Some experiences with the diversity in word problems

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Introduction

The critical reading of a text is one of the abilities, which cultivation stands in the last years in the centre of attention of teachers. This ability can be improved by solving word problems in mathematics, if an accident or a mistake in thinking of pupils occures. But this can be reached also purposely by solving such word problems, whose texts can be understood in different ways, where each of them is correct and tolerable by the text. Let us call this type of word problems the fuzzy word problems.

Unique interpretation of the text is a standart atribute of word problems in mathematics. From this point of view fuzzy word problems were seen as uncorrect, uncomplete or faulty. Recently, some papers appeared, which are related to the variety of solutions of word problems (see [2]). This inspired us to study a variety of interpretations of a word problem. Some of our ideas will be published in [1]. If only one interpretation is possible (in the case of standard word problems) it is very difficult to determine and to describe the key moments of grasping a word problem. They usually appear only by a mistake in a solution. In fuzzy word problems we can detect them by various but correct interpretations occured in solutions. In this contribution we therefore analyze elements, which are significant for the interpretation of a text. It is useful to know about their existence, so we could easier find the source of a mistake, in case of a faulty solution. We recognized, that exactly fuzzy word problems can help us to understand the whole process of grasping a word problem by pupils and so to determine and describe factors, which are relevant for choosing an interpretation of an text. We performed an investigation, which results are described below. In the conclusion of this contribution we have formulated a plan of research of factors, whose we have found they take part in choosing the interpretation of the text of a word problem.

Texts of the word problems

An investigation of reactions on fuzzy problems was performed in the framework of The club of math teachers, where each month about twenty teachers from primary and secondary schools meet. The concrete word problems were chosen among of word problems, whose teachers had encountered in their praxis, and considered them as fuzzy word problems. Some texts needed to be modified. Original fuzzy word problem, sent by a teacher was:

> *Suppose that a cube has edge 6 cm. Create a prism with the base 3 cm × 4 cm from this cube. Find its surface area.*

Teacher's original two solutions were attached:

The first solution: *If to create means to cut, then the heights of both solids are equal and thus the surface area is 108 cm²*.
The second solution: *If to create means to mould, then the volumes of both solids are equal and hence the surface area of the prism is 276 cm²*.
We have reformulated this problem into the problem B below.

Teachers, who have wanted to take part in an investigation, obtained texts of next three word problems.

A: *Suppose 12 same cubes. How many prisms can you compose from them?*

B: *Suppose a cube has edge 5 cm. Create a prism with the base 3 cm ×4 cm from this cube. Find its maximal height.*

C: *The garden for hens is 5 m long and 3 m wide. From three sides, there is a fencing, and on the fourth side stands a hen-house. In order not to let hens sneak the fence, it is necessary to line the garden. You can use four 2 m long boards, one 3 m long board and one 5 m long board. How can you do this? Draw possibilities of lining.*

In the centre of our observation there is the word problem B. The problem C is unimportant for this article. On opposite to this, the word problem A can not be ignored, as we will see.

Instructions to solutions

These three word problems had to be given to pupils (from the 5^{th} class of an elementary school to the 1st class of a secondary school) in the written form, without loud reading. Intonation, various volume of a voice or accent on some word could direct pupils at grasping of a word problem.

Hereby, we asked teachers to warn pupils not to ask questions on these word problems. They must find alone an approach to each vagueness in the text of a word problem. The teachers were not allowed to explain or to precise the texts of these three word problems. We wanted to be sure that the pupils are not influenced by teachers or by hearing any questions from classmates.

Next instruction was, to divide the time of solving to all three word problems equally, if possible. We wanted each pupil to solve each word problem. Pupils had 45 minutes for it.

Finally, teachers had to ask pupils to comment their solutions, so that we could more precisely analyse the obtained solutions. Teachers should not specify the type of a commentary. Pupils could decide on their own, how to express themselves (words, pictures). This let us understand their solutions more completely. Sketches and auxiliary calculations also had to be added to their solutions.

Aims and results

The main aims of our investigation were

- to understand the process of grasping a word problem by pupils and to explain the influence of various factors on choosing an interpretation (current subject matter, life experience, solving of a related problem, signal word in the text of the word problem, solving of a preceding problem),
- to acquire notion of pupils' reactions on fuzzy word problems (what feelings they experience, what attitudes they take),
- to gain study material for preparation of a research with the intention to identificate factors significant for choosing an interpretation, and explain how exactly they take part on choosing an interpretation of a word problem.

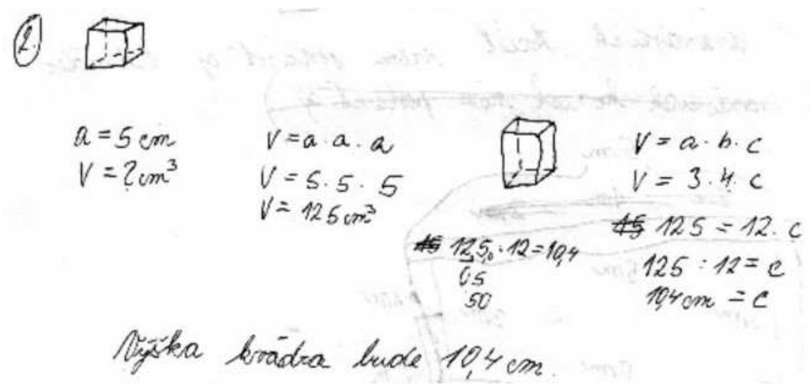
We present some of our results on the problem B in the following. Having prepared those fuzzy word problems, we have expected that pupils will interpret the problem B in one or more ways, named by us as "Volume", "Cutting", "125 cubes", "Stacking". We neglected the frequency of occurrence of these interpretations. Questions were: Will pupils notice the diversity in the text of this word problem? Will they interpret and solve this word problem in one or more ways? We have expected that life experience of pupils is the main factor relevant for choosing an interpretation.

We have obtained 263 solutions from 321 pupils. Next table shows numbers of pupils in particular classes:

Class	5	6	7	8	9	1
Pupils	92	71	42	43	58	15

1. "Volume"

Sample:



What is the height of a prism with the base 3 cm × 4 cm, if its volume is equal to the volume of a cube with the edge 5 cm ?

Model solution:
The volume of a cube with the edge 5 cm is $5^3 = 125$ cm^3 *. The volume of a prism with the base 3 cm × 4 cm, and height v is 3 × 4 v = 12× v. Because those volumes are equal, it is 125 = 12 × v and so v = 125/12 = 10,416 cm.*

2. "Cutting"

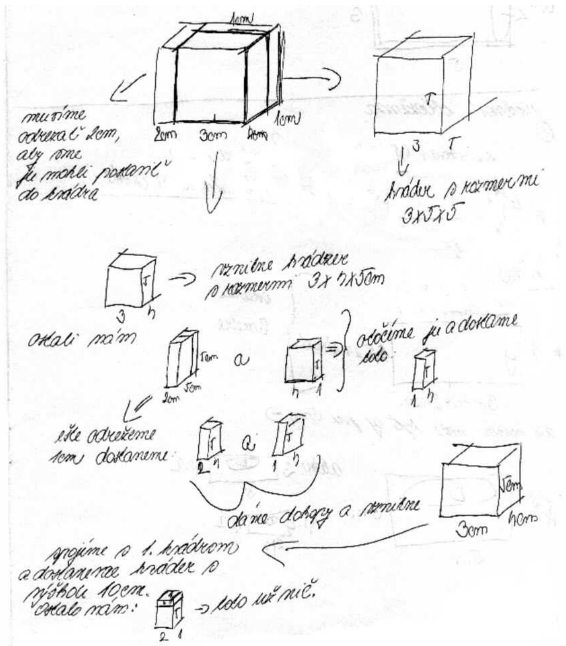
What is the maximal possible height of a prism with the base 3 cm × 4 cm, which was cut from a cube with the edge 5 cm ?

To state the precise value means to have a serious knowledge and skills of higher mathematics. Nevertheless, great number of pupils cut the cube paralelly with its sides and found an answer to the simplified problem.

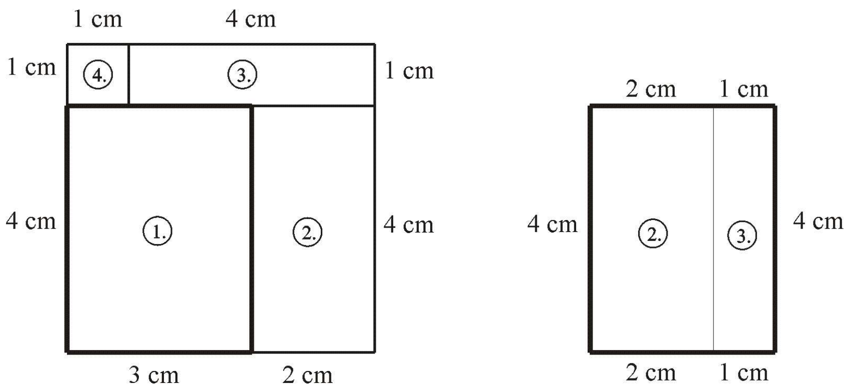
Sample (translated from a slovak written sample):
It is 5 cm, because the cube has the edges 5 cm. So the height of the prism cannot be longer.

3. "Stacking"

What is the maximal possible height of a prism with the base 3 cm × 4 cm, which arised by cutting a cube with the edge 5 cm into four prisms and stacking three of them?
Sample:



Model solution:



The cube will be divided into four 5 cm high prisms (1. - 4.) with their bases 4 cm × 3 cm, 2 cm × 4 cm, 1 cm × 4 cm and 1 cm × 1 cm, respectively. The prisms 2. and 3. will be composed to the prism with the base 3 cm × 4 cm and the height 5 cm. The same prism as the prism 1. arises. We put these two same prisms on each other and we obtain a prism with the base 3 cm $\times$ 4 cm and the height 10 cm.

4. "It is impossible"

What is the maximal possible height of a prism with the base 3 cm $\times$ 4 cm, which can be created by cubes with the edge 5 cm ?

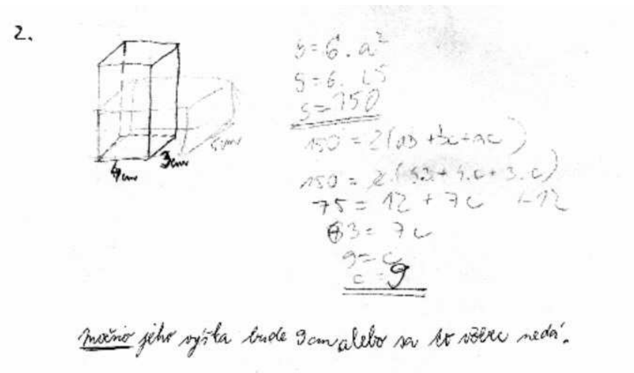
Model solution (translated from a slovak written sample):
Such a prism can not be created, because the edge of a cube is longer than 3 or 4 cm.

5. "Surface area"

What is the height of a prism with the base 3 cm $\times$ 4 cm, if its surface area is equal to the surface area of a cube with the edge 5 cm ?

Model solution:

Sample:



The surface area of a cube with the edge 5 cm is $6 \times 5^2 = 150cm^2$*. The surface area of a prism with the base 3 cm $\times$ 4 cm, and height v is*

$$2(3.4 + 3.v + 4 \times v) = 24 + 14 \times v.$$

Because those surface areas are equal, we have

$$150 = 24 + 14 \times v \text{ and so } v = (150 - 24)/14 = 9cm.$$

6. "125 cubes"

What is the maximal possible height of a prism with the base 3 cm $\times$ 4 cm, which arised from a cube with the edge 5 cm by cutting into cubes with the edge 1 cm ?

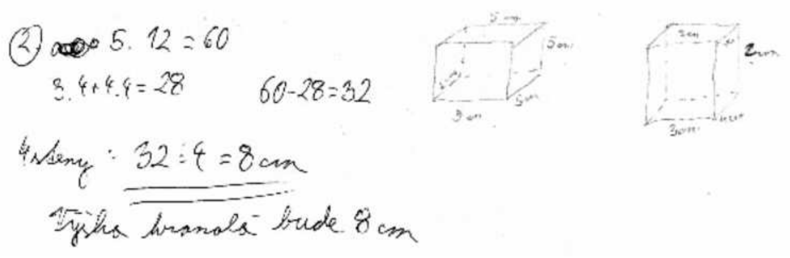
Model solution (translated from a slovak written sample):
By cutting a cube with the edge 5 cm arised 5^3 = 125 small cubes with the edge 1 cm. For the base and for each 1 cm of a height of a prism, 3 $\times$ 4 = 12 small cubes are necessary. Because 125/12 = 10 where the remainder is 5, the maximal possible height of a such prism is 10 cm.

7. "Edges"

What is the height of a prism with the base 3 cm $\times$ 4 cm, if the sum of all its edges is equal to the sum of all edges of a cube with the edge 5 cm ?

Model solution:
The sum of all edges of a cube is 12$\times$5 = 60 cm. The sum of all edges of a prism with the base 3 cm $\times$ 4 cm, and height v is 4$\times$3 + 4$\times$4 + 4$\times$v = 28 + 4$\times$v. Because those sums are equal, we have 60 = 28 + 4$\times$v and so v = (60 - 28)/4= =8 cm.

Sample:



Conclusions

Pupils of the 5^{th} class do not learn the term of prism, and so only one from 92 pupils was able to solve this problem. In opposite to this, in the 1^{st} class of the secondary school 9 from 15 pupils solved this problem as "Volume", one as "Cutting" and rest of solutions was misty. So we decided to analyze only solutions obtained from 6^{th} to 9^{th} class of elementary schools. From these 214 pupils, 186 of them began to solve this word problem. The others did not start solving. 110 pupils grasped this word problem and their interpretations were recognizable.

Analysis showed some suprising facts. Pupils have found three more interpretations of this word problem, namely "Surface area", "Edges", "It is impossible". Only few pupils noticed the diversity in this problem and interpreted it in more than one way.

The next table shows a review of numbers of all seven interpretations in each class.

Interpretation	class 6	class 7	class 8	class 9	sum
Volume	14	3	14	20	51
Cutting	7	6	13	7	33
Stacking	3	0	0	7	10
It is impossible	1	1	4	3	9
Surface area	2	3	0	2	7
125 cubes	1	0	1	3	5
Edges	1	1	0	2	4

We also recognized the following elements significant for choosing an interpretation:

- current subject matter (among of all 22 pupils of a class 14 have interpreted this word problem correctly and 13 from them as "Volume"),
- solving of a preceding problem (9 pupils decided for "It is impossible" and 5 for "125 cubes").

The teacher, sending the original word problem, and the authors of the paper did not aware of the importance of signal words. We suppose the word *surface area* as well as *edge* is a signal word for interpreting of corresponding texts.

In the future it would be useful

- to compare pupils by the dispersion of their interpretations; we expect it to decrease with the increasing age of pupils,
- to observe dependence on the preceding solved problem; Problem A brings up the interpretations "125 cubes" and "It is impossible",
- to study influence of the current subject matter; for instance to have just learnt volume formulas of solids,
- to investigate the reactions before and after the contact with fuzzy word problems; the number of interpretations increases.

References

[1] Harminc, M.: Slovné úlohy s rôznym výkladom, Dva dny s didaktikou matematiky, Praha 2006 (to appear).

[2] Hejný, M.: Rozmanitost řešení žáků jako diagnostický nástroj edukačního stylu, Letná škola z teórie vyučovania matematiky PYTAGORAS 2005, 19-31.

[3] Hejný, M., Michalcová, A.: Skúmanie matematického riešiteľského postupu, Bratislava 2001.