C&C

HIGH TATRAS, SLOVAKIA

Igor Fabrici, Mirko Horňák, Stanislav Jendroľ ed.

Workshop

Cycles and Colourings 2006

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Dear Participant,

welcome to the Fifteenth Workshop Cycles and Colourings. Except for the first workshop in the Slovak Paradise (Čingov 1992), the remaining thirteen workshops took place in the High Tatras (Nový Smokovec 1993, Stará Lesná 1994–2003, Tatranská Štrba 2004–2005).

The series of C&C workshops is organised by combinatorial groups of Košice and Ilmenau. Apart of dozens of excellent invited lectures and hundreds of contributed talks the scientific outcome of our meetings is represented also by special issues of journals Tatra Mountains Mathematical Publications and Discrete Mathematics (TMMP 1994, 1997, DM 1999, 2001, 2003). We are ready to prepare a special issue of DM devoted to the forthcoming workshop. Please do not forget that the deadline for submitting papers to this special issue is October 31, 2006.

The scientific programme of the workshop consists of 50 minute lectures of invited speakers and of 20 minute contributed talks. This booklet contains abstracts as were sent to us by the authors.

Invited speakers:

Guantao Chen,	Georgia State University, Atlanta, GA, USA
Ronald J. Gould,	Emory University, Atlanta, GA, USA
Jan van den Heuvel,	London School of Economics, London, UK
Tommy Jensen,	Alpen-Adria University, Klagenfurt, Austria
Tomáš Kaiser,	University of West Bohemia, Plzeň, Czech Republic
Oriol Serra,	Technical University of Catalonia, Barcelona, Spain
Margit Voigt,	University of Applied Sciences, Dresden, Germany

Have a pleasant and successfull stay in Tatranská Štrba.

Organising committee:

Igor Fabrici
Jochen Harant
Erhard Hexel
Mirko Horňák
Stanislav Jendroľ (chair)
Štefan Schrötter

Contents

Maximal nontraceable oriented graphs

Susan van Aardt

D is a maximal nontraceable (MNT) oriented graph if D is not traceable but for any two nonadjacent vertices u and v in D, $D + uv$ is traceable.

We characterize MNT oriented graphs that are not walkable (unilaterally connected). Furthermore we show that every walkable MNT oriented graph D can be described by $D = (T_1 + S) + T_2$ where T_i, $n(T_i) \geq 0$, $i = 1, 2$, are tournaments and where S, $n(S) \geq 3$, is a certain "significant" MNT oriented graph.

We characterize acyclic and unicyclic MNT oriented graphs as well as the strong component digraphs of MNT oriented graphs. This enables us to characterize MNT oriented graphs of order n that have size $\binom{n}{2} - 1$ and we show that no MNT oriented graph of order n has size $\binom{n}{2} - 2$. We also show the maximum size of a *strong* MNT oriented graph of order n is $\binom{n}{2} - 3$.

Total labelings of graphs

Martin Bača

Suppose G is a finite graph with vertex-set $V(G)$ and edge-set $E(G)$. An (a, d)-*edge-antimagic total labeling* on G is a one-to-one map f from $V(G) \cup E(G)$ onto the integers $1, 2, \ldots, |V(G)| + |E(G)|$ with the property that the edge-weights $w(uv) = f(u) + f(v) + f(uv)$, $uv \in E(G)$, form an arithmetic progression starting from a and having common difference d. Such a labeling is called *super* if the smallest labels appear on the vertices.

The aim of the talk is to present recent results on super (a, d)-edge-antimagic total labelings of disjoint union of multiple copies of complete graph and complete bipartite graph.

Elementary bipartite graphs and unique colorability

Gábor Bacsó

The Clique-Pair-Conjecture (**CPC**) states that a uniquely colorable perfect graph, different from a clique, contains two maximum size cliques having a two element symmetric difference. One can make an auxiliary graph B from a minimal counterexample for the **CPC** (if any exists), this B is bipartite.

We prove that B is elementary. Furthermore, we give a characterization of the minimal counterexamples, in terms of the constructed auxiliary graphs.

On F-decompositions of graphs

Mieczysław Borowiecki
(joint work with Katarzyna Jesse-Józefczyk)

Let $c : V(G) \to \{\text{red, blue}\}$ be a partial function, and let denote by $R(G) = \{v \in V(G) : c(v) = \text{red}\}$, and by $B(G) = \{v \in V(G) : c(v) = \text{blue}\}$. The function c is called a *partial colouring* of G. If $e = uv \in E(G)$ and $c(u) \neq c(v)$ then the edge e we call 2-*coloured.* Let us denote by $E_2(G)$ the set of all 2-coloured edges of G.

Let $F = (X_F, Y_F; E(F))$ be a bipartite graph with a natural ordered bipartition (X_F, Y_F), where X_F, Y_F are colour classes. Such bipartition will be identified with a proper 2-colouring such that each vertex of X_F is coloured red, and each vertex of Y_F is coloured blue, i.e., $X_F = R(F)$, $Y_F = B(F)$.

Let G be a graph and F a bipartite graph with at least 2 vertices.

An ordered partition (V_1, V_2) of $V(G)$ will be called the F-*free decomposition* or briefly the F-*decomposition* of G if there is a partial 2-colouring of $V(G)$ which satisfies the following conditions:

(1) $V_1 \supseteq R(G)$ and $V_2 \supseteq B(G)$,
(2) $E_2(G)$ is a cut-set of G,
(3) The graph F is not an induced subgraph of $G' = (R(G) \cup B(G), E_2(G))$,
(4) $|R(G)| \geq |X_F|$ and $|B(G)| \geq |Y_F|$.

A graph G is *totally decomposable* with respect to the F-decomposition if every induced subgraph of G with at least $|X_F| + |Y_F|$ vertices has an F-decomposition.

Totally F-decomposable graphs are specially interesting because the class of all totally F-decomposable graphs is an induced hereditary property.

Graphs which are totally F-decomposable, where $F = K_2 + K_1$, (also called totally *modular decomposable*) coincide with the class of all *cographs* (P_4-free graphs), see [4]. For $F = K_{1,2}$, a graph G is totally F-decomposable if and only if G is K_3-free, see [3]. Graphs which are totally *split decomposable* are characterized in [1]. Namely, G is totally split decomposable if and only if G is distance-hereditary. The class of totally *generalized join* decomposable graphs ($F = 2K_2$) was considered in [2] but the graphs of this class are not completely characterized. It seems to be still an open problem up to now.

Some results and open problems on totally decomposable graphs with respect to an F-decomposition for $F \in \{K_{1,2} \cup K_1, P_4, C_4\}$ will be presented.

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Color-bounded hypergraphs, II: Interval hypergraphs and hypertrees

Csilla Bujtás
(joint work with Zsolt Tuza)

A color-bounded hypergraph is a hypergraph (set system) with vertex set X and edge set $\mathcal{E} = \{E_1, \ldots, E_m\}$, together with integers s_i and t_i ($1 \le s_i \le t_i \le |E_i|$) for $i = 1, \ldots, m$. A vertex coloring $\varphi : X \to \mathbb{N}$ is feasible if the number of colors occurring in E_i satisfies $s_i \le |\varphi(E_i)| \le t_i$, for every $i \le m$.

In the talk we point out that hypertrees — hypergraphs admitting a representation over a (graph) tree where each hyperedge E_i induces a subtree of the base tree — play a central role concerning the possible numbers of colors that can occur in a feasible coloring. We also consider interval hypergraphs and circular hypergraphs, where the underlying graph is a path or a cycle, respectively. Sufficient conditions are given for a 'continuous' chromatic spectrum; i.e., when each number of colors can be realized between minimum and maximum. Algorithmic complexity for the colorability is studied, too.

Path factors in cubic graphs

Roman Čada

A conjecture due to Akiyama and Kano states that a 3-connected cubic graph of order divisible by 3 has a P_3-factor.

We will discuss recent results related to this conjecture.

Finding long cycles in 3-connected graphs

Guantao Chen

The study of the longest cycles of sparse graphs evolved from development of the Four Color Theorem. In 1931, Whitney proved that every 4-connected planar triangulation contains a Hamilton cycle. Tutte generalized this result to all 4-connected planar graphs. Furthermore, there are infinitely many 3-connected planar graphs that do not contain any Hamilton cycles. A good lower bound on the length of 3-connected graphs has been a subject of extensive research.

The length of the longest cycle in G, denoted by $c(G)$, is called the circumference of G. When studying paths in polytopes, Moon and Moser in 1963 conjectured that $c(G) \geq \alpha n^{\log_3 2}$ *for all 3-connected planar graph of order* n*, where* $\alpha > 0$ *is a universal constant.* They also showed the bound $n^{\log_3 2}$ is best possible. Ten years later, Grünbaum and Walther made the same conjecture for 3-connected **cubic** planar graphs. Recently, working on graph minors, Thomas made the following conjecture: *There exist two functions* $\alpha(t)$ *and* $\beta(t) > 0$ *such that, for any integer* $t \geq 3$ *and for any* 3*-connected graph* G *with no* $K_{3,t}$*-minor,* $c(G) \geq \alpha(t) n^{\beta(t)}$. Furthermore, Seymour and Thomas conjectured that $\beta(t)$ in the Thomas conjecture can be a constant independently from t. In this talk, we will report the solutions of these conjectures and the progress thus far for the following conjecture of Jackson and Wormald: *there exists a function* $\alpha(d) > 0$ *such that* $c(G) \geq \alpha(d) n^{\log_{d-1} 2}$ *for any positive integer* $d \geq 4$ *and any 3-connected graph* G *with maximum degree at most* d.

Decomposition of complete bipartite graphs into trails

Sylwia Cichacz

It has been showed by Horňák and Woźniak that any bipartite graph $K_{a,b}$, where a, b are even is decomposable into closed trails of prescribed even lengths.

We show that complete directed bipartite graph $\overleftrightarrow{K}_{a,b}$ is decomposable into directed closed trails of even lengths greater than 2, whenever these lengths sum up to the size of the digraph $\overleftrightarrow{K}_{a,b}$. We use this result to prove that complete bipartite multigraphs can be decomposed in a similar manner.

We consider also the corresponding question for open trails. We show when the complete bipartite graph $K_{a,b}$ or digraph $\overleftrightarrow{K}_{a,b}$ is decomposable into open trails of arbitrarily lengths whenever these lengths sum up to the size of the graph $K_{a,b}$ ($\overleftrightarrow{K}_{a,b}$). Let $K'_{a,a} := K_{a,a} - I_a$ for any 1-factor I_a. We also prove similar theorem for $K'_{a,a}$ with odd a.

The total outer-connected domination in trees

Joanna Cyman

Let $G = (V, E)$ be a graph. A set $D \subseteq V(G)$ is a total outer-connected dominating set if D is total dominating in G and $G[V(G) - D]$ is connected. The total outer-connected domination number of G, denoted by $\gamma_{tc}(G)$, is the smallest cardinality of a total outer-connected dominating set of G. We show that if T is a tree of order n, then $\gamma_{tc}(T) \geq \lceil \frac{2n}{3} \rceil$. Moreover, we constructively characterize the extremal trees T of order n achieving this lower bound.

On super (a,d)-edge antimagic total labeling of disconnected graphs

Dafik

(joint work with Martin Bača, Mirka Miller and Joe Ryan)

A graph G of order p and size q is called an (a, d)-*edge-antimagic total* if there exist a bijection $f : V(G) \cup E(G) \to \{1, 2, \ldots, p + q\}$ such that the edge-weights, $w(uv) = f(u) + f(v) + f(uv), uv \in E(G)$, form an arithmetic sequence with first term a and common difference d. Such a graph G is called *super* if the smallest possible labels appear on the vertices. In the talk we present super (a, d)-edge-antimagic total properties of disconnected graphs mC_n and mP_n.

The crossing numbers of graphs

Emília Draženská

The *crossing number*, $cr(G)$, of a graph G is the minimum number of pairwise intersections of edges in a drawing of G in the plane. Computing the crossing number of a given graph is in general an elusive problem, and the crossing numbers of few families of graphs are known. Most of them are Cartesian products of special graphs.

The crossing numbers of the Cartesian products of all four vertex graphs with cycles are determined. There are known the crossing numbers of the Cartesian products of cycles with a lot of graphs of order five. The crossing numbers of the Cartesian products of cycles and six vertex trees, except the graph $K_{1,5}$, are determined.

We extend the previous results and give the exact values of the crossing numbers of the Cartesian products of another six vertex graphs with cycles.

k-chromatic number of graphs on surfaces

Zdeněk Dvořák
(joint work with Riste Škrekovski)

In 1890 Heawood established an upper bound for the chromatic number of a graph embedded on a surface of Euler genus $g \geq 1$. This upper bound became known as the Heawood number $H(g)$. Almost a century later, Ringel and Youngs proved that the Heawood number $H(g)$ is in fact the maximum chromatic number as well as the maximum clique number of graphs embbeded on a surface of Euler genus $g \geq 1$ beside the Klein bottle. We present a Heawood type formula for the edge disjoint union of k graphs that are embedded on a given surface $\mathcal{S}$.

On some three color Ramsey numbers for paths and cycles

Robert Fidytek

For given graphs $G_1, G_2, \ldots, G_k, k \geq 2$, the *multicolor Ramsey number* $R(G_1, G_2, \ldots, G_k)$ is the smallest integer n such that if we arbitrarily color the edges of complete graph on n vertices with k colors, there is always a monochromatic copy of G_i colored with i, for some $1 \leq i \leq k$. Let P_k (resp. C_k) be the path (resp. cycle) on k vertices. In the talk we show that $R(P_3, P_k, C_m) = 2k - 1$ for odd k and $k \geq m \geq 3$. In addition, we provide the exact values for Ramsey numbers $R(P_3, P_k, C_4)$ for all integers $k \geq 3$, and $R(P_4, P_k, C_m) = 2k + 1$ for all odd integers $m \geq 3$ and $k \geq m$. In the talk we present new other results in this field as well as some conjectures.

A new perspective on the Path partition conjecture

Marietjie Frick

The order of a longest path in a graph G is denoted by $\lambda(G)$. If the difference between the order and the detour order of G equals p, we say that G is p-deficient. A 0-deficient graph is called traceable. The following conjecture, which was formulated in 1981 but has not yet been settled, is referred to as the Path Partition Conjecture (PPC):

PPC: If G is any graph and (a, b) any pair of positive integers such that

$$a + b = \lambda(G) \tag{1}$$

then G has a vertex partition (A, B) such that

$$\tau(\langle A\rangle) \leq a \text{ and } \tau(\langle B\rangle) \leq b. \tag{2}$$

A vertex partition (A, B) satisfying (2) is called an (a, b)-partition. If the PPC is true, it would be "best possible" since, for example, the complete graph K_n has no (a, b)-partition if $a+b < \lambda(K_n)$. However, if G is any connected, 1-deficient or 2-deficient graph, then G has an (a, b)-partition for every pair of positive integers such that $a + b = \lambda(G) - 1$. Thus it may well be that "something stronger" than the PPC holds for connected, nontraceable graphs. These considerations led to the following definition.

Definition. The *path partition function* $f : \mathbb{Z}^+ \cup \{0\} \to \mathbb{Z}$ is defined by: $f(p)$ is the greatest integer for which every p-deficient graph G has an (a, b)-partition for every pair of positive integers (a, b) such that $a + b = \tau(G) - f(p)$.

The PPC is equivalent to the conjecture that $f(p) \geq 0$ for all $p \geq 0$.

We show that $-p \leq f(p) \leq 1$ for all $p \geq 0$. Moreover, $f(0) = 0$, $f(1) = f(2) = 1$ and $0 \leq f(3) \leq 1$.

A look at cycles containing specified elements

Ron Gould

The study of cycles in graphs has a long and involved history. Over the years there has been a clear evolution in the type of question being asked and the variety of results possible. Early cycle work centered on showing the existence of certain cycles, especially hamiltonian cycles, 2-factors, or other special types of cycles. These questions evolved as the types of conditions imposed on the graph changed, growing less restrictive.

Later, questions about the existence of cycles of specified lengths (pancyclic systems, or 2-factors where the cycle lengths were specified, or at least the number of cycles was specified), drew attention.

Even though this is really an old question, lately, more and more attention has been paid to placing specified elements of the graph on cycles. Sometimes these elements are sets of vertices, sometimes sets of edges or more generally paths, and other times a combination of both (linear forests).

The purpose of this talk is to provide an overview of the typical questions being asked about cycles containing specified elements of a graph, results that have been obtained, and questions that can and should still be asked. We hope to provide a framework of natural questions in this area. In doing so it is hoped that many more natural questions will become apparent. Time restrictions make it impossible to mention all known results, but hopefully we will provide a significant and useful view of this broad and interesting area.

Our general framework will consist of four fundamental parts:

[1] The type of cycle or cycle system of interest.

Here there are many natural choices: hamiltonian cycles, long cycles, short cycles, 2-factors, pancyclic graphs, and arbitrary cycles all come immediately to mind.

[2] The type of specified elements to be placed on the cycle(s).

As mentioned above, choices include sets of specified vertices of a particular size, sets of independent edges of a particular size, sets of independent paths (usually containing a certain total number of edges), or a combination of these, namely linear forests (specified as to total number of edges and paths).

[3] The conditions that allow formation of our cycle(s).

Here connectivity, minimum degree and degree sums, neighborhood unions and many others have all been used. Combinations of these have also been considered and offer a vast variety of possibilities.

[4] Any additional properties we wish our specified elements to have on these cycle(s).

Some of the properties that will be considered are:

- order: (having the elements occur in a particular order around the cycle),
- orientation: (having the edges or paths occur in a particular direction),
- distance between elements: (having the elements be distributed around the cycle(s)).

It is hoped that we can demonstrate results and questions involving each of these parts. It is also hoped that the reader will see even more that can and should be asked based on this framework.

Big light graphs in some classes of polyhedra

Róbert Hajduk

(joint work with Roman Soták)

Let $\mathcal{G}$ be a family of graphs and let G be a connected graph which is isomorphic to a proper subgraph of at least one member of $\mathcal{G}$. Let $\varphi(G, \mathcal{G})$ be a smallest integer with the property that every graph $H \in \mathcal{G}$ which has a subgraph isomorphic with G also contains a subgraph K such that $K \cong G$ and, for every vertex $v \in V(K)$, $\deg_G(v) \leq \varphi(G, \mathcal{G})$ holds. (If such a number does not exist, we put $\varphi(G, \mathcal{G}) = \infty$.) We say that the graph G is light in the family $\mathcal{G}$ if $\varphi(G, \mathcal{G}) < \infty$.

We denote by $\mathcal{G}_c(\delta, \rho)$ class of all c–connected, planar graphs with minimum degree at least δ and minimum face size at least ρ. If $\max\{\delta, \rho\} \leq 4$, then the only light graphs in $\mathcal{G}_3(\delta, \rho)$ are paths. For $\mathcal{G}_3(3,5)$ and $\mathcal{G}_3(5,3)$ there are also other light graphs, which are mostly on a small number of vertices.

We study existence of light graphs in $\mathcal{G}_3(3,5)$ and $\mathcal{G}_3(5,3)$ with big number of vertices, particularly, those one that consist of a long path attached to a small graph.

A regional version of Tutte's Theorem concerning Hamiltonian cycles in 4-connected planar graphs

Jochen Harant

(joint work with Stefan Senitsch)

Let G be a graph, $X \subseteq V(G)$ and $G[X]$ be the subgraph of G induced by X. A set $S \subset V(G)$ *splits* X if the graph H obtained from G by removing S contains two components both containing a vertex of X. Let $\kappa(X)$ be infinity if $G[X]$ is complete or the minimum cardinality of a set $S \subset V(G)$ splitting X. Note that G is k-connected if and only if $\kappa(V(G)) \geq k$.

We prove: If G is a planar graph and $\kappa(X) \geq 4$ for $X \subseteq V(G)$ then G contains a cycle C with $X \subseteq V(C)$.

Mixing colour(ing)s in graphs

Jan van den Heuvel

(joint work with Luis Cereceda and Matthew Johnson)

For a graph G and a positive integer k, the k-colour graph of G, denoted $\mathcal{C}(G;k)$, is the graph that has the proper k-vertex-colourings of G as its vertex set, and two k-colourings are joined by an edge in $\mathcal{C}(G;k)$ if they differ in colour on exactly one vertex of G. We are interested in the properties of this colour graph $\mathcal{C}(G;k)$. In particular we consider the question : given a graph G and a positive integer k, is $\mathcal{C}(G;k)$ connected ?

In the talk we discuss some first results on the research in this area. We concentrate on small values of k (including the case that $k = \chi(G)$). And we consider the complexity of several decision problems related to the connectivity of the colour graph.

Cycle structure and colourings

Tommy Jensen

It is not surprising that some amount of information about the cycles of a graph is often very helpful in determining its chromatic properties. After all, the chromatic polynomial encodes precise information about both the cycles and the colourings of a graph. Thus in a sense, these properties are two sides of the same coin.

We survey recent developments in the study of how knowledge of the cycle structure may be used to deduce colouring properties. This includes noteworthy results and old and new open problems related to induced cycles, Hamilton cycles, cycles in planar graphs, and more.

The Dominating cycle conjecture

Tomáš Kaiser

Our talk is an overview of problems and results related to the Dominating cycle conjecture. Recall that a cycle C in a graph G is *dominating* if each edge of G has at least one endvertex on C. A cubic graph is *cyclically 4-edge-connected* if it is 3-edge-connected and all of its edge-cuts of size 3 are trivial. The Dominating cycle conjecture, due to Fleischner and Jackson [3], is the following:

Conjecture 1 *Every cyclically 4-edge-connected cubic graph contains a dominating cycle.*

We begin by recalling several equivalent conjectures, stated in the 1980s by various authors [2, 6, 8].

We will then focus on more recent development. One interesting line of research involves the *contractibility technique*, introduced by Ryjáček and Schelp [7]. We discuss its application in [1] to show that in proving Conjecture 1, one may restrict to graphs containing no cycles of length ≤ 4.

Another concept we investigate is that of a *coverable set*, whose definition in [5] originated from the study of colorings of planar graphs without short monochromatic cycles [4]. As shown in [5], this notion allows one to formulate (a mild generalization of) Conjecture 1 as a covering problem, and reveals an unexpected link to questions such as the Four color problem.

Other related topics, such as the existence of Hamilton cycles in graphs without forbidden induced subgraphs, will also be discussed.

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Symmetry of fulleroids

František Kardoš

Fulleroids are cubic convex polyhedra with faces of size 5 or greater. They are suitable models for carbon molecules. We study the symmetry group of fulleroids with only pentagonal and n-gonal faces, especially connections between the symmetry group and the face size n. Necessary and sufficient conditions for existence of fulleroids with prescribed symmetry group depending on the number n are presented. In our contribution we focus at skewed and pyramidal symmetry types and low symmetry groups.

On a tree partition problem

Peter Katrenič

Let $\mathcal{I}$ denote a class of all simple finite mutually non-isomorphic graphs. A property of graphs is any subset $\mathcal{P}$ of $\mathcal{I}$. If $T = (V, E)$ is a tree then we define the property $-T$ as the set $\{G \in \mathcal{I} : G$ does not contain T as a subgraph$\}$. For an arbitrary vertex v of T we consider a partition of T into two trees T_1, T_2, so that $V(T_1) \cap V(T_2) = \{v\}$, $V(T_1) \cup V(T_2) = V(T)$, $E(T_1) \cap E(T_2) = \emptyset$, $E(T_1) \cup E(T_2) = E(T)$. We deal with the following problem:

Given a graph G belonging to $-T$. Is it true that for any partition T_1, T_2 of T there exists a partition V_1, V_2 of $V(G)$ such that $G[V_1] \in -T_1$ and $G[V_2] \in -T_2$? This problem provides a natural generalization of Path Partition Conjecture and some other partition problems studied by different autors.

$[r,s,t]$-chromatic numbers of stars

Arnfried Kemnitz

Given non-negative integers r, s, and t, an $[r,s,t]$-coloring of a graph G with vertex set $V(G)$ and edge set $E(G)$ is a mapping c from $V(G) \cup E(G)$ to the color set $\{0, 1, \ldots, k-1\}$ such that $|c(v_i) - c(v_j)| \geq r$ for every two adjacent vertices v_i, v_j, $|c(e_i) - c(e_j)| \geq s$ for every two adjacent edges e_i, e_j, and $|c(v_i) - c(e_j)| \geq t$ for all pairs of incident vertices and edges, respectively. The $[r,s,t]$-chromatic number $\chi_{r,s,t}(G)$ of G is defined to be the minimum k such that G admits an $[r,s,t]$-coloring.

This is an obvious generalization of all classical graph colorings since c is a vertex coloring if $r = 1$, $s = t = 0$, an edge coloring if $s = 1$, $r = t = 0$, and a total coloring if $r = s = t = 1$, respectively.

We present general bounds for $\chi_{r,s,t}(G)$ as well as exact values for certain parameters. Moreover, we completely determine the $[r,s,t]$-chromatic numbers for stars.

Generalized domatic numbers of graph products

Monika Kijewska

We discuss k-ply and k-tuple domatic numbers of graph products and derive new bounds for these parameters of graphs, for an arbitrary $k \geq 1$.

Suppose that G is a simple graph. We consider two generalizations of the domatic number. Namely, k-ply domatic number, denoted by $d^k(G)$, and k-tuple domatic number, denoted by $d_k(G)$ ([1], [2], [3]).

We report our bounds for the above mentioned numbers of the following of graph products: the contraction of one clique into a new vertex, the join of two graphs by in common vertex and the join of two graphs by the bridge. Next, we give a full or partial characterizations for graphs for which the bounds are attained.

In the talk we also state the difference between k-ply domatic number of each of the mentioned graph products and its bound for this one. The analogous result is presented for $k-$tuple domatic number of considering graph products.

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On the crossing numbers of categorical and strong products

Marián Klešč

The *crossing number* $cr(G)$ of a simple graph G with $|V|$ vertices and $|E|$ edges is defined as the minimum number of crossings among all possible projections of G on the $\mathbb{R}^2$ plane. The investigation on the crossing number of graphs is a classical and however very difficult problem. Garey and Johnson have proved that the problem to determine the crossing number of graphs is NP-complete. Because of this difficulty, presently we only know the crossing number of some special graphs, for example: the complete graphs with small number of vertices, the complete bipartite graphs of less number of vertices in one bipartite partion, certain generalized Petersen graphs, and some Cartesian products of graphs.

In 1973, Harary, Kainen and Schwenk proved that toroidal graphs can have arbitrarily large crossing number. For the Cartesian product $C_m \times C_n$ of the cycles of sizes m and n they conjectured that its crossing number is $(m-2)n$, for all $3 \leq m \leq n$, but only for m, n satisfying $n \geq m$, $m \leq 6$, this equality was shown. It was recently proved by L. Glebsky and G. Salazar (2004) that the crossing number of $C_m \times C_n$ equals its long–conjectured value at least for $n \geq m(m+1)$. There are also known exact crossing numbers of Cartesian products of paths, cycles or stars with all graphs of order four and with several graphs of order five.

It thus seems natural to inquire about crossing numbers for some other products of graphs. In the talk, we give the exact values of crossing numbers for some infinite families of graphs and we find the upper bound of crossing numbers for some other graphs. All these graphs are obtained as categorical product and as strong product of two graphs.

Colorings of quadrangulations of the torus and the Klein bottle

Daniel Kráľ

(joint work with Robin Thomas)

Motivated by a question of Thomassen, we show that a triangle-free quadrangulation of the torus is 3-colorable if and only if it does not contain the Cayley graph for the group $\mathbb{Z}_{13}$ and the set $\{1, 5\}$ as a subgraph. Our result yields a proof of the conjecture of Archdeacon et al. that every quadrangulation of the torus with representativity at least six is 3-colorable.

We also show that every triangle-free quadrangulation of the Klein bottle with even meridian walk that does not contain a non-contractible separating 4-cycle is 3-colorable. This yields a proof of another conjecture of Archdeacon et al.

that every quadrangulation of the Klein with representativity at least five is 3-colorable.

In the proofs of both of our results, we first establish that a minimal counterexample must be a 4-regular quadrangulation of the surface and we then apply the characterization of such quadrangulations by Thomassen.

On the crossing numbers of products of power graphs

Daniela Kravecová

The *crossing number*, $cr(G)$, of a graph G is the minimal number of pairwise intersections of nonadjacent edges in any drawing of G in the plane. Finding the exact value of crossing number of a given graph is in general difficult problem. There are known several exact results on the crossing numbers of the Cartesian product of a 5-vertex graphs with paths, cycles and stars. The crossing numbers and upper bounds for few families of graphs ($S_m \times P_n,\ S_m \times C_n$) are known.

In the talk we give several exact values of the crossing numbers for the Cartesian products of power graphs and for some other families of these products.

Trestles and walks in $K_{1,r}$-free graphs

Roman Kužel

For any integer $r > 1$, an *r-trestle* is a 2-connected graph F with maximum degree $\Delta(F) \leq r$. We say that a graph G has an r-trestle if G contains a spanning subgraph which is an r-trestle. A graph G is called *$K_{1,r}$-free* if G has no $K_{1,r}$ as an induced subgraph. This concept can be viewed as an interesting variation on the notion of hamiltonian cycle. Another such variation is a concept of k-walks, where a k-walk in a graph G is a closed spanning walk visiting each vertex at most k times, where $k \geq 1$ is an integer.

We show that every 2-connected $K_{1,r}$-free graph has an r-trestle and that every bridgeless graph of maximum degree Δ has a spanning $\lceil (\Delta + 1)/2 \rceil$-walk. Both bounds are optimal. By combining these two results we can conclude, that every 2-connected $K_{1,r}$-free graph has a $\lceil (r + 1)/2 \rceil$-walk.

2-perfect trails system, total colourings and latin squares

Martin Mačaj

We present a bijection between 2-perfect trail systems on n vertices and total colourings of K_n. We also determine number of all 2-perfect trail systems on 7 and 9 vertices.

On shortest cycle covers of cubic graphs

Edita Máčajová
(joint work with Martin Škoviera)

In 1985 Alon and Tarsi conjectured that every bridgeless graph has a cycle cover of total length at most $(7/5)|E|$, the extremal case being reached by the Petersen graph. The conjecture is known to imply the Cycle Double Cover Conjecture and is related to other important conjectures in graph theory. The purpose of this talk is to show that the conjecture of Alon and Tarsi is true for all bridgeless cubic graphs of girth at least 30.

Colourings of graphs with prescribed cycle lengths

Stephan Matos Camacho

In 1992 Gyárfás confirmed the well known conjecture of Erdős and Bollobás, that graphs with exactly k distinct odd cycle lengths are $2k+1$-colourable unless they are isomorphic to a K_{2k+2}. Schiermeyer and Mihók generalised this statement to graphs with k distinct odd and s distinct even cycle lengths in 2004. They proved that these graphs are colourable with at most $\min\{2k+2, 2s+3\}$ colours.

If we focus on graphs with exactly 2 distinct odd cycle lengths this estimation turns out to be not sharp if there is no triangle in the graph. This problem will be discussed in the talk.

Starting with the already known statements on graphs with k distinct odd cycle lengths, we will discuss the case $k = 2$ more precisely. Most attention we will pay on graphs having two consecutive odd cycle lengths. We will show that such graphs are 4-colourable if the odd cycles have length greater than 3.

k-hereditarily dominated graphs

Danuta Michalak

A graph G is *k-hereditarily dominated* by a class $\mathcal{D}$ of connected graphs if for each connected induced subgraph of G there exists a dominating induced subgraph containing at most k connected components and each of them belongs to $\mathcal{D}$.

For $k = 1$ and various classes of connected graphs, several papers have been published in which this problem is the focus of study. Finally, 1-hereditarily dominated graphs were characterized by Zs. Tuza [3], and independently by G. Bacsó [1].

Let $\mathcal{D}$ be a class of all cliques. For $k = 1$, the first result of this type can be found in Wolk's paper [5] where $\mathcal{D} = \{K_1\}$ and the class of hereditarily one-vertex-dominated graphs was characterized in terms of the forbidden induced subgraphs P_4, C_4. The next result was given by G. Bacsó and Zs. Tuza [2], and independently by M. B. Cozzens and L. L. Kelleher [4], for dominating cliques. In this case the family of forbidden subgraphs consists of P_5 and C_5.

A graph G is 2-hereditarily dominated by a class $\mathcal{D} = \{K_n : n \geq 1\}$ if for each connected induced subgraph of G there exists a dominating induced subgraph containing at most two disjoint cliques.

P_5 and C_5 are 2-hereditarily dominated by the family of cliques. We give some properties of forbidden subgraphs for 2-hereditarily dominated graphs for $\mathcal{D} = \{K_n : n \geq 1\}$.

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Generalizations of Brooks' Theorem - a short survey

Peter Mihók

Let $\mathcal{P}$ be a property of graphs. A graph G is vertex $(\mathcal{P}, k)$-*colourable* if the vertex set $V(G)$ of G can be partitioned into k sets $V_1, V_2, \ldots, V_k$ such that the subgraph $G[V_i]$ of G belongs to $\mathcal{P}$, $i = 1, 2, \ldots, k$. If $\mathcal{P}$ is a hereditary property, then the set of *minimal forbidden subgraphs* of $\mathcal{P}$ is defined as follows: $\boldsymbol{F}(\mathcal{P}) = \big\{G : G \notin \mathcal{P}$ but each proper subgraph H of $G \in \mathcal{P}\big\}$.

For the class $\mathcal{O}^k$ of all k-colorable graphs the set $\boldsymbol{F}(\mathcal{O}^k)$ consists of all $(k+1)$-edge critical graphs.

The well-known Brooks' Theorem states that each connected graph G other than a clique or an odd cycle is in $\mathcal{O}^{\Delta}$, where $\Delta = \Delta(G)$ is the maximum degree of G.

In our talk we will present a short survey of several results concerning the generalizations and proofs of Brooks' Theorem.

Diregularity of digraphs of out-degree three and defect two

Mirka Miller
(joint work with Dafik and Slamin)

It is easy to show that any digraph with out-degree at most $d \geq 2$, diameter $k \geq 2$ and order $n = d + d^2 + \cdots + d^k - 1$, that is, two less than Moore bound must have all vertices of out-degree d.

In other words, the out-degree of the digraph is constant $(= d)$. However, establishing the diregularity or otherwise of the in-degree of such a digraph is not easy. Indeed, when the diameter is 2, there exist both diregular and non-diregular digraphs of out-degrees 2 and 3. However, for diameter $k \geq 3$ and out-degree at most two it has been proved that every digraph of order two less than the Moore bound is diregular.

In this talk we consider the diregularity of digraphs of out-degree at most three, and order two less than the Moore bound.

Upper bound for maximal vertex and face degree of oblique graphs

Jozef Miškuf

A polyhedral graph G is said to be *oblique* if it has no two faces of the same type. A k-gonal face α is of type $\langle b_1, b_2, \ldots, b_k \rangle$, if the vertices incident with α in cyclic order have degrees $\langle b_1, b_2, \ldots, b_k \rangle$ and this is the lexicographic minimum of all such sequences available for α. The set of all oblique graphs imbeded into orientable surfaces was proved to be finite by Walther and Voigt.

Upper bound for maximal vertex degree and maximal face degree of oblique graphs are presented. It is also proved that there are finitely many oblique graphs imbeded into nonorientable surfaces.

Dynamics of iterated path graphs

Ľudovít Niepel

For a given graph G and a positive integer r the *r-path graph*, $P_r(G)$, has for vertices the set of all paths of length r in G. Two vertices are adjacent when the intersection of the corresponding paths forms a path of length $r-1$, and their union forms either a cycle or a path of length $r+1$ in G. Let $P_r^k(G)$ be the k-iteration of r-path graph operator on a connected graph G. Let H be a subgraph of $P_r^k(G)$. The k-history $P_r^{-k}(H)$ is a subgraph of G that is induced by all edges that take part in the recursive definition of H. We present some general properties of k-histories and give a complete characterization of graphs that are k-histories of vertices of 2-path graph operator.

Determining the diameter of a graph by the method of "hanging"

Agnieszka Patyk
(joint work with Jerzy Topp)

We consider a method of measuring the diameter of a graph called "*hanging*": let's hang graph $G = (V, E)$ by any vertex, say $x \in V$. Let $y \in V$ be the farthest vertex from x. Now, let's hang G by vertex y and let $z \in V$ be the farthest vertex from y. Does $d_G(y, z) = \text{diam}(G)$? Which classes of graphs can be measured in that way and for which ones this method is inappropriate?

Let $G = (V, E)$ be a connected graph and $x \in V$ be one of its vertices. The *eccentricity* of x is $e_G(x) = \max\{d_G(x, y) : y \in V\}$. The *peripherum* of graph G

is $D(G) = \{x \in V : e_G(x) = \mathrm{diam}(G)\}$. The *hanging of G by x* is a function $h_x : V \to \mathbb{N}$ that associates to every $y \in V$ the value $d_G(x, y)$ called the *level* of y in the hanging h_x. Let $L^i(x)$ denote the set of vertices with level i in h_x. Certainly, the largest i such that $L^i(x) \neq \emptyset$ is equal to the eccentricity $e_G(x)$ of x. We will explore some classes of 2-*hangable* graphs for which $\forall\, x \in V : e_G(x) = \mathrm{diam}(G)$.

The concept of diameter of a graph is not only of theoretical interest but also arises in natural manner in applications, eg. in the problem of designing communication networks and in the problem of the distribution of service points. The traditional algorithms for finding shortest paths between all vertices in graph $G = (V, E)$ take $O(|V|^3)$ time, while for 2-*hangable* graphs computing the diameter of a graph takes just $O(|V|^2)$ steps.

On (Δ,2,2)-Graphs

Guillermo Pineda-Villavicencio

(joint work with Mirka Miller)

The Degree/Diameter problem consists of determining the largest possible number of vertices in a graph of given maximum degree Δ and diameter D. It is well known that the general upper bound, called Moore bound $M(\Delta, D)$, for the order of such graphs is attainable only for certain special values of maximum degree and diameter. Therefore, we are interested in studying the existence of large graphs of given maximum degree Δ, diameter D and order $M(\Delta, D)$ - δ, that is, (Δ, D, δ)-graphs.

In the talk we discuss recent results about structural properties of $(\Delta, 2, 2)$-graphs. We prove the nonexistence of such graphs for infinitely many values of Δ and we give some necessary conditions for their existence. Finally, we present some related open problems in this research area.

Cyclic Kirkman triple systems

Alexander Rosa

A parallel class in a Steiner triple system (STS) (V, B) is a set of pairwise disjoint triples which partition the set of elements V. An STS (V, B) is resolvable if one can partition the set of triples B into parallel classes; any such partition R is a resolution. A Kirkman triple system (KTS) (V, B, R) is an STS (V, B) together with a particular resolution R. It is cyclic if there exists a permutation of V consisting of a single cycle of length $|V|$ which preserves both the set of blocks B, and the resolution R.

We survey the known and new results on the existence and nonexistence of cyclic KTSs, with emphasis on constructing and enumerating cyclic KTSs of small orders.

A new upper bound for the chromatic number of a graph

Ingo Schiermeyer

For a connected graph G of order n, the clique number $\omega(G)$, the chromatic number $\chi(G)$ and the independence number $\alpha(G)$ satisfy $\omega(G) \leq \chi(G) \leq n - \alpha(G) + 1$. We will show that $\chi(G) \leq \frac{n+\omega+1-\alpha}{2}$, which is the arithmetic mean of the previous lower and upper bound. Moreover, if G contains no $K_{\omega+3} - C_5$ and is not a split graph, then $\chi(G) \leq \frac{n+\omega-\alpha}{2}$.

For a connected graph G, the clique number $\omega(G)$, the chromatic number $\chi(G)$ and the maximum degree $\Delta(G)$ satisfy $\omega(G) \leq \chi(G) \leq \Delta(G)+1$. In 1941 Brooks has shown that complete graphs and odd cycles are the only graphs attaining the upper bound $\Delta(G) + 1$.

In 1998 Reed posed the following

Conjecture: For any graph G of maximum degree Δ and clique number ω,

$$\chi(G) \leq \left\lceil \frac{\Delta + 1 + \omega}{2} \right\rceil .$$

We will report about recent progress on proving this conjecture for special graph classes and compare the upper bounds.

Non-regular supermagic graphs

Andrea Semaničová

(joint work with Jaroslav Ivančo)

A graph is called supermagic if it admits a labeling of the edges by pairwise different consecutive integers such that the sum of the labels of the edges incident with a vertex is independent of the particular vertex. In this talk we deal with non-regular supermagic graphs. We introduce some constructions of supermagic non-regular graphs using supermagic labeling of some regular graphs and $(a, 1)$-antimagic labeling of some graphs.

On-line altitude of graphs

Gabriel Semanišin

(joint work with Ján Katrenič)

A *linear ordering* of the edges of a graph $G = (V, E)$ is an injective mapping $f : V \longrightarrow \{1, 2, \dots\}$. If f is a linear ordering of a graph G then f*-ascent* of G is a path P for which f increases along the edge sequence of P. By $\text{alt}(G, f)$ we denote the maximum length of f-ascents of G with respect to f. The *altitude* of G, denoted by $\text{alt}(G)$, is defined as the minimum of $\alpha(G, f)$ taken over all linear orderings of the edges of G.

We study the on-line version of the problem, i.e. the structure of a graph G is presented step by step and our goal is to determine the value of the on-line altitude of graph. We present some general bounds and the estimations for some particular classes of graphs. Moreover we discuss some related complexity problems.

On the chromatic number of circulant graphs

Oriol Serra

Every graph can be embedded in a circulant graph with reasonable size. This is a reason why restriction to this particular family of graphs does not make the computation of the chromatic number substantially easier. Even so this computation can be achieved for circulant graphs of small degree $d \leq 4$, and general bounds can be given for larger degrees. The conections with problems in number theory makes the study of the chromatic number of circulant graphs (and of distance graphs over the integers) particularly interesting. We will describe some of these connections and give an overview of the main results in the area.

Nowhere-zero 3-flows in Cayley graphs of nilpotent groups

Martin Škoviera

(joint work with Mária Nánásiová)

Thirty years ago Tutte conjectured that every bridgeless graph with no 3-edge-cuts has a nowhere-zero 3-flow. This conjecture is closely related to the conjecture of Alspach, Liu and Zhang (1996) that every Cayley graph of valency at least 3 admits a nowhere-zero 4-flow, and to the well-known conjecture that every Cayley graph of valency at least 3 is hamiltonian.

In the talk we verify Tutte's 3-Flow Conjecture on Cayley graphs of nilpotent groups, improving a recent result of Potočnik et al. (2005).

Domination hypergraphs of graphs

Martin Sonntag

(joint work with Ang Lu)

Let $G = (V, E)$ be an undirected graph. A set $e \subseteq V$ *dominates* G iff every vertex $v \in V \setminus e$ has a neighbour in e. We generalize the well-known concept of the *domination graph* in the following way: $\mathcal{DH}(G) = (V, \mathcal{E})$ is the *domination hypergraph* of the graph G iff

$$\mathcal{E} = \{e \mid e \subseteq V \ \wedge \ e \text{ is a minimal dominating set of } G\}.$$

Basic properties of domination hypergraphs of certain classes of graphs, e.g. paths, cycles and graphs with articulation vertices, are investigated. For some classes of graphs we present algorithms to construct their domination hypergraphs.

Finally, we discuss the relationship between domination hypergraphs of digraphs and domination hypergraphs of undirected graphs.

On 2-walks in chordal planar graphs

Jakub Teska

A 2-walk is a closed spanning trail which uses every vertex at most twice. A graph is said to be chordal if each cycle different from 3-cycle has a chord. The *toughness* of a non-complete graph is $t(G) = \min(\frac{|S|}{c(G-S)})$, where the minimum is taken over all nonempty vertex sets S, for which $c(G - S) \geq 2$ and $c(G - S)$ denotes the number of components of the graph $G - S$. We prove that every chordal planar graph G with toughness $t(G) > \frac{3}{4}$ has a 2-walk. Then we show the existence of an infinite class of 2-connected chordal planar graphs with toughness $\frac{1}{2}$ and without a 2-walk, followed by conjectures and open problems.

Color-bounded hypergraphs, I: General results

Zsolt Tuza

(joint work with Csilla Bujtás)

We generalize the concept of 'mixed hypergraphs' to the following model. A *color-bounded hypergraph* is a hypergraph (set system) with vertex set X and edge set $\mathcal{E} = \{E_1, \ldots, E_m\}$, where each edge E_i is associated with two integers s_i and t_i $(1 \leq s_i \leq t_i \leq |E_i|)$. A vertex coloring $\varphi : X \to \mathbb{N}$ is considered to be feasible if the number of colors occurring in E_i satisfies $s_i \leq |\varphi(E_i)| \leq t_i$, for all $i \leq m$. (Another related concept deals with the number of *occurrences* of colors inside each edge.)

In the earlier model of mixed hypergraphs, introduced by Voloshin (1993), the 'C-edges' and 'D-edges' have $(s_i, t_i) = (1, |E_i|-1)$ and $(s_i, t_i) = (2, |E_i|)$, respectively. Recently, Drgas-Burchardt and Lazuka (2005) considered the case of arbitrarily specified s_i with $t_i = |E_i|$ for all $i \leq m$.

In the talk we discuss the similarities and differences between our general model and the more particular earlier ones. An important issue is the chromatic spectrum — strongly related to the chromatic polynomial — which is the sequence whose k-th element is the number of allowed colorings with precisely k colors. Problems concerning algorithmic complexity are considered, too.

List colorings of graphs

Margit Voigt

Let G be a simple graph and $\mathcal{L} = \{L(v) \mid v \in V(G)\}$ be an assignment of lists of admissible colors for its vertices. We say that G is *$\mathcal{L}$-list colorable* if the vertices of G can be properly colored (i.e. adjacent vertices receive distinct colors) so that each vertex v is colored with a color from $L(v)$. If all lists of $\mathcal{L}$ have the same size k, $\mathcal{L}$ is called a k-assignment. The *list chromatic number* (or *choice number*) $\chi_\ell(G)$ of G is the minimum integer k such that G is $\mathcal{L}$-list colorable for every k-assignment $\mathcal{L}$. The graph G is called *k-choosable* if $\chi_\ell(G) \leq k$.

The talk summarizes some results and open problems in this very rich field of research including Brooks-type theorems, precoloring extensions and choosability of planar graphs.

The nonclassical mixed domination Ramsey numbers

Renata Zakrzewska

The nonclassical mixed domination Ramsey number $v(m, G)$ is the smallest integer p such that every 2-coloring of the edges of K_p with color red and blue, $\Gamma(B) \geq m$ or there is a blue copy of graph G, where B is the subgraph of K_p induced by blue edges; $\Gamma(G)$ is the maximum cardinality of a minimal dominating set of a graph G. We give exact values for numbers $v(m, K_3 - e)$, $v(3, P_k)$, $v(3, C_k)$. In addition, we give exact values and bounds for numbers $v(3, K_n - e)$, where $n \geq 3$.

Cyclic chromatic number

Jana Zlámalová

(joint work with Mirko Horňák)

The *cyclic chromatic number* $\chi_c(G)$ of a graph G embedded to a surface is the minimum number of colours in such a colouring of vertices of G that any two vertices of G sharing a common face receive different colours.

Let $\Delta^*(G)$ denote the maximum face size of G. Plummer and Toft conjecture that $\chi_c(G) \leq \Delta^*(G) + 2$ for any 3-connected plane graph G. It is known that the conjecture is true for $\Delta^*(G) \leq 4$ and for $\Delta^*(G) \geq 22$.

Here it is proved that the conjecture is true for $\Delta^*(G) \geq 18$.

Domination in special constructions of graphs

Maciej Zwierzchowski

(joint work with Magdalena Bohonos)

Let D be a total dominating set of a graph G. The cardinality of a smallest total dominating set of a graph G is called the total domination number of G and it is denoted by $\gamma_t(G)$. We determine the total domination number with respect to special constructions of graphs: the duplication of a subset of G as also a partial join. As a corollary we obtain the result concerning the domination and the total domination number of Mycielski's construction and generalized Mycielski's construction.

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Programme of the Conference

Sunday		
14:00 - 22:00	Registration	
18:00 - 21:00	Dinner	

Monday		
07:30 - 08:30	Breakfast	
08:40 - 08:45	Opening	
08:45 - 09:35	Kaiser T.	The Dominating cycle conjecture
09:40 - 10:00	van Aardt S.	Maximal nontraceable oriented graphs
10:05 - 10:25	Škoviera M.	Nowhere-zero 3-flows in Cayley graphs of nilpotent groups
10:30 - 10:55	Coffee break	
10:55 - 11:15	Kemnitz A.	$[r, s, t]$-chromatic numbers of stars
11:20 - 11:40	Kráľ D.	Colorings of quadrangulations of the torus and the Klein bottle
11:45 - 12:05	Bacsó G.	Elementary bipartite graphs and unique colorability
12:10 - 12:30	Problem session	
12:30 - 13:15	Lunch	
14:45 - 15:35	Voigt M.	List colorings of graphs
15:40 - 16:00	Kijewska M.	Generalized domatic numbers of graph products
16:05 - 16:25	Bača M.	Total labelings of graphs
16:30 - 16:55	Coffee break	
16:55 - 17:15	Michalak D.	k-hereditarily dominated graphs
17:20 - 17:40	Dvořák Z.	k-chromatic number of graphs on surfaces
17:45 - 18:05	Klešč M.	On the crossing numbers of categorical and strong products
18:10 - 18:30	Katrenič P.	On a tree partition problem
18:30 - 19:15	Dinner	
20:00 -	Welcome party	

Tuesday		
07:30 - 08:30	Breakfast	
08:45 - 09:35	GOULD R.	A look at cycles containing specified elements
09:40 - 10:00	BOROWIECKI M.	On F-decompositions of graphs
10:05 - 10:25	FRICK M.	A new perspective on the Path partition conjecture
10:30 - 10:55	Coffee break	
10:55 - 11:15	HARANT J.	A regional version of Tutte's Theorem concerning Hamiltonian cycles in 4-connected planar graphs
11:20 - 11:40	MILLER M.	Diregularity of digraphs of out-degree three and defect two
11:45 - 12:05	SONNTAG M.	Domination hypergraphs of graphs
12:10 - 12:30	BRANDT S.	tba
12:30 - 13:15	Lunch	
14:45 - 15:35	SERRA O.	On the chromatic number of circulant graphs
15:40 - 16:00	NIEPEL Ľ.	Dynamics of iterated path graphs
16:05 - 16:25	DAFIK	On super (a, d)-edge antimagic total labeling of disconnected graphs
16:30 - 16:55	Coffee break	
16:55 - 17:15	ZLÁMALOVÁ J.	Cyclic chromatic number
	KRAVECOVÁ D.	On the crossing numbers of products of power graphs
17:20 - 17:40	KUŽEL R.	Trestles and walks in $K_{1,r}$-free graphs
	FIDYTEK R.	On some three color Ramsey numbers for paths and cycles
17:45 - 18:05	ZAKRZEWSKA R.	The nonclassical mixed domination Ramsey numbers
	HAJDUK R.	Big light graphs in some classes of polyhedra
18:10 - 18:30	KARDOŠ F.	Symmetry of fulleroids
18:30 - 19:15	Dinner	
20:00 - 21:20	Videopresentation C&C 2005	

Wednesday	
07:00 - 08:30	Breakfast
08:30 - 16:00	Trip
19:00 - 20:00	Dinner

Thursday		
07:30 - 08:30	Breakfast	
08:45 - 09:35	Chen G.	Finding long cycles in 3-connected graphs
09:40 - 10:00	Schiermeyer I.	A new upper bound for the chromatic number of a graph
10:05 - 10:25	Mihók P.	Generalizations of Brooks' Theorem – a short survey
10:30 - 10:55	Coffee break	
10:55 - 11:15	Rosa A.	Cyclic Kirkman triple systems
11:20 - 11:40	Tuza Zs.	Color-bounded hypergraphs, I: General results
11:45 - 12:05	Bujtás Cs.	Color-bounded hypergraphs, II: Interval hypergraphs and hypertrees
12:10 - 12:30	Semanišin G.	On-line altitude of graphs
12:30 - 13:15	Lunch	
14:45 - 15:35	Jensen T.	Cycle structure and colourings
15:40 - 16:00	Cichacz S.	Decomposition of complete bipartite graphs into trails
16:05 - 16:25	Teska J.	On 2-walks in chordal planar graphs
16:30 - 16:55	Coffee break	
16:55 - 17:15	Máčajová E.	On shortest cycle covers of cubic graphs
17:20 - 17:40	Pineda Villavicencio G.	On $(\Delta,2,2)$-graphs
17:45 - 18:05	Cyman J.	The total outer-connected domination in trees
18:10 - 18:30	Zwierzchowski M.	Domination in special constructions of graphs
19:00 -	Farewell party	

Friday		
07:30 - 08:30	Breakfast	
08:45 - 09:35	van den Heuvel J.	Mixing colour(ing)s in graphs
09:40 - 10:00	Mačaj M.	2-perfect trails system, total colourings and latin squares
10:05 - 10:25	Patyk A.	Determining the diameter of a graph by the method of "hanging"
10:30 - 10:55	Coffee break	
10:55 - 11:15	Čada R.	Path factors in cubic graphs
11:20 - 11:40	Matos Camacho S.	Colourings of graphs with prescribed cycle lengths
11:45 - 12:05	Draženská E.	The crossing numbers of graphs
12:10 - 12:30	Miškuf J.	Upper bound for maximal vertex and face degree of oblique graphs
12:30 - 13:15	Lunch	