GEMS 2005

Igor Fabrici, Stanislav Jendroľ, Tomáš Madaras ed.

Workshop

Graph Embeddings and Maps on Surfaces 2005

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Preface

Welcome to the workshop "GEMS 2005", the fourth Slovak workshop on graph embeddings and maps on surfaces. The history of this workshop dates back to 1994, when the first GEMS workshop took place at Donovaly, Slovakia. Other GEMS workshops in Slovakia were held with the period of 4 years in Banská Bystrica in 1997, and in Bratislava in 2001. During the last decade, each year there were conferences devoted to embeddings and maps also in Slovenia (in the years congruent to 3 mod 4), New Zealand (in the years congruent 0 mod 4) and in Arizona and Portugal.

The aim of the workshop is to bring together people interested in various aspects of graphs embedded on surfaces; among the topics of the common interest, there are representativity and graph minors, embeddings with constraints, various combinatorial properties of embedded graphs, symmetrical embeddings, regular maps and hypermaps, symmetries of graphs, group action graphs, and convex polytopes and polytopal graphs.

The venue of the 2005 GEMS workshop will be the Hotel Horizont in Stará Lesná, a small village on the foot of the beautiful High Tatra Mountains. These are the highest mountain range in Slovakia, situated in the northern part of the country and providing a natural boundary between Slovakia and Poland. The beauty of the High Tatra Mountains is admired every year by tens of thousands of tourists from all over the world. The mountains offer numerous hiking opportunities and views of astounding beauty.

The scientific program of the workshop consists of one-hour lectures of invited speakers and 20-minute contributed talks presented by other participants. This booklet contains abstracts of invited lectures and contributed talks as were sent to us by the authors.

The list of invited speakers includes:

Marston Conder, University of Auckland, New Zealand
Gareth A. Jones, University of Southampton, U.K.
Alexander Mednykh, Novosibirsk State University, Novosibirsk, Russia
Bojan Mohar, University of Ljubljana, Slovenia
David B. Surowski, Shanghai American School, Shanghai, PR China
Thomas W. Tucker, Colgate University, Hamilton, NY, U.S.A.
Alexander Zvonkin, LaBRI University Bordeaux, France

We wish all participants a pleasant and fruitful stay in High Tatras.

Organising committee:

Igor Fabrici
Stanislav Jendroľ (chair)
Tomáš Madaras
Štefan Schrötter

Cayley maps on general surfaces

Marcel Abas

In this presentation we introduce a theory of Cayley maps on surfaces that can be non-orientable and with non-trivial boundary components. We also present an application of the theory to Hamiltonian embeddings of complete graphs.

Crossing number critical graphs

Drago Bokal

Crossing number of a graph G counts the smallest possible number of crossings in a drawing of G. A crossing-number-critical graph is a graph G for which any proper subgraph has strictly smaller crossing number than G. Several infinite families of crossing critical graphs with a given crossing number are known and, as Pinontoan and Richter pointed out, they can all be obtained from smaller graphs, tiles, which are joined together in a ring-like structure.

In the talk we present a construction of crossing number cricital graphs, which combines critical graphs into new ones. We conjecture that for every graph G there exists an infinite family of crossing number critical graphs, all having the same crossing number and containing G as an induced subgraph. We give strong evidence to support this conjecture. The ring structure observed in the known families plays an important role in the new construction, but we prove that there can be many such rings involved as substructures in a critical graph.

Induced non-spearating cycles and double-rays in 4-connected graphs

C. Paul Bonnington, Primož Potočnik, Carsten Thomassen

We present some results about induced non-separating cycles in 4-connected graphs which leads to a characterisation of non-planar 4-connected graphs (motivated in part by Tutte's characterisation of 3-connected planar graphs. We also discuss the role of induced non-separating double rays in infinite 4-connected graphs.

Restricted regularity in hypermaps

Antonio Breda d'Azevedo

Regularity has had different meanings in the past. Uniformity (that is, vertices, edges and faces having constant valencies, say l, $m = 2$ and n, respectively), for instance, was the regularity in the early studies of regular polyhedra of genus one and two by Errera (1922). Although this weak form of regularity is enough to describe "stronger forms" of regularity in the sphere such as "rotary" and "reflexible", this feature is exclusive to this surface; no other compact surface has such similar behaviour. As noticed by Brahana (1926) for a surface of genus one (Anchor Ring), it gives only a certain measure of "regularity" (read "orientable regularity"). But Brahana itself took regularity to mean "orientable regularity" (that is, "rotary" or "orientable direct regularity").

There is a well known correspondence between hypermaps and co-compact subgroups of the free product $\Delta = C_2 * C_2 * C_2$. In this correspondence, hypermaps correspond to conjugacy classes of subgroups of Δ, and hypermap coverings to subgroup inclusions. Regular hypermaps with extra symmetries, namely G-symmetric regular hypermaps, where G is a group of the outer automorphism $Out(\Delta)$ of the triangle group Δ, can be seen as an extension of regularity. Here we move in the opposite direction and restrict regularity to normal subgroups Θ of Δ of finite index. This restricted form of regularity generalises the notion of regularity to some non-regular objects and cast light upon some of the weaker forms of regularity studied earlier.

Reduced regularity is not at all new. An orientably regular map, or hypermap, is the most known restricted form of regularity, in this case restricted to orientation preserving automorphisms (Δ^+-regularity). Less familiar is the orientable bipartite-regularity that appeared in the medial maps of regular oriented maps studied by Archdeacon, Širáň and Škoviera (1992), an orientable regularity restricted to bipartite-face-preserving automorphisms. Other forms of restricted regularity can be found in older literature, though not directly. Some maps appearing as geometrical illustrations of groups in Burnside's monograph (1955) are non-regular (in the "reflexible" and "orientable" sense), and even non-uniform. Fig. 10 in the Burnside's book, for example, illustrates a non-regular map which is $\langle R_2, (R_0R_1)^3 \rangle^{\Delta}$-regular in this restricted sense of regularity.

On the other side, any group with any set of generators corresponds to a restrictly regular hypermap. More generally yet, any thin chamber system (Surowski, 2001), or any combinatorial map (Vince, 1983), or any thin residually connected incidence structure (Tits (1974) and Vince (1983)), correspond to a Θ-conservative hypermap.

Chiral regular maps and related matters

Marston Conder

Regular maps (symmetric embeddings of connected arc-transitive graphs on surfaces) fall into three classes: reflexible regular maps on orientable surfaces, reflexible regular maps on non-orientable surfaces, and orientably-regular maps that are irreflexible (or *chiral*). In this talk I will explain the relationship between the class of chiral regular maps and normal subgroups of triangle groups, give some examples of small genus, and describe constructions for three different infinite families. An open question is for which g does there exist an orientably-regular chiral map of genus g.

Spherical and toroidal bipartite-regular hypermaps

Antonio Breda d'Azevedo, Rui Duarte

A hypermap is a four-tuple $\mathcal{H} = (\Omega; r_0, r_1, r_2)$, where Ω is a non-empty finite set and r_0, r_1 and r_2 are fixed-point free involutory permutations of Ω such that the permutation group $\mathrm{Mon}(\mathcal{H}) := \langle r_0, r_1, r_2 \rangle$ acts transitively on Ω. Topologically, a hypermap $\mathcal{H}$ is a cellular imbedding of a hypergraph $\mathcal{G}$ in a compact connected surface $\mathcal{S}$.

The hypervertices of $\mathcal{H}$ are the $\langle r_1, r_2 \rangle$-orbits in Ω. $\mathcal{H}$ is bipartite if we can split Ω in two sets of hypervertices $\Omega_\circ$ and $\Omega_\bullet$ such that $\Omega_\circ r_0 = \Omega_\bullet$. Topologically, a bipartite hypermap $\mathcal{H}$ is a cellular imbedding of a bipartite hypergraph $\mathcal{G}$ in a compact connected surface $\mathcal{S}$.

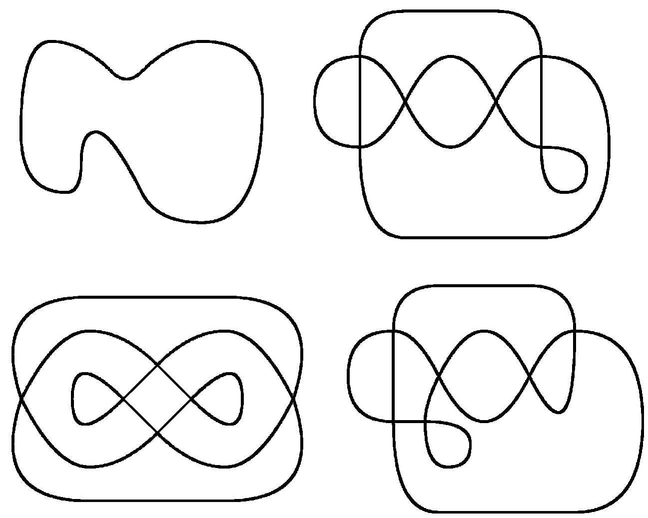
An automorphism of $\mathcal{H}$ is a permutation ψ of Ω such that $r_i\psi = \psi r_i$. $\mathcal{H}$ is regular if for all $\omega, \sigma \in \Omega$ there is an automorphism ψ of $\mathcal{H}$ such that $\omega\psi = \sigma$ and $\mathcal{H}$ is bipartite-regular if for all $\omega, \sigma \in \Omega_\circ$ (or $\Omega_\bullet$) there is an automorphism ψ of $\mathcal{H}$ such that $\omega\psi = \sigma$, i.e, $\mathcal{H}$ is regular if the automorphism group of $\mathcal{H}$ acts transitively on Ω and is bipartite-regular if the $\Omega_\circ$-preserving automorphism group of $\mathcal{H}$ acts transitively on $\Omega_\circ$.

In the present talk we show how the bipartite-regular hypermaps on the sphere are constructed from spherical regular hypermaps. On the torus the scenery is different, there are bipartite-regular hypermaps which are not constructed from regular hypermaps in the same way as the spherical bipartite-regular hypermaps.

When does a curve bound a distorted disk?

Jack E. Graver

Consider a closed curve in the plane that does not intersect itself; by the Jordan Curve Theorem, it bounds a distorted disk. Now consider a closed curve that intersects itself, perhaps several times. Is it the boundary of a distorted disk that overlaps itself? If it is, is that distorted disk essentially unique? The question of when an immersion of the circle can be extended to an immersion of a disk has been studied by several people, notably Titus [C. J. Titus, *The combinatorial topology of analytic functions on the boundary of a disk*, Acta Math. 106 (1961), 45-64.] and Blank [S. J. Blank, *Extending immersions of the circle*, Dissertation, Brandeis University, Mass. (1967)]. We will discuss their work and then we will develop combinatorial techniques for answering these questions.



Three of these curves bound distorted disks; one does not.
Also, one of the three bounds two distinct distorted disks.

From graph colourings to constraint satisfaction, and back

Pavol Hell

Graph colourings may be viewed as special constraint satisfaction problems. The class of k-colouring problems enjoys a well known dichotomy of complexity - these problems are polynomial time solvable when $k \leq 2$, and NP-complete when $k \geq 3$. For general constraint satisfaction problems such dichotomy was conjectured by Feder and Vardi, but has still not been proved in full generality. We discuss some results and techniques related to this 'dichotomy conjecture'. We survey the effects of a new notion of 'fullness', describing how it affects the complexity of constraint satisfaction problems, and their dichotomy. Full constraint satisfaction problems may then be specialized back to graph colourings, and this leads to a new set of problems in graph theory, related to the study of graph perfection.

On crossing-critical graphs

Petr Hliněný

We review the crossing number problem, and some known results and open questions concerning the structure of crossing-critical graphs. Namely we look at the conjecture of [Richter, Salazar, and Thomassen]: A k-crossing-critical graph has bounded bandwidth in k. We show that an analogous statement is deeply false in the projective plane.

Self-dual chiral polyhedra

Isabel Hubard

Chiral polyhedra have maximal symmetry of rotation. Self-dual chiral polyhedra appear in two different kinds. Some examples and properties of the extended groups (the groups of dualities and automorphisms) of such polyhedra will be discussed.

Exponents of maps and regular embeddings of canonical double covers of graphs

Miroslav Hužvár

Let $M = (F; \lambda, \varrho, \tau)$ be an embedding of a graph K. If the map $M^t = (F; \lambda\tau^j, (\varrho\tau)^{e-1}\varrho, \tau)$ with the same underlying graph K is isomorphic to M, for some integer e coprime with the valency of K, and for some $j \in Z_2$, we say the expression $t = ep^j$ to be an *unoriented exponent* of M.

The integer part e of the unoriented exponent coincides with an exponent of the embedded graph K whereas the Petrie part p^j reflects the significant role the Petrie operation plays in study of unoriented maps equipped with a high degree of symmetry. This concept seems to be useful for the construction and classification of regular embeddings on nonorientable surfaces. Following the works on oriented exponents by Roman Nedela and Martin Škoviera we prove analogies of several their results for unoriented maps.

We apply the unoriented exponent theory to the classification of regular maps of canonical double covering graphs. From a regular map M of a connected non-bipartite graph K it is possible to derive a regular map M_t of its canonical double cover $\tilde{K}$, for any involutory integer exponent $t = e$ and any mixed exponent $t = ep$ such that $e^2 \equiv -1$ (modulo valency of K). Conversely, in case the base graph K is stable, every regular map of $\tilde{K}$ that meets some natural additional conditions can be obtained from a regular map of K with this construction. Finally, we show the conditions under which two regular maps of $\tilde{K}$ derived from regular maps of the base graph K with use of involutory integer exponents are isomorphic.

Subgraphs of bounded degrees of their vertices in embedded graphs

Stanislav Jendroľ

It is well known that every planar graph contains a vertex of degree at most 5. Kotzig proved that every 3-connected planar graph contains and edge with degree sum of its end vertices at most 13. Recently Fabrici and Jendroľ proved that every 3-connected planar graph G that has a path on k vertices also contains a path P_k on k vertices such that every vertex of this path has, in G, the degree at most $5k$. In our talk we give a survey of analogous results for graphs embedded into surfaces.

Regular maps and their groups

Gareth A. Jones

I will discuss the relationship between regular maps and their groups, and in particular I will give some examples of how results and techniques from finite and infinite group theory can be used to classify and construct examples of maps and hypermaps (compact and noncompact) with specific properties.

Combinatorial classification of 3-manifolds of genera zero and one

Ján Karabáš, *Roman Nedela*

Theory of crystallisations, developed in early 70's of last century, allows us to use combinatorial methods to attack the classification problem of n-manifolds. The classification of 3-manifolds, as most interesting instance, is not known in general in present. Many methods are occupied to attack this problem. We introduce pure combinatorial method to classify 3-manifolds of genera zero and one.

Symmetries of fulleroids

Stanislav Jendroľ, *František Kardoš*

Fulleroids were defined as cubic (trivalent) polyhedral maps with two types of faces only – the pentagons and n-gons ($n > 5$). The relationship between the symmetry group of a fulleroid and the number n has been studied. The results concerning existence or nonexistence of such fulleroids for various admissible symmetry groups, depending on number n, are presented. The existence is demonstrated by finding at least one example (or an infinite series of them), the nonexistence is proved using some symmetry invariants.

On the crossing numbers of join graphs

Marián Klešč

The *crossing number,* $cr(G)$ of a graph G is the minimum number of pairwise intersections of edges (at a point other than a vertex) in a drawing of G in the plane. Computing the crossing number of a given graph is in general an elusive problem, and the crossing numbers of very few infinite families of graphs are known. It was recently proved by L. Glebsky and G. Salazar (2004) that the crossing number of the Cartesian product $C_m \times C_n$ of the cycles of sizes m and n equals its long–conjectured value, namely $(m-2)m$, at least for $n \geq m(m+1)$. There are known exact crossing numbers of Cartesian products of paths, cycles or stars with all graphs of order four and with several graphs of order five. It thus seems natural to inquire about the crossing numbers for some other products of graphs.

The join $G+H$ of two vertex–disjoint graphs G and H is obtained from the union of G and H by adding all edges between every vertex of G and every vertex of H. For $|E(G)| = m$ and $|E(H)| = n$, the edge set of $G+H$ is the union of edges of the graphs G, H, and the complete bipartite graph $K_{m,n}$. It has been long–conjectured that the crossing number $cr(Km,n)$ of the complete bipartite graph $K_{m,n}$ equals the Zarankiewicz Number $Z(m,n) := \lfloor\frac{m-1}{2}\rfloor\lfloor\frac{m}{2}\rfloor\lfloor\frac{n-1}{2}\rfloor\lfloor\frac{n}{2}\rfloor$ (1954). This conjecture has been verified by D. J. Kleitman for $\min\{m,n\} \leq 6$ (1970) and by R. D. Woodall for the special cases $7 \leq m \leq 8$, $7 \leq n \leq 10$ (1993). Using these results, we give the exact values of crossing numbers for several infinite families of join graphs.

Regular hamiltonian embeddings of $K_{n,n}$

Martin Knor

In a map, a flag is an edge together with an incident vertex and one of the two faces. An embedding of a graph is regular if for every pair of flags, say a and b, there is an automorphism of the embedded graph mapping a to b. We prove that if $n \not\equiv 0 \pmod 8$ then there is a unique regular hamiltonian embedding of $K_{n,n}$ in an orientable surface. On the other hand, if $n \equiv 0 \pmod 8$ then there are exactly two embeddings of this type.

On the 1-chromatic number of nonorientable surfaces with large genus

Volodymyr P. Korzhyk

The 1-chromatic number $\chi_1(S)$ of a surface S is the maximum chromatic number of all graphs which can be drawn on the surface so that each edge is crossed by no more than one other edge.

For a surface S, denote

$$R(S) = \left\lfloor \frac{9 + \sqrt{81 - 32E(S)}}{2} \right\rfloor,$$

where $E(S)$ is the Euler characteristic of the surface.

G. Ringel (1981) gave the following upper bound

$$\chi_1(S) \leq M(S),$$

where $M(S)$ is $R(S) - 1$ if $R(S)$ is even and

$$\left\lfloor \frac{13 + \sqrt{169 - 72E(S)}}{3} \right\rfloor = R(S) - 1,$$

and $M(S)$ is $R(S)$ in all other cases. Ringel conjectured that the upper bound is least, but he suspected that there are several exceptions in the lower genus area.

We prove that:

(a) There is an integer $Q > 0$ such that

$$M(N_q) - 1 \leq \chi_1(N_q) \leq M(N_q)$$

for all $q \geq Q$;

(b) $\chi_1(N_q) = M(N_q)$ for about 7/12 of all nonorientable surfaces N_q.

The results are obtained by using index one current graphs.

Regular orientable embeddings of Frobenius graphs

Jin Ho Kwak, Yan Wang

A graph is called Frobenius if it is a connected orbital regular graph of a Frobenius group. A Frobenius map is a regular Cayley map whose underlying graph is Frobenius. In this talk, we show that almost all low rank Frobenius graphs admit regular embeddings and enumerate non-isomorphic Frobenius maps for a given Frobenius graph. For some Frobenius groups, we classify all Frobenius maps derived from these groups. As a result, we construct some Frobenius maps with trivial exponent groups.

Classifications of reflexible and self-Petrie dual regular orientable embeddings of $K_{n,n}$

Jin Ho Kwak, Young Soo Kwon

In this talk, we classify reflexible and self-Petrie dual regular orientable embeddings of complete bipartite graphs. The classifications shows that for any natural number n, say $n = 2^a p_1^{a_1} p_2^{a_2} \cdots p_k^{a_k}$ ($p_1, p_2, \ldots, p_k$ are distinct odd primes and $a_i > 0$ for each $i \geq 1$), there are 2^ℓ distinct reflexible regular orientable embeddings of the complete bipartite graph $K_{n,n}$ up to isomorphism, where $\ell = 0$ if $a = 0$; $\ell = k$ if $a = 1$; $\ell = k + 1$ if $a = 2$ and $\ell = 2k + 3$ if $a \geq 3$. And, it is shown that there are 2^t distinct self-Petrie dual regular orientable embeddings of $K_{n,n}$ up to isomorphism, where $t = 0$ if $a = 0$; $t = k$ if $a = 1$; $t = k + 1$ if $a = 2$ and $t = k + 2$ if $a \geq 3$.

The 'orbicyclic' arithmetic function

Valery A. Liskovets

We analyze the multiplicativity and other arithmetic properties of the multivariate function $E(m_1, \ldots, m_r)$ introduced recently by A. Mednykh and R. Nedela for counting unrooted maps on oriented surfaces.

It is defined in terms of the bivariate alternating von Sterneck function (Ramanujan sum) and counts, up to two elementary factors (one of which is the multiplicative Jordan function $\phi_{2g}(n)$), order preserving epimorphisms (known also as smooth or surface-kernel epimorphisms) from the fundamental group of a "cyclic" orbifold onto a cyclic group. In particular, the function $E(m_1, \ldots, m_r)$ is represented as the product of its primary factors, which are simple non-negative multivariate functions. We also establish necessary and sufficient conditions of its non-vanishing, which prove to be equivalent to familiar Harvey's conditions that characterize possible branching data of finite cyclic groups acting on Riemann surfaces.

Tessellations and automorphism actions on 3-manifolds associated to the $[5, 3, 5]$ Coxeter group

Gareth A. Jones, Cormac Long

Just as automorphism groups of maps on compact surfaces arise as finite quotients of triangle groups, isometry groups of hyperbolic 3-manifolds and their tessellations arise as finite quotients of certain rank 4 Coxeter groups. Here we classify the epimorphisms from the $[5, 3, 5]$ Coxeter group onto the simple groups $L_2(q)$ in order to study the resulting manifolds and their tessellations and isometry groups.

Regular maps from Cayley graphs III: t-balanced Cayley maps

Ľubica Líšková, Martin Mačaj, Martin Škoviera

The class of t-balanced Cayley maps (Martino and Schultz, 1998) is a natural generalisation of balanced and antibalanced Cayley maps introduced and studied by Širáň and Škoviera (1992, 1994). The present paper continues this study by investigating the distribution of inverses, automorphism groups, and exponents of t-balanced Cayley maps. The methods are based on the use of t-automorphisms of groups with sign-structure which extend the notion of an antiautomorphism crucial for antibalanced Cayley maps. As an application, a new series of non-standard regular embeddings of complete bipartite graphs $K_{n,n}$ is constructed for each n divisible by 8.

The structure of 1-planar graphs

Igor Fabrici, Tomáš Madaras

A graph is called 1-planar if it can be drawn in the plane so that each its edge is crossed by at most one other edge. We study the existence of subgraphs of bounded degrees in 1-planar graphs. It is shown that each 1-planar graph contains a vertex of degree at most 7; we also prove that each 3-connected 1-planar graph contains an edge whose endvertices are of degrees at most 20, and we present similar results concerning bigger structures in 1-planar graphs with additional constraints.

Counting coverings and unrooted maps on closed surface

Alexander Mednykh

We suggest a general method to calculate the number of conjugacy classes of subgroups of given index in an arbitrary finitely generated group. Then we apply this result to find the number of non-equivalent coverings over surface with given multiplicity and the number of unrooted maps on closed surface with given number of edges.

Coloring locally planar graphs

Bojan Mohar

In the early 1990's, Thomassen proved that graphs embedded in a fixed surface with sufficiently large edge-width are 5-colorable. Today, this result is well understood and extended in several directions. In this talk it will be explained why there is so similar behaviour between chromatic properties of planar and locally planar graphs on orientable surfaces, and why this similarity is slightly weaker on nonorientable surfaces. Several new result in this area will be presented, including approximate flow-coloring duality, 5-list-colorings and hardness results about coloring problems on Eulerian triangulations.

Regular embeddings of complete bipartite graphs

Roman Nedela

It is well-known that for each integer n, the complete bipartite graph $K_{n,n}$ has at least one regular embedding in an orientable surface described by Biggs and White as a Cayley map for the group Z_{2n}. Nedela, Škoviera and Zlatoš proved that there is exactly one regular embedding of $K_{p,p}$, p a prime. This result was generalised by Jones, Nedela and Škoviera proving that there is a unique regular embedding of $K_{n,n}$ if and only if square of a prime does not divide n and $p \nmid q-1$ for any two prime divisors of n. Such prime divisors of n are called *disjoint*. Note that the set of n admitting a unique regular embedding of $K_{n,n}$ coincides with the set of n such that there is a unique group of order n. If n is a power of an odd prime the authors proved that there are p^{e-1} non-isomorphic regular embeddings of $K_{n,n}$ ($n = p^e$ a power of an odd prime) are of type $\{2n, n\}$ and of genus $(n-1)(n-2)/2$. The method of the proof uses the following fact: if $\mathcal{M}$ is an oriented regular map with the underlying graph $K_{n,n}$ for some n then the subgroup $G = Aut_0^+\mathcal{M} < Aut(\mathcal{M})$ of automorphisms of $\mathcal{M}$, preserving vertex-colours, factorises as a product of two disjoint cyclic groups of order n. Generally, the structure of groups which are products of two cyclic groups is not known. The main obstacle seems to be a lack of information in the case when the considered product is not metacyclic. When n is an odd prime power, a result of Huppert implies that such a group G must be metacyclic, and this enabled us to classify the possibilities for G, and hence for $\mathcal{M}$. When $p = 2$, however, Huppert's result does not apply, and indeed for each $e \geq 2$ there are regular embeddings of $K_{n,n}$ ($n = 2^e$) for which G is not metacyclic, so these are not direct analogues of the maps arising when p is odd. Nevertheless, the techniques used in the odd case can also be applied here, giving a partial classification for $p = 2$, provided one restricts to those embeddings for which G is metacyclic. Recently, the regular embeddings of complete bipartite graphs with $G = HK$, where $H \cong K \cong Z_{2^e}$ are isomorphic cyclic 2-groups, were classified Du, Kwak, Jones, Nedela and Škoviera. It turned out that given $e > 2$ there are exactly 4 regular maps with non-metacyclic partition stabiliser $G < Aut(\mathcal{M})$. Hence we have

Theorem Let $n = 2^e$ be a power of two, $e > 2$. Then there are exactly $2^{e-2}+4$ oriented regular embeddings of $K_{n,n}$.

For $n = 2$ there is one, and for $n = 4$ there are two regular embeddings of $K_{n,n}$, respectively.

The classification of regular embeddings of complete bipartite graphs is still not completed. Some new results were proved by using a different approach. In particular, Kwak and Kwon proved that for n odd there is exactly one (reflexible) regular embedding of $K_{n,n}$, namely the standard one constructed by Biggs and White with $G = Z_n \times Z_n$ and $Aut(\mathcal{M}) \cong (Z_n \times Z_n) : Z_2$.

Decomposition of maps of high symmetry

Alen Orbanić

A parallel product of maps was introduced by Wilson [W] and it is an elegant tool for producing new rooted maps form old ones. A product can be viewed as one component of a permutation voltage cover over one map using the another map as a permutation voltage. Some interesting properties and constructions using parallel product are introduced. Several ways of quotienting maps are considered together with conditions for automorphisms to project. Conditions for a decomposition of a highly symmetrical map into the parallel product of two or more highly symmetrical maps are introduced. The maps that cannot be decomposed in that way are called *minimal*. All minimal reflexive maps up to 100 edges are classified by use of a computer. Some special voltage assignments on highly symmetrical maps are introduced together with some of their properties.

[W] S. E. Wilson, *Parallel Products in Groups and Maps*, J. of Algebra **167** (1994) 539–546

Infinitely many hypermaps of a given genus

Daniel Pinto

It is conjectured that given positive integers l, m, n with $l^{-1}+m^{-1}+n^{-1} < 1$ and an integer $g \geq 0$ there are infinitely many nonisomorphic compact orientable hypermaps of type (l, m, n) and of genus g. A couple of constructions will be presented proving the result for some particular types of hypermaps.

Triangle group homomorphisms and hypermap operations

Anton Prowse

It is well known that the classical map operation of duality is one of six which arise from outer automorphisms of Grothendieck's *cartographic group*

$$\mathscr{C}_2 = \langle\, t_0, t_1, t_2 \,|\, t_0^2 = t_1^2 = t_2^2 = (t_0 t_2)^2 = 1 \,\rangle,$$

the extended triangle group of type $[\infty, 2, \infty]$ which acts as a discrete group of isometries of the hyperbolic plane and which governs all maps. We may equally well consider generalized hypermap operations defined to arise from arbitrary homomorphisms between standard and extended triangle groups. A classification of such homomorphisms (subject to the constraint that the canonical generators may not map to hyperbolic elements) is given and some corresponding operations illustrated.

Quotient genus of covalence sequences of Cayley tessellations

Jana Šiagiová

Let C be a cyclic sequence of covalences appearing around a vertex of a Cayley tessellation of the plane. In general, there may exist many non-isomorphic Cayley tessellations with covalence sequence equal to C. The quotient genus of C is defined to be the least genus of a one- vertex quotient of a Cayley tessellation with covalence sequence C.

In the talk we present bounds as well as exact results related to the quotient genus of covalence sequences of Cayley tessellations.

Exponents of Cayley maps

Ľubica Staneková

A Cayley map is a Cayley graph embedded in an oriented surface in such a way that the cyclic order of generators is the same at each vertex. The distribution of inverses of a Cayley map is the involution indicating the position of mutually inverse generators in the cyclic order at a vertex. An exponent of a Cayley map is a number e with the property that, roughly speaking, the Cayley map is isomorphic to its 'e-fold rotational image'.

In our contribution we present results on exponents of Cayley maps, with emphasis on the construction of Cayley maps with given exponents and given distribution of inverses.

Counting connected cyclic principal derived algebraic maps

David Surowski

The study of regular algebraic maps continues unabated, with one of the fundamental problems being that of classifying all regular maps of a given genus. As any such map can be regarded as a principal derived map (with coefficients in the group of covering transformations), and since the principal map construction is essentially cohomological in nature, we may avail ourselves of the plethora of results and techniques from homological algebra.

If the covering map has a finite cyclic covering group of transformations, the above machinery is particularly amenable to counting arguments. The regularity and connectivity constraints have their corresponding homological formulations which in turn lend to a rather explicit count of such maps. The unrestricted count is a bit complex, but simplifies considerably in two special cases of particular interest: the unramified and totally ramified covering cases. For example, if $\mathcal{M}$ is a regular orientable map of genus g, then the number of $\mathcal{M}$-isomorphism classes of unramified coverings $\mathcal{M}' \to \mathcal{M}$ with $\mathcal{M}'$ connected and whose group of covering transformation is cyclic of order n is given by the sum

$$\sum_{l|n} \mu(n/l) l^{2g}/\phi(n),$$

where μ is is the Möbius function and ϕ is Euler's totient function.

Regular maps on a given surface

Jozef Širáň

A regular map is a cellular embedding of a graph on a surface, such that the automorphism group of the embedding acts regularly on the set of flags. We give a brief survey of the state-of-the-art of classification of regular maps on a given surface and present new results in this area.

Distinguishing maps

Thomas W. Tucker

The action of a group A on a set X is called k-distinguishable if the set X can be colored with k colors so that there is no element of A preserving the coloring, other than the identity. An interesting case is the automorphism group of a map acting on the vertices of the map. We show that all but finitely many such actions are 2-distinguishable. In the orientation-preserving case, we can list the exceptions: the tetrahedron, the octahedron, the quadrilater embedding of K_5 in the torus, and the embedding of K_7 in the torus. In the general case, we know of many exceptions, all with 10 or fewer vertices, but we do not have the full list. We also discuss the situation where one requires the coloring to be proper (adjacent vertices get different colors). Finally, we discuss other classes of group actions where we find or expect that all but finitely many are 2-distinguishable.

On maps with the face incident with all vertices

Milan Tuhársky

It is already known that every outerplanar graph with minimum vertex-degree at least two contains an edge e with the sum of degrees of its endvertices at most 6. This sum is called the *weight* of e. We investigated similar problem for graphs embeddable in an orientable surface $\mathbb{S}_g$. We prove that every map G on $\mathbb{S}_g$ containing a face incident with all vertices of G contains an edge of weight depending on the genus g of $\mathbb{S}_g$; in particular: If G is a map on $\mathbb{S}_g$ with vertex degree at least 2 containing a face incident with every vertex of G then there is an edge e in G of weight at most 10 if G is the map on the torus or an edge e of weight at most $4g + 5$ if G is the map on $\mathbb{S}_g, g \geq 2$.

Embeddings of cubic graphs

Andrej Vodopivec

I will present some results about embedding of cubic graphs. The research is motivated by the Grünbaum conjecture which states that if a cubic graph admits a polyhedral embedding in an orientable surface, then it is 3-edge-colourable. I will show that small cycles in a cubic graph G are facial cycles in a polyhedral embedding of G. Using this results the conjecture can be verified for some families of non 3-edge-colourable graphs. I will show that for every non-orientable surface $\mathbb{N}$ (except possibly for the Klein bottle) there exists a non 3-edge-colourable graph, which can be in $\mathbb{N}$ so that the embedding is polyhedral. I will conclude my talk with embeddings of non 3-edge-colourable graphs in the torus.

Counting unrooted maps on the plane

Valery A. Liskovets, Timothy R. Walsh

A planar map is a 2-cell embedding of an undirected connected planar graph, loops and parallel edges allowed, on the sphere. A plane map is a planar map with a distinguished outside ("infinite") face. A rooted map is a map with a distinguished oriented edge (not necessarily incident to the outside face) and an unrooted map is an equivalence class of maps under orientation-preserving homeomorphism which, in the case of a plane map, fixes the distinguished face. Previously we obtained formulae for the number of unrooted planar maps of various classes, including all maps, non-separable maps, eulerian maps and loopless maps with n edges; M. Bousquet, G. Labelle and P. Leroux did so for unrooted planar and plane maps with two faces. In this article, using the same technique we obtain closed formulae for counting unrooted plane maps of all these classes and their duals. The corresponding formulae for rooted maps are known to be all sum-free; the formulae that we obtain for unrooted maps contain at most a sum over the divisors of n. Numerical tables for all these types of plane maps, rooted and unrooted, with up to 20 edges are also provided.

Tetravalent edge-transitive graphs having large vertex-stabilizers

Primož Potočnik, Steve Wilson

Fundamental to the use of computers in constructing a census of symmetric and semi-symmetric graphs of degree three is the fact that vertex stabilizers in symmetry groups of such maps are bounded in size. No such fact is known to hold for tetravalent graphs and in fact it is known that no such result can hold. To be more precise, call a family of edge-transitive tetravalent graphs *troublesome* provided that, for every positive M, there is a graph Γ in the family such that in every subgroup G of $\mathrm{Aut}(\Gamma)$ which is transitive on the edges of Γ, the stabilizer G_v of the vertex v has size greater than M. In this talk, we shall present three troublesome families of graphs: Wreath graphs, certain Rose Window graphs and certain partial line graphs of dihedral Cayley graphs.

Topological classification of polynomials

Alexander Zvonkin

The study of the topological classification of complex polynomials was begun in 19th century by Lüroth (1871), Clebsch (1873), and Hurwitz (1891). In 1970, it was shown that the problem can be reduced to the study of orbits of an action of the braid groups on certain planar combinatorial tree-like structures called *cacti*. In the talk, we will give a general account on the current state of the problem and will try to elucidate an intricate interplay between the three approaches to its solution: enumerative combinatorics (I. Goulden, D. Jackson), some results related to classification of finite groups (W. Feit, G. Jones), and computer experimentation. We will also formulate two conjectures which, if they turn out to be true, must settle the problem for the “indecomposable” polynomials (those that cannot be represented as a composition of non-linear polynomials of smaller degrees). The case of composite polynomials “essentially” reduces to that of indecomposable ones.

At the end, we will also give some examples concerning the topological classification of meromorphic functions of higher genera having a single pole.

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Programme of the workshop

Sunday		
14:00 - 22:00	Registration	
18:00 - 22:00	Dinner	

Monday		
07:30 - 08:30	Breakfast	
08:40 - 08:45	Opening	
08:45 - 09:45	*Marston CONDER*	Chiral regular maps and related matters
09:50 - 10:15	Coffee break	
10:15 - 10:35	*Jin Ho KWAK*	Regular orientable embeddings of Frobenius graphs
10:40 - 11:00	*Young Soo KWON*	Classifications of reflexible and self-Petrie dual regular orientable embeddings of $K_{n,n}$
11:05 - 11:25	*Roman NEDELA*	Regular embeddings of complete bipartite graphs
11:30 - 11:40	Break	
11:40 - 12:00	*Jozef ŠIRÁŇ*	Regular maps on a given surface
12:05 - 12:25	*Martin KNOR*	Regular Hamiltonian embeddings of $K_{n,n}$
12:30 - 13:00	Lunch	
13:00 - 15:00	Time off	
15:00 - 16:00	*David B. SUROWSKI*	Counting connected cyclic principal derived algebraic maps
16:05 - 16:25	Coffee break	
16:25 - 16:45	*Ľubica STANEKOVÁ*	Exponents of Cayley maps
16:50 - 17:10	*Jana ŠIAGIOVÁ*	Quotient genus of covalence sequences of Cayley tessellations
17:15 - 17:35	*Marcel ABAS*	Cayley maps on general surfaces
17:40 - 18:00	*Martin MAČAJ*	Regular maps from Cayley graphs III: t-balanced Cayley maps
18:05 - 18:25	*Miroslav HUŽVÁR*	Exponents of maps and regular embeddings of canonical double covers of graphs
18:30 - 19:00	Dinner	
20:00 -	Welcome party	

Tuesday		
07:30 - 08:30	Breakfast	
08:45 - 09:45	*Bojan MOHAR*	Coloring locally planar graphs
09:50 - 10:15	Coffee break	
10:15 - 10:35	*Jack E. GRAVER*	When does a curve bound a distorted disk?
10:40 - 11:00	*Paul BONNINGTON*	Induced non-spearating cycles and double-rays in 4-connected graphs
11:05 - 11:25	*Marián KLEŠČ*	On the crossing numbers of join graphs
11:30 - 11:40	Break	
11:40 - 12:00	*Volodymyr KORZHYK*	On the 1-chromatic number of nonorientable surfaces with large genus
12:05 - 12:25	*Pavol HELL*	From colourings to constrained satisfaction, and back
12:30 - 13:00	Lunch	
13:00 - 15:45	Research problems afternoon	
15:45 - 16:45	*Alexander MEDNYKH*	Counting coverings and unrooted maps on closed surface
16:50 - 17:15	Coffee break	
17:15 - 17:35	*Valery A. LISKOVETS*	The 'orbicyclic' arithmetic function
17:40 - 18:00	*Timothy R. WALSH*	Counting unrooted maps on the plane
18:05 - 18:25	*Andrej VODOPIVEC*	Embeddings of cubic graphs
18:30 - 19:00	Dinner	

Wednesday	
07:30 - 08:30	Breakfast
08:30 - 14:00	Trip
14:00 - 16:00	Lunch
16:00 - 17:00	Return from the trip
17:00 - 19:00	Time off
19:00 - 19:30	Dinner

Thursday		
07:30 - 08:30	Breakfast	
08:45 - 09:45	*Thomas W. TUCKER*	Distinguishing maps
09:50 - 10:15	Coffee break	
10:15 - 10:35	*Steve WILSON*	Tetravalent edge-transitive graphs having large vertex-stabilizers
10:40 - 11:00	*Antonio BREDA D'AZEVEDO*	Restricted regularity in hypermaps
11:05 - 11:25	*Isabel HUBARD*	Self-dual chiral polyhedra
11:30 - 11:40	Break	
11:40 - 12:00	*Tomáš MADARAS*	The structure of 1-planar graphs
12:05 - 12:25	*Cormac LONG*	Tessellations and automorphism actions on 3-manifolds associated to the $[5, 3, 5]$ Coxeter group
12:30 - 13:00	Lunch	
13:00 - 15:00	Time off	
15:00 - 16:00	*Gareth A. JONES*	Regular maps and their groups
16:05 - 16:25	Coffee break	
16:25 - 16:45	*Rui DUARTE*	Spherical and toroidal bipartite-regular hypermaps
16:50 - 17:10	*Daniel PINTO*	Infinitely many hypermaps of a given genus
17:15 - 17:35	*Anton PROWSE*	Triangle group homomorphisms and hypermap operations
17:40 - 18:00	*Drago BOKAL*	Crossing number critical graphs
18:05 - 18:25	*Alen ORBANIĆ*	Decomposition of maps of high symmetry
19:00 -	Farewell party	

Friday		
07:30 - 08:30	Breakfast	
08:45 - 09:45	*Alexander ZVONKIN*	Topological classification of polynomials
09:50 - 10:15	Coffee break	
10:15 - 10:35	*Stanislav JENDROL'*	Subgraphs of bounded degrees of their vertices in embedded graphs
10:40 - 11:00	*Petr HLINĚNÝ*	On crossing-critical graphs
11:05 - 11:25	*František KARDOŠ*	Symmetries of fulleroids
11:30 - 11:40	Break	
11:40 - 12:00	*Ján KARABÁŠ*	Combinatorial classification of 3-manifolds of genera zero and one
12:05 - 12:25	*Milan TUHÁRSKY*	On maps with the face incident with all vertices
12:30 - 13:00	Lunch	