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C&C
HIGH TATRAS, SLOVAKIA

Igor Fabrici, Mirko Horňák, Stanislav Jendroľ ed.

Workshop
Cycles and Colourings 2009

September 06 – 11, 2009 Tatranská Štrba

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Dear Participant,

welcome to the Eighteenth Workshop Cycles and Colourings. Except for the first workshop in the Slovak Paradise (Čingov 1992), the remaining sixteen workshops took place in the High Tatras (Nový Smokovec 1993, Stará Lesná 1994–2003, Tatranská Štrba 2004–2008).

The series of C&C workshops is organised by combinatorial groups of Košice and Ilmenau. Apart of dozens of excellent invited lectures and hundreds of contributed talks the scientific outcome of our meetings is represented also by special issues of journals Tatra Mountains Mathematical Publications and Discrete Mathematics (TMMP 1994, 1997, DM 1999, 2001, 2003, 2006, 2008 – under preparation).

The scientific programme of the workshop consists of 50 minute lectures of invited speakers and of 20 minute contributed talks. This booklet contains abstracts as were sent to us by the authors.

Invited speakers:

Mieczysław Borowiecki	University of Zielona Góra, Zielona Góra, Poland
Gyula O. H. Katona	Hungarian Academy of Sciences, Budapest, Hungary
Daniel Král'	ITI, Charles University, Prague, Czech Republic
Andrzej Ruciński	Adam Mickiewicz University, Poznań, Poland
Jozef Širáň	Slovak University of Technology, Bratislava, Slovakia
Eberhard Triesch	RWTH Aachen University, Aachen, Germany
Richard M. Wilson	California Institute of Technology, Pasadena, CA, USA

Have a pleasant and successful stay in Tatranská Štrba.

Organising Committee:

Igor Fabrici
Jochen Harant
Erhard Hexel
Mirko Horňák
Stanislav Jendroľ (chair)
Dieter Rautenbach
Štefan Schrötter

Contents

Extremal problems on some versions of the chromatic number

Gábor Bacsó

(joint work with Mihály Hujter and Zsolt Tuza)

The relation of three graph parameters, the traditional chromatic number ($\chi(G)$), the Grundy number ($\Gamma(G)$) and the achromatic number ($a(G)$) is discussed.

It can be easily seen that $\chi(G) \leq \Gamma(G) \leq a(G)$ is valid. The following statement is known: given three numbers $f \leq g \leq h$, there exists a graph G_0 with $\chi(G_0) = f$, $\Gamma(G_0) = g$ and $a(G_0) = h$ (with a few exceptions).

– However, even good estimations are not known for the minimum of $|V(G_0)|$. This is the subject of the lecture.

Crossings and colorings

János Barát

(joint work with Géza Tóth)

Mike Albertson conjectured that if a graph G has chromatic number r then its crossing number is at least as much as the crossing number of K_r. Apparently, this conjecture is closely related to the Hajós conjecture. If a graph G satisfies the Hajós conjecture, then Albertson's conjecture also holds for G. Erdős and Fajtlowicz [2] proved that almost all graphs are counterexamples to the Hajós conjecture. Therefore, there should be enough ground against Albertson's conjecture as well. However, all results seems to talk for the validity of the conjecture so far.

Albertson, Cranston, and Fox [1] verified the conjecture for $r \leq 12$. They also showed that any counterexample to Albertson's conjecture must have less than $4r$ vertices.

In this talk, we present the following improvements of these results: we verify the conjecture for $r \leq 16$, and show that any counterexample has at most $3.57r$ vertices. Our results are based on the theory of critical graphs settled by Gallai [3], Kostochka and Stiebitz [4] and on variations of the crossing lemma proved by Pach et al. [5].

References

[1] M. Albertson, D. Cranston, J. Fox, Crossings, Colorings and Cliques, Electron. J. Combin. 16 (2009), #R45.

[2] P. Erdős, S. Fajtlowicz, On the conjecture of Hajós. Combinatorica 1 (1981), 141–143.

[3] T. Gallai, Kritische Graphen. II., Magyar Tud. Akad. Mat. Kutat Int. Közl. 8 (1963), 373–395 (german).
[4] A. V. Kostochka, M. Stiebitz, Excess in colour-critical graphs, In: Graph Theory and Combinatorial Biology (Balatonlelle 1996), Bolyai Soc. Math. Stud. 7 (1999), 87–99.
[5] J. Pach, R. Radoičić, G. Tardos, G. Tóth, Improving the crossing lemma by finding more crossings in sparse graphs, Discrete Comput. Geom. 36 (2006), 527–552.

Generalized graph colourings with local constraints

Mieczysław Borowiecki

We consider finite undirected graphs without loops or multiple edges. Let $\mathcal{I}$ denote the class of all graphs. A *property (class) of graphs* is any nonempty class of graphs from $\mathcal{I}$ which is closed under isomorphisms. A property $\mathcal{P}$ is called (*induced*) *hereditary* if every (induced) subgraph of any graph with property $\mathcal{P}$ also has property $\mathcal{P}$.

Many difficult (NP-hard) optimization problems on graphs become tractable when restricted to some classes of graphs, usually to hereditary properties. A large part of these problems can be expressed in the vertex partitioning formalism, i.e., by partitioning of the vertices of a given graph into subsets $V_1, \ldots, V_k$ called *colour classes*, satisfying certain constraints either internally or externally, or both: internally and externally. This requirements may be conveniently captured by symmetric k-by-k matrix $\boldsymbol{M}$ in which the diagonal entries $m_{ii} = \mathcal{P}_i$ encode the internal restrictions on the sets V_i and the off-diagonal entries $m_{ij} = \mathcal{P}_{ij}$ $(i \neq j)$ encode the restriction on the edges between V_i and V_j.

Concepts which are modeled by $\boldsymbol{M}$-partitions fall naturally into the three types, each of them will be represented in this work by some problems.

Small(est) edge sets meeting all triangles

Csilla Bujtás
(joint work with S. Aparna Lakshmanan and Zsolt Tuza)

Given a graph G we consider the smallest cardinality $\tau_\triangle$ of an edge set containing at least one edge from every triangle of the graph. This parameter can be bounded in terms of the largest number $\nu_\triangle$ of mutually edge-disjoint triangles of G.

It was conjectured by Tuza in 1981 that $\tau_\triangle \leq 2\nu_\triangle$ holds for every graph. The conjecture has been verified for several graph classes but the problem is still open in general. We prove it for further classes of graphs. Moreover, we describe sufficient conditions for the equality $\tau_\triangle = \nu_\triangle$.

Some antimagic results

Roman Čada

A conjecture due to Hartsfield and Ringel (1990) says that all graphs (except K_2) are antimagic (a graph with m edges is antimagic if the integers $1, \ldots, m$ can be assigned in a 1-1 manner to edges in such a way that for every two distinct vertices their associated sums of labels of all edges incident with them are different).

Until now several results of antimagicness of particular graph classes (e.g. dense graphs) and some graphs obtained from specific constructions are known. We give some particular results supporting the conjecture. We also deal with labellings of hypergraphs.

Looseness of plane graphs

Július Czap

(joint work with Stanislav Jendroľ, František Kardoš and Jozef Miškuf)

All considered graphs are finite, loops and multiple edges are allowed. Let $G = (V, E, F)$ be a connected plane graph with the vertex set V, the edge set E and the face set F.

A *k-colouring* of a graph G is a mapping $\varphi : V \to \{1, \ldots, k\}$.

For a face $f \in F$ we define $\varphi(f)$ to be the set of colours used on the vertices incident with the face f. A face f is called *loose* if $|\varphi(f)| \geq 3$.

Question: What is the minimum number of colours $ls(G)$ that any surjective vertex colouring of a connected plane graph G with $ls(G)$ colours enforces a loose face?

The invariant $ls(G)$ of a plane graph G is called the *looseness* of G.

We prove that the looseness of a connected plane graph G equals 2 plus the maximum number of vertex disjoint cycles in the dual graph G^*.

We also show upper bounds on the looseness of graphs based on the edge connectivity, the girth of the dual graphs and other basic graph invariants. Moreover, we present infinite classes of graphs where these equalities are attained.

References

[1] J. Czap, S. Jendroľ, F. Kardoš and J. Miškuf, Looseness of plane garphs, Preprint 2009.

Acyclic colourings of graphs

Anna Fiedorowicz

An *acyclic edge k-colouring* of a graph G is a proper edge k-colouring of G such that there are no bichromatic cycles. In other words, for every two distinct colours i and j, the subgraph induced in G by all the edges which have either colour i or j is acyclic. The *acyclic chromatic index* of G is the minimum k such that G has an acyclic edge k-colouring, denoted by $\chi'_a(G)$.

In 1991, Alon, McDiarmid, and Reed, introduced the acyclic chromatic index of a graph and proved that $\chi'_a(G) \leq 64\Delta(G)$ for any graph G of maximum degree $\Delta(G)$. Later, Molloy and Reed improved this bound to $16\Delta(G)$. Unfortunately, these bounds are not sharp. In fact, it was conjectured by Alon, Sudakov and Zaks that $\chi'_a(G) \leq \Delta(G) + 2$ for all graphs G. This conjecture has been by now verified only for some special classes of graphs. In general, the problem of computing the acyclic chromatic index of a graph is NP-complete.

In this talk we present upper bounds for the acyclic chromatic index of some classes of graphs. Moreover, we discuss algorithmic aspects of acyclic edge colourings.

Upper bounds on the sum of the squares of the degrees of a triangle-free *k*-chromatic graph

Jochen Harant

(joint work with Steffi Pflugradt)

For a simple triangle-free k-chromatic graph G with $k \geq 2$ we prove the upper bound $m(n - f(k-2))$ on the sum of the squares of the degrees of G, where n, m, and $f(l)$ are the order of G, the size of G, and the minimum order of a triangle-free l-chromatic graph, respectively. Consequences of this bound are discussed.

On a packing colouring of the square lattice

Přemysl Holub

(joint work with Roman Soukal)

The concept of a packing colouring is related to the frequency assingment problem. The packing chromatic number $\chi_p(G)$ of a graph G is the smallest integer k such that the vertex set $V(G)$ can be partitioned into disjoint classes $X_1, \ldots, X_k$, where vertices in X_i have pairwise distance greater than i. In this note we improve the upper bound on the packing chromatic number of the square lattice.

Groups, graphs, and distance two colorings

Robert E. Jamison

A coloring of the vertices of a graph G is a *distance two coloring* of G iff any two vertices at distance two or less in G are given different colors. The minimum number of colors required by a distance two coloring is often denoted by $\chi_2(G)$. Since any two vertices in a closed neighborhood in a graph G are at distance at most two in G, we get the trivial lower bound $1 + \Delta(G) \leq \chi_2(G)$. We say that G is *tight* iff equality holds. In this talk, we will study tightness, especially for products of complete graphs, trees, and cycles. In general determining the distance two chromatic number can be rather difficult. Tightness provides an easy way to obtain the distance two chromatic number $\chi_2(G)$ since the bound $1+\Delta(G)$ is easily calculated. In particular we will show that the product of trees, each with even maximum degree, is always tight.

Coloring cycles

Gyula O. H. Katona

Motivated by the title of the conference, I suggested the following problem in Stará Lesná in 2002. Determine the minimum number of colors needed to color all cycles of length k (fixed, ≥ 3) in K_n in such a way that edge-disjoint cycles get different colors. For $k = 3$ this is equivalent to the following problem. Partition the family of all three-element subsets (triangles) of an n-element set in such a way that each class consists of either subsets of a 4-element set or some triangles containing a two-element set. Zsolt Tuza and I determined this minimum as

$$\left\lfloor \frac{(n-1)^2}{4} \right\rfloor \quad (6 \leq n).$$

Generalizations are also considered.

Some algorithmic notes on the Minimum rainbow subgraph problem

Ján Katrenič
(joint work with Ingo Schiermeyer)

We consider the Minimum Rainbow Subgraph problem (MRS): Given a graph G of n vertices, whose edges are coloured with p colours. Find a subgraph $F \subseteq G$ of minimum order and with p edges such that each colour occurs exactly once.

In this talk we show that for each $\epsilon > 0$ there is a polynomial time approximation algorithm on MRS with performance ratio $1 + (0.5 + \epsilon)\Delta$. This improves the ratio obtained in [1]. We also discuss parameterized complexity for MRS depending on the size of p and classify several cases of the problem for which the optimal solution can be found in time $O(2^p n^{O(1)})$ or $O(p! n^{O(1)})$, respectively.

References

[1] S. Matos Camacho, I. Schiermeyer, Z. Tuza, Approximation algorithms for the minimum rainbow subgraph problem, Preprint 2008.

Hamiltonicity in vertex-deleted hypercubes

Arnfried Kemnitz
(joint work with Heiko Harborth)

All cases such that a hypercube with 0, 1, or 2 arbitrary deleted vertices contains a hamiltonian path between any two of the remaining vertices and all cases such that a hypercube with up to 7 arbitrary deleted vertices contains a hamiltonian cycle are determined.

Maximum weight of a connected graph of given order and size

Maria Koch
(joint work with Ingo Schiermeyer, Stanislav Jendroľ and Mirko Horňák)

The weight of an edge $e = xy$ of a graph G is $w(e) := \deg_G(x) + \deg_G(y)$ and the weight of G is $w(G) := \min(w(e) : e \in E(G))$. For a positive integer n, $m \in \{0, \ldots, \binom{n}{2}\}$ and a graph property $\mathcal{P}$ let

$$w(n, m, \mathcal{P}) := max(w(G) : |V(G)| = n, |E(G)| = m, G \in \mathcal{P}).$$

At Czechoslovak Symposium on Combinatorics, Graphs and Complexity in 1990 Erdős posed the problem of determining $w(n, m, \mathcal{I})$ for the most general property $\mathcal{I}$ of all graphs. The problem has been solved first partially by Ivančo and Jendroľ and then completely by Jendroľ and Schiermeyer.

$$G \in \mathcal{C} \leftrightarrow G \text{ is connected.}$$

This talk will present partial results for $w(n, m, \mathcal{C})$.

Linear programming proofs in graph theory

Daniel Kráľ

Linear programming offers variety of powerful tools and results. In this talk, we will focus on applications in graph theory which are related to the perfect matching polytope of a graph, the convex hull of the characteristic vectors of its perfect matchings. We will survey a variety of results, ranging from results on the numbers of perfect matchings in graphs through existence of special matchings in cubic bridgeless graphs to Erdős-Pósa-type results on odd cycles in planar graphs.

Partitioning a graph into a dominating set, a total dominating set, and something else

Christian Löwenstein
(joint work with Michael A. Henning and Dieter Rautenbach)

A recent result of Henning and Southey [1] implies that every connected graph G of minimum degree at least 3 has a dominating set D and a total dominating set T which are disjoint. We show that the Petersen graph is the only such graph for which $D \cup T$ necessarily contains all vertices of the graph G.

References

[1] M. A. Henning, J. Southey, A note on graphs with disjoint dominating and total dominating set, Ars Combin. 89 (2008), 159–162.

On exact scale-free graphs

Tomáš Madaras

A graph G is called *scale-free* if its degree distribution follows (at least asymptotically) a power law (that is, the relative number $P(k)$ of vertices of given degree k is asymptotically equal to $k^{-\gamma}$ for some constant γ, usually being positive). This graph family is widely studied from the nineties of 20th century since it was found that many real-world networks of different nature (for example, World Wide Web, protein-protein interaction networks, collaboration and citation networks, semantic networks) show the scale-free behaviour. Nevertheless, the general notion of a scale-free graph admits many interpretations leading to different definitions that are used in the literature; this ambiguity often follows from statistical methods that are frequently used for study of this graph family.

In our contribution, we propose more "traditional" and deterministic approach to scale-free graphs by defining the *exact scale-free graphs*, for which the equality $P(k) = ck^{-\gamma}$ (with constants c, γ) is satisfied for each degree k of the degree sequence. We present selected preliminary results on exact scale-free graphs concerning their existence, constructions, range of degree sequence and estimations on number of edges.

Some approximation algorithms on the Minimum rainbow subgraph problem

Stephan Matos Camacho

(joint work with Ingo Schiermeyer and Franziska Heinicke)

It is widely anticipated that a comprehensive knowledge on variations in the human genome is the key to predicting risk of a variety of complex diseases. A very common form of such genomic variations are single nucleotide polymorphisms (SNPs). Arising from this, we are interested in finding a set $\mathcal{H}$ of haplotypes explaining a given set of $\mathcal{G}$ of genotypes, where $\mathcal{H}$ has minimum cardinality. We reformulated this biological problem into a graph theoretical one as follows:

The Minimum Rainbow Subgraph problem (MRS)
Given a graph G, whose edges are coloured with p colours, find a subgraph $H \subseteq G$ of G of minimum order $r^*(G)$ with $|E(H)| = p$ such that each colour occurs exactly once.

Since the decision problem in finding a set H of given size is NP-complete, we are interested in good and fast approximation algorithms. This talk will be dedicated to such algorithms. We although will present some experimental results obtained for random graphs and biological data.

Hereditary properties of graphs and monotone invariants

Peter Mihók

(joint work with Janka Oravcová and Roman Soták)

We will consider monotone graph invariants such as chromatic number, colouring number, clique number, independence number, etc. We will show how these invariants relate to the structure of additive and hereditary graph properties. Then fractional and circular invariants will be considered. Let $\mathcal{P}$ be an additive and hereditary property of graphs and $r, s \in \mathbb{N}$. A circular $(\mathcal{P}, r, s)$-colouring of a graph G is an assignment $f : V(G) \to [0, r-1]$, such that edges of G, consisting of vertices $u, v \in V(G)$, for which $|f(u) - f(v)| < s$ or $|f(u) - f(v)| > r - s$, induce a subgraph of a graph G with the property $\mathcal{P}$. We will also present some basic properties of generalized circular chromatic number of graphs.

Domination in products of digraphs

Ľudovít Niepel

(joint work with Martin Knor)

Let $G = (V, E)$ be a digraph. Vertex u dominates vertex v if arc $uv \in E$. Set $D \subseteq V(G)$ is a dominating set of G if each vertex from $V - D$ is dominated by at least one vertex in D. The minimum cardinality $\gamma(G)$ of a dominating set of G is dominating number of G. Set T is a total dominating set of G if each vertex in V is dominated by at least one vertex from T. The minimal cardinality of a total dominating set $\gamma_t(G)$ is total dominating number of G. The cross product $G \times H$ of G and H is the digraph with vertex set $V(G \times H) = V(G) \times V(H)$ and $(u, v)(u', v') \in E(G \times H)$ if and only if $uu' \in E(G)$ and $vv' \in H$. We shall present lower and upper bounds of dominating and total dominating numbers of products of digraphs in terms of dominating and packing numbers of their factors.

On the complexity of paths avoiding forbidden pairs

Ondřej Pangrác

(joint work with Petr Kolman)

Given a graph $G = (V, E)$, two fixed vertices $s, t \in V$ and a set F of pairs of vertices (called *forbidden pairs*), the *problem of a path avoiding forbidden pairs* is to find a path from s to t that contains at most one vertex from each pair in F.

The problem is known to be NP-complete in general and a few restricted versions of the problem are known to be in P. We study the complexity of the problem for directed acyclic graphs with respect to the structure of the forbidden pairs.

Minimum degree and density of binary sequences

Dieter Rautenbach

(joint work with Stephan Brandt, Janina Müttel, and Friedrich Regen)

For $d, k \in \mathbb{N}$ with $k \leq 2d$, let $g(d,k)$ denote the infimum density of binary sequences $(x_i)_{i\in\mathbb{Z}} \in \{0,1\}^{\mathbb{Z}}$ which satisfy the minimum degree condition $\sum_{j=1}^{d}(x_{i+j} + x_{i-j}) \geq k$ for all $i \in \mathbb{Z}$ with $x_i = 1$. We reduce the problem to determine $g(d,k)$ to a combinatorial problem related to the generalized k-girth of a graph G which is defined as the minimum order of an induced subgraph of G of minimum degree at least k. Extending results of Kézdy and Markert, and of Bermond and Peyrat, we present a minimum mean cycle formulation which allows to determine $g(d,k)$ for small values of d and k. For odd values of k with $d+1 \leq k \leq 2d$, we conjecture $g(d,k) = \frac{k^2-1}{2(dk-1)}$ and show that this holds for $k \geq 2d-3$.

Complexity of shortest cycle packings

Friedrich Regen

(joint work with Dieter Rautenbach)

We study the problems to find a maximum packing of shortest edge-disjoint cycles in a graph of given girth g (g-ESCP) and its vertex-disjoint analogue g-VSCP. In the case $g = 3$, Caprara and Rizzi (2001) have shown that g-ESCP can be solved in polynomial time for graphs with maximum degree 4, but is APX-hard for graphs with maximum degree 5, while g-VSCP can be solved in polynomial time for graphs with maximum degree 3, but is APX-hard for graphs with maximum degree 4.

For $g \in \{4,5\}$, we show that both problems allow polynomial time algorithms for instances with maximum degree 3, but are APX-hard for instances with maximum degree 4. For each $g \geq 6$, both problems are APX-hard already for graphs with maximum degree 3.

Ramsey numbers for k-uniform hypercycles

Andrzej Ruciński

Graph Ramsey theory is nowadays a popular and well studied branch of graph theory. Much less in known about Ramsey numbers of hypergraphs. In this talk I will focus on recent developments concerning Ramsey numbers of hypercycles. Unlike graphs, there are several ways to define cycles in hypergraphs: there is the general definition due to Berge, and its numerous restricted versions, like loose and tight cycles, in particular.

I will present estimates of Ramsey numbers for Berge cycles in k-uniform hypegraphs due to A. Gyárfás, J. Lehel G. Sárközy, E. Szemerédi, and R. Schelp, as well as for loose and tight k-cycles due to P. Haxell, T. Łuczak, Y. Peng, V. Rödl, M. Simonovits, J. Skokan, and myself. On the way, an interesting connection with matchings will be shown.

The minimum rainbow subgraph problem

Ingo Schiermeyer
(joint work with Ján Katrenič and Stephan Matos Camacho)

Our research was motivated by the pure parsimony haplotyping problem: Given a set $\mathcal{G}$ of genotypes, the haplotyping problem consists in finding a set $\mathcal{H}$ of haplotypes that explains $\mathcal{G}$. In the pure parsimony haplotyping problem (PPH) we are interested in finding a set $\mathcal{H}$ of smallest possible cardinality. The pure parsimony haplotyping problem can be described as a graph colouring problem as follows:

The Minimum Rainbow Subgraph problem (MRS)
Given a graph G, whose edges are coloured with p colours. Find a subgraph $H \subseteq G$ of G of minimum order $r^*(G)$ with $|E(H)| = p$ such that each colour occurs exactly once.

If G is a graph with maximum degree $\Delta(G)$, then

$$\frac{2p}{\Delta(G)} \leq r^*(G) \leq 2p.$$

In this talk we will present improved lower and upper bounds for the minimum order $r^*(G)$ of a rainbow subgraph of G.

We will also show that the MRS can be approximated in polynomial time with an approximation ratio of $\min\{\frac{5}{6}\Delta, 1 + \frac{2}{3}\Delta\}$ for graphs with maximum degree Δ. This approximation can be obtained using the algorithm Maximum New Colour and the algorithm presented in [1].

References

[1] S. Matos Camacho, I. Schiermeyer, Z. Tuza, Approximation algorithms for the minimum rainbow subgraph problem, Preprint 2008.

Local computation of vertex colorings

Jens Schreyer
(joint work with Thomas Böhme)

We investigate algorithms for vertex colorings, which are local in the following sense. Let G be a simple finite graph of order n. Each vertex of G is associated with an agent, who decides for a color out of the set $\{1, ..., k\}$. The global structure of G is unknown to any of the vertices. Moreover, each agent may observe the colors chosen by the neighboring vertices, but there is no other means of communication between the agents. Now the graph is colored in rounds. In each round every vertex chooses a color. We look for algorithms, that guarantee a stable proper coloring after a finite number of rounds, where stable means that the coloring is not changed in the following rounds. The main result will be an algorithm that guarantees with probability $1 - \delta$ a stable proper coloring with k colors within $O(n \log \frac{n}{\delta})$ rounds, where $k-1$ is any constant bound on the coloring number $col(G)$.

Large vertex-transitive and Cayley graphs of given degree and diameter

Jozef Širáň

In their seminal 1960 paper, Hoffman and Singleton initiated research into the *degree-diameter problem*, which is to determine the largest order of a graph of a given diameter and degree. Despite five decades of intense activity and a number of deep results, the problem is still largely open.

In the past two decades, research has subdivided into more narrow areas, one of which is determination of the largest order of a vertex-transitive and a Cayley graph, respectively, of a given degree and diameter.

In our lecture we will survey fundamental techniques and results in this vertex-transitive and Cayley version od the degree-diameter problem, including possible new directions of research in this field.

Facial non-repetitive edge colouring of semiregular polyhedra and spider graphs

Erika Škrabuľáková

(joint work with Stanislav Jendroľ)

A sequence $r_1, r_2, \ldots, r_{2n}$ such that $r_i = r_{n+i}$ for all $1 \leq i \leq n$, is called a *repetition*. A sequence S is called *non-repetitive* if no subsequence of consecutive terms of S is a repetition. Let G be a graph whose edges are coloured. A trail in G is called *non-repetitive* if the sequence of colours of its edges is non-repetitive. If G is a plane graph, a *facial non-repetitive edge-colouring* of G is an edge-colouring such that any *facial trail* is non-repetitive. We denote $\pi'_f(G)$ the minimum number of colours of a facial non-repetitive edge-colouring of G. We proved in [1] that $\pi'_f(G) \leq 8$ for any connected plane graph and $\pi'_f(G) \leq 7$ for any 3-connected plane graph. In [2] we have determined the facial Thue chromatic index $\pi'_f(G)$ for graphs of semiregular polyhedra. It is either 3 or 4. We also show that for spider graphs $SW(m,n)$ it holds $4 \leq \pi'_f(SW(m,n)) \leq 6$.

References

[1] F. Havet, S. Jendroľ, R. Soták, E. Škrabuľáková, Facial non-repetitive edge-colouring of plane graphs, Preprint 2009.

[2] S. Jendroľ, E. Škrabuľáková, Facial non-repetitive edge-colouring of semiregular polyhedra, Preprint 2009.

Circular defective edge colouring of graphs

Roman Soták

(joint work with Peter Mihók and Janka Oravcová)

A refinement of the standard (classic/regular) edge colouring is a circular edge colouring. Another generalization of the standard edge colouring is so-called f-colouring, which is a colouring of edge set $E(G)$ of a graph G such that each color appears at each vertex $v \in V(G)$ at most $f(v)$ times. The combination of these two optimalizations is a circular defective edge colouring. We will introduce this new edge colouring and concentrate on determination of its basic properties.

On a min-max problems concerning weights of edges for particular graphs families — hereditary properties

Martin Tajboš

For a graph $G = (V, E)$ and $e = \{x, y\} \in E(G)$ the weight of e is defined as $w(e) = deg(x) + deg(y)$. Erdős asked the question what is the minimum weight of an edge e of a graph G having n vertices and m edges? Let $\mathcal{G}(n, m)$ be the family of all graphs having n vertices and m edges. Motivated by Erdős's question Ivančo, Jendroľ [1] and Jendroľ, Schiermeyer [2] solved the problem

$$W(n,m) = \max_{G \in \mathcal{G}(n,m)} \left\{ \min_{e \in E(G)} w(e) \right\}$$

$$w(n,m) = \min_{G \in \mathcal{G}(n,m)} \left\{ \max_{e \in E(G)} w(e) \right\}$$

Let $\mathcal{P}$ be a given property of graphs and $\mathcal{G}(\mathcal{P})$ be the family of all graphs having property $\mathcal{P}$. Then

$$w(\mathcal{P}) = \min_{G \in \mathcal{G}(\mathcal{P})} \left\{ \max_{e \in E(G)} w(e) \right\}.$$

We study the behavior of $w(\mathcal{P})$ with respect to different hereditary properties of graphs, i.e. properties that are closed with respect to taking subgraphs.

References

[1] J. Ivančo, S. Jendroľ, On extremal problems concerning weights of edges of graphs, Coll. Math. Soc. J. Bolyai 60, Sets, Graphs and Numbers, Budapest 1991, 399–410.

[2] S. Jendroľ, I. Schiermeyer, On a max-min problem concerning weights of edges, Combinatorica 21 (2001), 351–359.

On *r*-trestles and toughness

Jakub Teska

An r-trestle is a 2-connected factor with maximum degree at most r. The toughness of a non-complete graph is $t(G) = \min(\frac{|S|}{c(G-S)})$, where the minimum is taken over all nonempty vertex sets S, for which $c(G - S) \geq 2$ and $c(G - S)$ denotes the number of components of the graph $G - S$. Tkáč and Voss [3] generalized the Chvátal's conjecture for trestles, which is still open for every $r \geq 2$.

Conjecture: For every integer r greater than one, there is a real number $t_r > 0$ such that every t_r-tough graph has an r-trestle.

We find a lower bound on the number t_r from the previous conjecture for arbitrary $r \geq 3$ by constructing graphs with relatively high toughness having no r-trestle. We also present new results concerning the existence of an r-trestle in K_4-minor free graphs.

References

[1] D. Bauer, H. J. Broersma, H. J. Veldman, Not every 2-tough graph is hamiltonian, Discrete Appl. Math. 99 (2000), 317–321.
[2] V. Chvátal, Tough graphs and hamiltonian circuits, Discrete Math. 5 (1973), 215–228.
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Search problems on graphs and hypergraphs

Eberhard Triesch

Given a finite graph or hypergraph $G = (V, E)$, what is the worst case complexity $L(G)$ of finding one or more unknown edges in E by performing certain tests? We study several variants of the problem, some of them with applications in the biosciences. For example, the tests might be given by subsets $W \subset V$ and give the information whether at least one of the unknown edges is contained in W. We present new algorithms for the case of graphs and 3-uniform hypergraphs and prove a conjecture by Du and Hwang in these cases. We also discuss other test families and open problems.

Some innocent-looking old unsolved problems on hamiltonian cycles

Zsolt Tuza

In the talk we discuss two open problems (one extremal, one algorithmic), which were raised decades ago and still are unsolved.

Domination number of cubic graphs with large girth

Jan Volec

We show that every n-vertex cubic graph with girth at least g has domination number at most $0.299871 \cdot n + O(n/g) < 3n/10 + O(n/g)$. This result improves previous bound $0.321216 \cdot n + O(n/g)$ of Rautenbach and Reed.

The directed case of decompositions of edge-colored complete digraphs

Richard M. Wilson
(joint work with Anna Draganova and Yukiyasu Mutoh)

We discuss the most general setting of the asymptotic existence question for decompositions of complete graphs. Given a set of r colors, denote by $K_n^{(\lambda_1,\lambda_2,\ldots,\lambda_r)}$ the directed graph with n vertices and where for any ordered pair x, y of distinct vertices and any color j, there are exactly λ_j edges of color j directed from x to y.

For any given family $\mathcal{G}$ of edge-colored digraphs, we give necessary and asymptotically sufficient conditions on n for the existence of decompositions of $K_n^{(\lambda_1,\lambda_2,\ldots,\lambda_r)}$ into subgraphs isomorphic to digraphs in $\mathcal{G}$. We will give examples and applications. These necessary and asymptotically sufficient conditions are of the form

$$n-1 \equiv 0 \pmod{\alpha} \qquad \text{and} \qquad n(n-1) \equiv 0 \pmod{\beta}$$

where α and β depend on $\mathcal{G}$. For some families $\mathcal{G}$, the parameters α and β are easy to compute and understand. But it is interesting that for some families $\mathcal{G}$, the parameters require involve complex calculations and may not be so elementary.

As one application, we give a short proof for the asymptotic existence of resolvable (v, k, λ)-BIBDs for any value of λ when certain necessary conditions are satisfied.

Closure operation for even factors on claw-free graphs

Liming Xiong

Ryjáček [1] defined a powerful closure operation $\mathrm{cl}(G)$ on claw-free graphs G. Very recently, Ryjáček, Yoshimoto and the author [2] developed the closure operation $\mathrm{cl}^{2f}(G)$ on claw-free graphs which preserves the (non)-existence of a 2-factor. In this paper, we introduce a closure operation $\mathrm{cl}^{se}(G)$ on claw-free graphs that generalizes the above two closure operations. The closure of a graph is unique determined and the closure turns a claw-free graph into the line graph of a graph containing no cycle of length at most 5 and no cycles of length 6 satisfying certain condition and no induced subgraph being isomorphic to the unique tree with the degree sequence 111133. We show that these closure operations on claw-free graphs all preserve the minimum number of components of an even factor. In particular, we show that a claw-free graph G has an even factor with at most k components if and only if $\mathrm{cl}^{se}(G)$ $(\mathrm{cl}(G), \mathrm{cl}^{2f}(G)$, respectively) has an even factor with at most k components. However, the closure operation does not preserve the (non)-existence of a 2-factor.

References

[1] Z. Ryjáček, On a closure concept in claw-free graphs, J. Combin. Theory Ser. B 70 (1997), 217–224.

[2] Z. Ryjáček, L. Xiong, K. Yoshimoto, Closure concept for 2-factors in claw-free graphs, Preprint.

Large Cayley graphs of diameter 2 and given degree

Mária Ždímalová

The 'Cayley restriction' of the degree diameter problem is to find the largest order of a Cayley graph of a given degree d and a given diameter k. Concentrating on the case $k = 2$, we let C_2 denote the set of all pairs (n, d) for which there exists a Cayley graph of order n, degree d and diameter 2. By the Moore bound we have $n \leq d^2 + 1$, but the best known constructions of infinite families with n 'large' only give pairs $(2q^2, 2q - 1) \in C_2$, where q is an odd prime power. In this contribution we will show that there exists a set of positive integers D of positive arithmetic density such that $(n, d) \in C_2$ for all $d \in D$ and $n \approx \frac{d^2}{3}$. We will also discuss possible improvements of this result.

Spanning cyclic subdivisions of vertex-disjoint cycles and chorded cycles in graphs

Shenggui Zhang
(joint work with Shengning Qiao)

Let G be a graph on $n \geq 3$ vertices and H be a subgraph of G such that each component of H is a cycle with at most one chord. In this talk we prove that if the minimum degree of G is at least $n/2$, then G contains a spanning subdivision of H such that only non-chord edges of H are subdivided. This gives a new generalization of the classical result of Dirac on the existence of Hamilton cycles in graphs.

On minimal graphs with a given difference between their chromatic and clique numbers

Krzysztof T. Zwierzyński

This presentation is based on [2] where the following problem has been considered.

Problem. For a simple graph G let $c = \chi(G) - \omega(G) \geq 1$. Define $n_{min}(c, \chi)$, the minimum number of vertices for graphs with a given c and $\chi \geq c + 2$.

It is known, that there exist graphs with the clique number $\omega = 2$ (i.e., triangle-free graphs) having arbitrarily large chromatic number. A construction of such graphs has been defined by Mycielski [5]. The family R of graphs that uses this construction and the join operation has been proposed. The order of graphs from R is an upper bound for $n_{\min}(c, \chi)$.

A graph G is minimally k-chromatic if $\chi(G) = k$ and deleting any of its edges yields decreasing its chromatic number. Let symbol $+$ denote the join operation for two graphs.

Theorem 1. If a graph G is minimally k-chromatic, then $H = G + K_1$ is minimally $(k + 1)$-chromatic.

Let for each $c \geq 1$, $R_c = (R_c(\chi))$ be the infinite sequence of graphs with $\chi \geq c+2$. For a given c the initial element of R_c is the Mycielski graph M_{c+2}.

Theorem 2. Graphs from the sequences R_1 and R_2 are minimally k-chromatic.

Conjecture. If G is minimally k-chromatic, then $M(G)$ (the Mycielski construction for G) is minimally $(k + 1)$-chromatic.

If the above conjecture is true, then the Mycielski graphs M_k are minimally k-chromatic and, consequently, every graph $R_c(\chi)$ is minimally χ-chromatic.

It is proved [3] that the smallest 4-chromatic triangle-free graph has eleven vertices, is unique, and is isomorphic to M_4. The minimum number of vertices of a triangle-free 5-chromatic graph is at least 19 [1]. However, for $\chi = 5$ the following is true [4]: $n_{\min}(2,5) = 11$, $n_{\min}(3,5) = 22$. Note, that using graphs defined in [4] the better upper bound for $n_{\min}(c,\chi)$, $c \geq 2$, $\chi \geq 5$, can be found.

Open problem. Define a construction that produces a graph with the preserved clique number and the chromatic number increased by two (or more).

References

[1] D. Avis, On minimal 5-chromatic triangle-free graphs, J. Graph Theory 3 (1979), 397–400.

[2] K. T. Balińska, K. T. Zwierzyński, On minimal nonperfect graphs, Studies in Automation and Information Technology, Poznań 2008, 7–12.

[3] V. Chvátal, The minimality of the Mycielski graph, Lecture Notes in Math. 406, Springer, Berlin 1974, 243–246.

[4] T. Jensen, F. R. Gordon, Small graphs with chromatic number 5: A computer search, J. Graph Theory 19 (2006), 107–116.

[5] J. Mycielski, Sur le coloriage des graphs, Colloq. Math. 3 (1955), 161–162.

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Programme of the Conference

Sunday	
16:00 - 22:00	Registration
18:00 - 21:00	Dinner

Monday		
07:30 - 08:45	Breakfast	
09:00 - 09:50	Borowiecki M.	Generalized graph colourings with local constraints
09:55 - 10:15	Bujtás Cs.	Small(est) edge sets meeting all triangles
10:15 - 10:45	Coffee break	
10:45 - 11:05	Niepel Ľ.	Domination in products of digraphs
11:10 - 11:30	Holub P.	On a packing colouring of the square lattice
11:35 - 11:55	Barát J.	Crossings and colorings
12:00 - 12:20	Regen F.	Complexity of shortest cycle packings
12:30 - 13:30	Lunch	
15:30 - 16:20	Ruciński A.	Ramsey numbers for k-uniform hypercycles
16:25 - 16:45	Xiong L.	Closure operation for even factors on claw-free graphs
16:45 - 17:15	Coffee break	
17:15 - 17:35	Teska J.	On r-trestles and toughness
17:40 - 18:00	Madaras T.	On exact scale-free graphs
18:05 - 18:25	Tajboš M.	On a min-max problems concerning weights of edges for particular graphs families - hereditary properties
18:30 - 19:30	Dinner	
20:00 -	Welcome party	

Tuesday		
07:30 - 08:45	Breakfast	
09:00 - 09:50	Wilson R. M.	The directed case of decompositions of edge-colored complete digraphs
09:55 - 10:15	Harant J.	Upper bounds on the sum of the squares of the degrees of a triangle-free k-chromatic graph
10:15 - 10:45	Coffee break	
10:45 - 11:05	Jamison R. E.	Groups, graphs, and distance two colorings
11:10 - 11:30	Bacsó G.	Extremal problems on some versions of the chromatic number
11:35 - 11:55	Schreyer J.	Local computation of vertex colorings
12:00 - 12:20	Tuza Zs.	Some innocent-looking old unsolved problems on hamiltonian cycles
12:30 - 13:30	Lunch	
15:30 - 16:20	Triesch E.	Search problems on graphs and hypergraphs
16:25 - 16:45	Zhang S.	Spanning cyclic subdivisions of vertex-disjoint cycles and chorded cycles in graphs
16:45 - 17:15	Coffee break	
17:15 - 17:35	Schiermeyer I.	The minimum rainbow subgraph problem
17:40 - 18:00	Matos Camacho S.	Some approximation algorithms on the Minimum rainbow subgraph problem
18:05 - 18:25	Katrenič J.	Some algorithmic notes on the Minimum rainbow subgraph problem
18:30 - 19:30	Dinner	
20:00 - 21:00	Videopresentation C&C 2008	

Wednesday	
07:30 - 08:30	Breakfast
08:30 - 16:00	Trip
19:00 - 20:00	Dinner

Thursday		
07:30 - 08:45	Breakfast	
09:00 - 09:50	Katona G. O. H.	Coloring cycles
09:55 - 10:15	Rautenbach D.	Minimum degree and density of binary sequences
10:15 - 10:45	Coffee break	
10:45 - 11:05	Kemnitz A.	Hamiltonicity in vertex-deleted hypercubes
11:10 - 11:30	Soták R.	Circular defective edge colouring of graphs
11:35 - 11:55	Pangrác O.	On the complexity of paths avoiding forbidden pairs
12:00 - 12:20	Fiedorowicz A.	Acyclic colourings of graphs
12:30 - 13:30	Lunch	
15:30 - 16:20	Širáň J.	Large vertex-transitive and Cayley graphs of given degree and diameter
16:25 - 16:45	Mihók P.	Hereditary properties of graphs and monotone invariants
16:45 - 17:15	Coffee break	
17:15 - 17:35	Löwenstein Ch.	Partitioning a graph into a dominating set, a total dominating set, and something else
17:40 - 18:00	Koch M.	Maximum weight of a connected graph of given order and size
18:05 - 18:25	Škrabuľáková E.	Facial non-repetitive edge colouring of semiregular polyhedra and spider graphs
19:00 -	Farewell party	

Friday		
07:30 - 08:45	Breakfast	
09:00 - 09:50	Kráľ D.	Linear programming proofs in graph theory
09:55 - 10:15	Zwierzyński K. T.	On minimal graphs with a given difference between their chromatic and clique numbers
10:15 - 10:45	Coffee break	
10:45 - 11:05	Čada R.	Some antimagic results
11:10 - 11:30	Ždímalová M.	Large Cayley graphs of diameter 2 and given degree
11:35 - 11:55	Volec J.	Domination number of cubic graphs with large girth
12:00 - 12:20	Czap J.	Looseness of plane graphs
12:30 - 13:30	Lunch	