_1^1 (analytic) sets in a Polish space. M. J. Souslin showed that $Borel = \Sigma_1^1 \cap \Pi_1^1$, and this arises us the question of describing Π_1^1 sets which are not Σ_1^1 sets. The basic concept is that of a Π_1^1 -complete set. A. S. Kechris used the notion Π_1^1 -complete sets on hyperspaces in his Dichotomy Theorem. He presented a proof based on game theory but we simplified it by using our generalization of the Hurewicz Theorem.

Non-inscribability and hard inscribability

Sergej Ševc

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Non-inscribable phenomenon as well as its more sophisticated parallel in the class of inscribable polyhedra is discussed. We call the latter hard inscribability. Much analogy and interconnections implied between non-inscribability and hard inscribability are revealed, chiefly by quantitative methods. The research is accomplished both in graph-theoretical and geometrical lines. Adequate measures, conditions, and presuppositions are defined. New results are compared with those obtained formerly on non-inscribability exponent and respective kinds of (non-)inscribability.

Invited lectures

Some polynomial solvable problems in discrete optimization

Štefan Berežný

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Many known polynomial solvable problems on graphs become an insolvable in polynomial time or known NP-complete problems are solvable in polynomial time, if we put some more appropriate conditions. One of condition could be the partition of edge set to the n disjoint categories. This condition may cause worsening of solvability of given problem.

We show some modifications of optimisation problems on graphs with categorization of edges, which can be solvable in polynomial time. It is needed to find the feasible edge set (spanning tree, a-b path, etc.), that it minimize the special type of objective functions with respect to choices categories in problems listed above.

Schur complement and Gram matrix in time series analysis

Martina Hančová

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The first goal of the lecture that is aimed for a wider mathematical audience is to explain fundamental ideas of time series analysis using one of important approaches — linear regression modeling. The second lecture goal is to introduce some special matrices and their properties, particularly the Schur complement and Gram matrix, which play the central role in getting the most of results of such time-series-analysis approach.

Let \mathcal{P} be a given property of graphs and $\mathcal{G}(\mathcal{P})$ be the family of all graphs having property \mathcal{P} . Then

$$W(\mathcal{P}) = \max_{G \in \mathcal{G}(\mathcal{P})} \left\{ \min_{e \in E(G)} w(e) \right\}.$$

We study the behaviour of $W(\mathcal{P})$ with respect to different hereditary properties of graphs, i.e. properties that are closed with respect to taking subgraphs.

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- [2] S. Jendroľ, I. Schiermeyer, *On a max-min problem concerning weights of edges*, Combinatorica, **21** (2001), 351–359.

Toho roku pozvanie prednášať prijali: dr. Berežný (FEI TU Košice), dr. Hančová (PF UPJŠ Košice), doc. Horňák (PF UPJŠ Košice), dr. Jirásková (MÚ SAV Košice), prof. Kalina (SvF STU Bratislava), prof. Kopka (PrF ÚJEP Ústí nad Labem), prof. Lukáčová-Medvid'ová (TU Hamburg-Harburg), prof. Podlubný (FBERG TU Košice) a dr. Semanišinová (PF UPJŠ Košice).

Prajeme Vám príjemný pobyt v Herľanoch

Organizačný výbor:
Stanislav Jendroľ
Ján Buša
Štefan Schrötter

17⁵⁰ – Jana Pócsová (ÚMV PF UPJŠ) *Qualitative evaluation of the probability of events (Galton desk at the mathematics lesson)*

18³⁰ – Večera

Piatok 24. 4. 2009

8³⁰ – Ingrid Semanišinová (ÚMV PF UPJŠ) *Diskrétna matematika vo vyučovaní*

9¹⁵ – Galina Jirásková (MÚ SAV Košice) *Cyklický posun regulárnych jazykov a jeho stavová zložitosť*

10⁰⁰ – Kávová prestávka

10³⁰ – Igor Podlubný (ÚRIVP BERG TU) *Maticový prístup k diskretizácii obyčajných a parciálnych diferenciálnych rovníc ľubovoľného reálneho rádu*

11²⁰ – Mária Lukáčová-Medvid'ová (TU Hamburg) *Numerické modelovanie prúdiacich tekutín*

12³⁰ – Obed

14⁰⁰ – Martin Kalina (KMaDG SvF STU Bratislava) *Podmieňovanie na nebolovských priestoroch*

15⁰⁰ – Jan Kopka (PF ÚJEP, Ústí nad Labem) *Některé strategie řešení matematických problémů (přeformulování, zobecnění)*

16⁰⁰ – Kávová prestávka

16³⁰ – Mirko Horňák (ÚMV PF UPJŠ) *Achromatický index kompletného grafu verzus konečné projektívne roviny*

18⁰⁰ – Večera

19⁰⁰ – Spoločenský večer

Sobota 25. 4. 2009

8⁴⁵ – Martina Hančová (ÚMV PF UPJŠ) *Schurov doplnok a Gramova matica v analýze časových radov*

9³⁰ – Kávová prestávka

10⁰⁰ – Štefan Berežný (KM FEI TU) *Niektoré polynomiálne riešiteľné problémy v diskretnej optimalizácii*

11⁰⁰ – Obed

Táto publikácia vychádza pri príležitosti 50. výročia založenia Univerzity Pavla Jozefa Šafárika v Košiciach a 40. výročia založenia Fakulty elektrotechniky a informatiky TU v Košiciach.

Editori: Ján Buša, Stanislav Jendroľ, Štefan Schrötter

ISBN 978-80-553-0187-7

Sadzba programom pdfT_EX

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