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Workshop
Cycles and Colourings 2007

September 02 – 07, 2007 Tatranská Štrba

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C&C

HIGH TATRAS, SLOVAKIA

Igor Fabrici, Mirko Horňák, Stanislav Jendroľ ed.

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http://umv.science.upjs.sk/c&c

Dear Participant,

welcome to the Sixteenth Workshop Cycles and Colourings. Except for the first workshop in the Slovak Paradise (Čingov 1992), the remaining fourteen workshops took place in the High Tatras (Nový Smokovec 1993, Stará Lesná 1994–2003, Tatranská Štrba 2004–2006).

The series of C&C workshops is organised by combinatorial groups of Košice and Ilmenau. Apart of dozens of excellent invited lectures and hundreds of contributed talks the scientific outcome of our meetings is represented also by special issues of journals Tatra Mountains Mathematical Publications and Discrete Mathematics (TMMP 1994, 1997, DM 1999, 2001, 2003, 2006 - under preparation).

The scientific programme of the workshop consists of 50 minute lectures of invited speakers and of 20 minute contributed talks. This booklet contains abstracts as were sent to us by the authors.

Invited speakers:

Evelyne Flandrin,	Paris-Sud University, Orsay, France
Mihály Hujter,	University of Technology and Economics, Budapest, Hungary
Katsuhiro Ota,	Keio University, Yokohama, Japan
Dieter Rautenbach,	Technical University, Ilmenau, Germany
Martin Škoviera,	Comenius University, Bratislava, Slovakia
Riste Škrekovski,	University of Ljubljana, Ljubljana, Slovenia
Xuding Zhu,	National Sun Yat-sen University, Kaohsiung, Taiwan

Have a pleasant and successfull stay in Tatranská Štrba.

Organising committee:

Igor Fabrici
Jochen Harant
Erhard Hexel
Mirko Horňák
Stanislav Jendroľ (chair)
Štefan Schrötter

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Guard sets in hypergraphs

Gábor Bacsó
(joint work with Zsolt Tuza)

Given a hypergraph $\mathcal{F}$ and a hyperedge F, a subset $B \subseteq F$ is a *guard set* of F if every hyperedge, intersecting F but not contained in F intersects B, too. A hyperedge may have several guard sets and multiple hyperedges may occur.

We call the intersection graph of the hypergraph $\mathcal{F}$ briefly its *line graph* and denote it by $L\mathcal{F}$.

A hypergraph is *c-small* if all of its hyperedges have some guard set of size at most c. A graph G is *c-representable* if there exists a c-small hypergraph $\mathcal{F}$ such that $L\mathcal{F} = G$. The minimum of c such that G is c-representable is called the *representation number* of G and is denoted by $c(G)$.

The so-called *Minimax Conjecture* states that the representation number of a graph can be expressed by the clique covering numbers of the subgraphs, induced by the neighborhood sets of the vertices.

We give partial results, in connection with this conjecture.

Ramsey numbers for disjoint union of some graphs

Halina Bielak

We give the Ramsey number for disjoint union of some graphs with a complete graph in the pair generalizing a result of Stahl [2] and Baskoro et al. [1].

References

[1] E. T. Baskoro, Hasmawati, H. Assiyatun, Note. The Ramsey Number for disjoint unions of trees, Discrete Math. 306 (2006), 3297-3301.
[2] S. Stahl, On the Ramsey Number $r(F, K_m)$ where F is a forest, Canad. J. Math. 27 (1975), 585-589.

Triangle-free graphs whose independence number equals the degree

Stephan Brandt

In a triangle-free graph, the neighbourhood of every vertex is an independent set. We will investigate the class $\mathcal{S}$ of triangle-free graphs where each vertex neighbourhood is a maximum independent set. Such a graph $G \in \mathcal{S}$ must be regular of degree $d = \alpha(G)$ and the fractional chromatic number must satisfy $\chi_f(G) = \frac{|G|}{\alpha(G)}$. We indicate that $\mathcal{S}$ is a rich family of graphs by determining the rational numbers c for which there is a graph $G \in \mathcal{S}$ with $\chi_f(G) = c$ except for a small gap, where we cannot prove the full statement. The statements for $c \geq 3$ are obtained by using, modifying, and reanalysing constructions of Sidorenko, Mycielski, and Bauer, van den Heuvel and Schmeichel, while the case $c < 3$ is settled by a recent result of Brandt and Thomassé. We will also investigate the relation between other parameters of certain graphs in $\mathcal{S}$ like chromatic number and toughness.

Set systems crossing all k-partitions, II.

Csilla Bujtás

(joint work with Zsolt Tuza)

In this two-part talk we introduce the concept of partition-crossing, and point out its various connections to important areas of combinatorics and graph theory via the investigation of an extremal function.

We say that a subset F of a finite set X *crosses* a k-partition $X_1 \cup X_2 \cup \cdots \cup X_k$ of X (where $X_i \neq \emptyset$ is assumed for all $1 \leq i \leq k$) if either every partition class contains at most one element of F (for $|F| \leq k$) or every partition class contains at least one element of F (for $|F| \geq k$); that is, F intersects exactly $\min(|F|, k)$ of the k partition classes.

It is quite natural to ask about the smallest possible number of r-element subsets of an n-element set X such that every k-partition of X is crossed by at least one of the selected subsets.

We denote this minimum by $f(n, k, r)$, and discuss its asymptotic behavior as $n \to \infty$. Considering specified values of k and r, or letting k and r get large with n, lower and upper bounds are obtained that yield tight or asymptotically tight estimates in many cases. The solutions lead us to intensively studied areas of combinatorics such as Balanced Incomplete Block Designs and Turán-type extremal problems on graphs and hypergraphs.

The function $f(n, k, r)$ has an equivalent description in terms of hypergraph coloring theory, too. From this point of view, every selected r-element set is considered as a hyperedge associated with an appropriately fixed upper bound ($r-1$ or $k-1$) on the number of colors occurring in it. Then $f(n, k, r)$ can be interpreted as the minimum number of hyperedges that can forbid all colorings of the hypergraph with more than $k - 1$ colors. By this connection, our results give an asymptotic solution to an open problem of 'mixed hypergraph theory', too.

Maximum cycle decomposition in linegraphs

Jan Degenhardt

Let $G = (V, E)$ be a graph. A decomposition $\mathcal{Z} = \{G_1, \ldots, G_q\}$ of G is a collection of subgraphs G_i of G $(i = 1, \ldots, q)$ such that all G_i are mutually edge-disjoint and $G = \bigcup_{i=1}^{q} G_i$. For an arbitrary decomposition $\mathcal{Z}$ the set $\mathcal{C}(\mathcal{Z}) \subseteq \mathcal{Z}$ denotes the family of circuits in $\mathcal{Z}$, i. e. $G_i \in \mathcal{C}(\mathcal{Z}) \iff G_i \in \mathcal{Z}$ and G_i is a circuit.

The lecture deals with the cydec number $\varkappa(G) = \max\{|\mathcal{C}(\mathcal{Z})| : \mathcal{Z}$ is a decomposition of $G\}$ for special classes of graphs: Let $L(G)$ be the linegraph of G and $DL(G)$ be the linegraph of the graph $\tilde{G}$ which arises from G by subdividing every edge of G. In this lecture we will discuss relations between $\varkappa(G)$ and $\varkappa(DL(G))$ and $\varkappa(L(G))$ respectively.

Unique factorization theorems

Ewa Drgas-Burchardt

An additive hereditary property of graphs is any class of simple graphs, which is closed under union, subgraphs and isomorphisms. Let $\mathbf{L}^a$ denote a class of all such properties. We introduce the following notions: to be graph-reducible over $\mathbf{L}^a$, to be $\mathcal{C}$-reducible over $\mathbf{L}^a$, for $\mathcal{C}$ being the subclass of all prime graphs without K_1. They refer to an arbitrary property in $\mathbf{L}^a$. Then we analyze uniqueness of $\mathcal{C}$-factorization of a property in $\mathbf{L}^a$ into $\mathcal{C}$-irreducible and graph-irreducible factors over $\mathbf{L}^a$, giving the unique factorization theorems. They generalize known results of this type obtained in [2], [6], [7], bringing together $\circ$-reducibility over $\mathbf{L}^a$ and $\vee$-reducibility in the lattice $(\mathbf{L}^a, \subseteq)$.

It is also shown that each graph can be uniquely constructed from prime graphs using a known product of graphs. This result offers a new insight to the modular decomposition tree for a graph.

References

[1] A. J. Berger, I. Broere, S. J. T. Moagi, P. Mihók, Meet- and join-irreducibility of additive hereditary properties of graphs, Discrete Math. 251 (2002), 11–18.

[2] A. Berger, P. Mihók, Prime ideals in the lattice of additive induced-hereditary graph properties, Discuss. Math. Graph Theory 23(1) (2003), 117–127.

[3] M. Borowiecki, P. Mihók, Hereditary properties of graphs, in: V. R. Kulli, ed., Advances in Graph Theory, Vishawa International Publication, Gulbarga, 1991, 41–68.

[4] A. Farrugia, Uniqueness and complexity in generalized coloring, PhD thesis, Waterloo, Ontario, Canada, 2003

[5] M. Habib, F. de Montgolfier, Ch. Paul, A simple linear-time modular decomposition algorithm for graphs, using order extention, Lecture Notes in Computer Science 3842 (2006), 343–354.
[6] J. Kratochvíl, P. Mihók, Hom-properties are uniquely factorizable into irreducible factors, Discrete Math. 213 (2000), 189–194.
[7] P. Mihók, G. Semanišin, R. Vasky, Additive and hereditary properties of graphs are uniquely factorizable into irreducible factors, J. Graph Theory 33 (2000), 44–53.

Cycles in graphs: around the hamiltonian problem

Evelyne Flandrin

We consider some results related to the hamiltonian problem: particular classes of graphs, cycles of given length, cycles through given vertices or edges. This survey does not pretend to be exhaustive.

A traceability conjecture for oriented graphs

Marietjie Frick
(joint work with Susan van Aardt, Jean Dunbar, Morten Nielsen and Ortrud Oellermann)

An oriented graph is called *k-traceable* if each of its induced subdigraphs of order k has a hamiltonian path. A graph of order n is called *traceable* if it is n-traceable. We conjecture that for $k \geq 2$ every k-traceable graph of order at least $2k - 1$ is traceable. We prove the conjecture for $k \leq 5$ and also show that the conjecture holds for certain classes of oriented graphs.

Recent progress on vertex-disjoint cycles in graphs

Shinya Fujita

Recent progress on vertex-disjoint cycles in graphs will be reviewed. In this talk, I will introduce some recent results concerning sharp degree conditions for the existence of a prescribed number of vertex-disjoint cycles in graphs. There are many known results in this thesis. One of them is concerning a graph partition problem by a prescribed number of vertex-disjoint cycles and degenerated cycles. As an example of this topic, I obtained the following result:

Theorem. Let k, n be integers with $2 \leq k \leq n$, and let G be a graph of order n. Suppose that $\max\{d_G(x), d_G(y)\} \geq (n-k+1)/2$ for any $x, y \in V(G)$ with $x \neq y$ and $xy \notin E(G)$. Then G can be partitioned into k vertex-disjoint subgraphs $H_1, \ldots, H_k$ such that H_i is a cycle or K_1 or K_2 for each $1 \leq i \leq k$ unless $k = 2$ and $G = C_5$ or $k = 3$ and $G = K_1 \cup C_5$.

In this talk, I will mention about this theorem and also I will introduce other latest results.

Hamiltonian cycles through prescribed edges of at least 4-connected maximal planar graphs

Frank Göring

(joint work with Jochen Harant)

In 1956, W. T. Tutte proved that a 4-connected planar graph is hamiltonian. Moreover, in 1997, D. P. Sanders extended this to the result that a 4-connected planar graph contains a hamiltonian cycle through any two of its edges.

It is shown that Sanders' result is best possible by constructing 4-connected maximal planar graphs with three edges of large distance such that any hamiltonian cycle misses one of them. If the maximal planar graph is 5-connected then such a construction is impossible.

On the azulenoids with large ring of 5-gons

Róbert Hajduk

(joint work with František Kardoš and Roman Soták)

Deza and Grishukhin studied 3-valent maps $M_n(p, q)$ consisting of a ring of n q-gons whose inner and outer domains are filled by p-gons. They described the conditions on (n, p, q) under which such map may exist and presented several infinite families of them. The open cases are, in particular, $M_n(7, 5)$ with $n > 28$ and $M_n(5, q)$ for $q \geq 8$. For the case $M_n(7, 5)$, we give all maps with $n \leq 52$ and existence for all even n. We also construct maps with odd n, $n = 87$. For all these maps we give also their symmetry groups. Among the maps we have found, there are several ones with non-isomorphic inner and outer domains, and, moreover, the one that has only trivial symmetry group.

On Ramsey minimal graphs

Mariusz Hałuszczak

(joint work with Ewa Drgas-Burchardt)

For a graph G and families of graphs $\mathcal{F}, \mathcal{H}$ we write $G \stackrel{v}{\to} (\mathcal{F}, \mathcal{H})$ to mean that if the vertices of G are coloured with two colours, say red and blue, then the red subgraph contains a copy of F or the blue subgraph contains a copy of H. The graph G is vertex $(\mathcal{F}, \mathcal{H})$-*minimal* (*Ramsey-minimal*) if $G \stackrel{v}{\to} (\mathcal{F}, \mathcal{H})$ but $G' \stackrel{v}{\not\to} (\mathcal{F}, \mathcal{H})$ for any proper subgraph G' of G. The class of all vertex $(\mathcal{F}, \mathcal{H})$-minimal graphs will be denoted by $\Re_v(\mathcal{F}, \mathcal{H})$. A. Berger in [1] showed that if $\mathcal{F}, \mathcal{H}$ contain only connected graphs then $\Re_v(\mathcal{F}, \mathcal{H})$ has infinitely many graphs.

In this talk we present new constructions of such families. They are based on the well-known Hajós' construction.

References

[1] A. J. Berger, Minimal forbidden subgraphs of reducible graph properties, Discuss. Math. Graph Theory 21(1) (2001), 111–117.

[2] M. Borowiecka-Olszewska, E. Drgas-Burchardt, P. Mihók, Minimal vertex Ramsey graphs and minimal forbidden subgraphs, Discrete Math. 286 (2004), 31–36.

[3] M. Borowiecki, P. Mihók, Hereditary properties of graphs, in: V. R. Kulli, ed., Advances in Graph Theory, Vishawa International Publication, Gulbarga, 1991, 41–68.

[4] A. Farrugia, Vertex-partitioning into fixed additive induced-hereditary properties is NP-hard, Electron. J. Combin. 11 (2004), R46.

[5] G. Hajós, Über eine Konstruction nicht n-färbbarer Graphen, Wiss. Z. Martin-Luther-Univ. Halle-Wittenberg Math.-Natur. Reihe 10 (1961), 116–117.

On vertices enforcing a Hamilton cycle

Erhard Hexel

(joint work with Igor Fabrici and Stanislav Jendroľ)

For a hamiltonian graph G a nonempty vertex set $X \subseteq V(G)$ is called a *Hamilton cycle spanning set* (in short *H-span*) of G if every cycle of G which contains each vertex of X is a Hamilton cycle. The number $h(G)$ denotes the cardinality of the smallest H-span of G. In the talk we present some properties of this invariant of hamiltonian graphs.

Precoloring extension: the first fifteen years

Mihály Hujter

Fifteen is the fifth triangle number that is the number of edges in the smallest graph which is not 5-colorable. The Precoloring Extension Problem (PrExt in short) was launched fifteen years ago. We show a few different branches of the research on PrExt. Five initial papers are listed below.

PrExt is more general than the usual chromatic number problem and less general than list-coloring:

Instance. An integer $k \geq 2$, a graph $G = (V, E)$ with $|V| \geq k$, a vertex subset $W \subseteq V$, and a proper k-coloring of G_W.

Question. Can this k-coloring be extended to a proper k-coloring of the whole graph G?

Many results are related to the complexity status of some versions of PrExt. Others give some new information on perfect graphs. The practical applications are also important (scheduling, memory allocation, geometrical packings).

During the last fifteen years more than one hundred authors studied some versions of PrExt. We will show a few old and a few new results.

References

[1] M. Biró, M. Hujter, Zs. Tuza, Precoloring extension. I. Interval graphs, Discrete Math. 100 (1992), 267–279.

[2] M. Biró, M. Hujter, Zs. Tuza, Cross fertilisation of graph theory and aircraft maintenance scheduling, Thirty-Second Annual Symposium (G. Davison, ed.), AGIFORS (Airline Group of the International Federation of Operational Research Societies), 1993, 307–317.

[3] M. Hujter, Zs. Tuza, Precoloring extension. II. Graph classes related to bipartite graphs, Acta Mathematica Universitatis Comenianae 62 (1993), 1–11.

[4] M. Hujter, Zs. Tuza, The number of maximal independent sets in triangle-free graphs, SIAM J. Discrete Math. 6 (1993), 284–288.

[5] M. Hujter, Zs. Tuza, Precoloring extension. III. Classes of perfect graphs, Combin. Probab. Comput. 5 (1996), 35–56.

Cyclic edge-cuts in fullerene graphs

František Kardoš

(joint work with Riste Škrekovski)

In this talk, we present recent results on cyclic edge-cuts in fullerene graphs. First, we show that the cyclic edge-cuts of a fullerene graph can be constructed from its trivial cyclic 5- and 6-edge-cuts using three basic operations. This result immediately implies the fact that fullerene graphs are cyclically 5-edge-connected. Next, we characterize a class of nanotubes as the only fullerene graphs with non-trivial cyclic 5-edge-cuts. A similar result is also given for cyclic 6-edge-cuts of fullerene graphs.

Graphs with no path-kernels

Peter Katrenič

Let $\tau(G)$ denote the number of vertices in a longest path of a graph $G = (V, E)$. A subset K of V is called a P_n-*kernel* of G if $\tau(G[K]) \leq n - 1$ and every vertex $v \in V \setminus K$ is adjacent to an end-vertex of a path of order $n-1$ in $G[K]$. We show that for every nonnegative integer k there exist a graph H with no $P_{\tau(H)-k}$-kernel.

Circular total colorings of graphs

Arnfried Kemnitz

A (k, d)-total coloring ($k, d \in \mathbb{N}$, $k \geq 2d$) of a graph G is an assignment c of colors $\{0, 1, \ldots, k-1\}$ to the vertices and edges of G such that $d \leq |c(x_i)-c(x_j)| \leq k-d$ whenever x_i and x_j are two neighbored elements (two adjacent edges, two adjacent vertices or an edge incident to a vertex).

If C is a circle of perimeter r then an r-circular total coloring of a graph G is defined as an assignment c' of open unit length arcs of C to the elements of G such that $c'(x_i) \cap c'(x_j) = \emptyset$ for every two neighbored elements x_i and x_j.

A (k, d)-total coloring induces a k/d-circular total coloring, and vice versa.

The circular total chromatic number $\chi''_c(G)$ is defined by

$$\begin{aligned}\chi''_c(G) &= \inf\{k/d : G \text{ has a } (k, d)\text{-total coloring}\}\\ &= \inf\{r : G \text{ has an } r\text{-circular total coloring}\}.\end{aligned}$$

It holds $\chi''(G) - 1 < \chi''_c(G) \leq \chi''(G)$ with equality for all type-1 graphs where $\chi''(G)$ is the total chromatic number of G.

We determine exact values of $\chi''_c(G)$ for different classes of type-2 graphs. Moreover, we determine infinite classes of graphs G such that $\chi''_c(G) < \chi''(G)$.

On the crossing numbers of join between stars and other graphs

Marián Klešč

The *crossing number* $cr(G)$ of a graph G is the minimum possible number of edge crossings in a drawing of G in the plane. The investigation on the crossing number of graphs is a classical and however very difficult problem. Garey and Johnson have proved that the problem to determine the crossing number of graphs is NP-complete. Because of this difficulty, only for some specific families of graphs the exact values of crossing numbers are known: the complete graphs with small number of vertices, the complete bipartite graphs of less number of vertices in one bipartite section, certain generalized Petersen graphs, and some Cartesian products of graphs.

Let G_1 and G_2 be two disjoint graphs. The *join* of G_1 and G_2, denoted by G_1+G_1, is obtained from vertex–disjoint copies of G_1 and G_2 by adding all possible edges between $V(G_1)$ and $V(G_2)$. For $|V(G_1)| = m$ and $|V(G_2)| = n$, the edge set of $G_1 + G_2$ is the union of disjoint edge sets of the graphs G_1, G_2, and the complete bipartite graph $K_{m,n}$. The crossing numbers for join of two paths, join of two cycles, and for join of path and cycle are given in [1]. In the talk, we give the exact values of crossing numbers for join of stars with some graphs of small order and we find the upper bounds for crossing numbers of join of stars with other graphs.

References

[1] M. Klešč, The join of graphs and crossing numbers, Electron. Notes Discrete Math. 28 (2007), 349–355.

Embeddings of tables of groups of order 8

Martin Knor

(joint work with M. J. Grannell and T. S. Griggs)

We consider orientable triangulations by the complete tripartite graph $K_{n,n,n}$. Such embeddings correspond to biembeddings of Latin squares. Formerly, using a computer, we obtained all such embeddings for $n \leq 7$, while now we concentrated on the case $n = 8$. We focussed our attention to the tables (Latin squares) of the 5 groups of order 8 and we found all their embeddings. Some features of these embeddings are discussed.

6-critical graphs on the Klein bottle

Daniel Kráľ

(joint work with Ken-ichi Kawarabayashi, Jan Kynčl and Bernard Lidický)

Thomassen [2] showed that the number of 6-critical graphs on every surface is finite. Recall that a graph G is 6-critical if it is 6-chromatic and any proper subgraph of G is 5-colorable. There are no 6-critical planar graphs and the only 6-critical projective planar graph is K_6. Thomassen [1] found a complete list of three 6-critical graphs on the torus and conjectured a list of 6-critical graphs on the Klein bottle. We refutate his conjecture by finding a complete list of nine 6-critical graphs on the Klein bottle (which have 18 non-isomorphic 2-cell embedding and one non-2-cell embedding).

References

[1] C. Thomassen, Five-Coloring Graphs on the Torus, Journal Comb. Theory Ser. B 62 (1994), 11–33.

[2] C. Thomassen, Color-Critical Graphs on a Fixed Surface, Journal Comb. Theory Ser. A 70 (1997), 67–100.

Cycles and colourings of sociosemantic graphs

Jorma Kyppö

This lecture is focused on sociosemantic graphs. Semantic graphs can provide the methods of visualizing the long-term memory. Sociometric graphs, sociograms, reflect the human inter-relationships. Sociosemantic graphs are perceived as the combination of both, the sociograms and semantic graphs. This presentation will provide a survey on different types of cycles and colourings which can be found in the sociosemantic graphs.

Chromaticity of certain hypergraphs

Ewa Łazuka

Let H be a hypergraph and λ be a positive integer. A *λ-coloring* of H is such a function $f\colon V(H) \to \{1, 2, \ldots, \lambda\}$ that for each edge $e \in E(H)$ there exist $x, y \in e$ for which $f(x) \neq f(y)$. The number of λ-colorings of H is given by a polynomial $f(H, \lambda)$ of degree $|V(H)|$ in λ, called *the chromatic polynomial* of H.

Hypergraphs H_1 and H_2 are said to be *chromatic equivalent* ($H_1 \sim H_2$), if $f(H_1, \lambda) = f(H_2, \lambda)$. The equivalence class determined by H under $\sim$ is denoted by $\langle H \rangle$. A hypergraph H is *chromatically unique* if $\langle H \rangle = \{H\}$. A class

$\mathcal{H}$ of hypergraphs is said to be *chromatically characterized* if for any $H \in \mathcal{H}$ the condition $\langle H \rangle = \mathcal{H}$ holds. Let $\mathcal{K}$ and $\mathcal{H}$ be two classes of hypergraphs. $\mathcal{H}$ is said to be chromatically characterized in $\mathcal{K}$ if for every $H \in \mathcal{H} \cap \mathcal{K}$ we have $\langle H \rangle \cap \mathcal{K} = \mathcal{H} \cap \mathcal{K}$.

We present some theorems and conjectures concerning the chromaticity of several types of hypergraphs.

References

[1] M. Borowiecki and E. Łazuka, Chromatic polynomials of hypergraphs, Discuss. Math. Graph Theory 20 (2000), 293–301.

[2] M. Borowiecki and E. Łazuka, On chromaticity of hypergraphs, Discrete Math. 307 (2007), 1418–1429.

[3] K. Dohmen, Chromatische Polynome von Graphen und Hypergraphen, Ph. D. thesis, Düsseldorf, 1993.

[4] R. P. Jones, Some results of chromatic hypergraph theory proved by "reduction to graphs", Colloque CNRS. Problèmes Combinatoires et Théorie des Graphes 260 (1976), 249–250.

[5] I. Tomescu, Chromatic coefficients of linear uniform hypergraphs, J. Combin. Theory (B) 72 (1998), 229–235.

[6] I. Tomescu, On the chromaticity of sunflower hypergraphs $SH(n, p, h)$, Discrete Math. 307 (2007), 781–786.

Packing chromatic number for lattices

Bernard Lidický

(joint work with Jiří Fiala)

The packing chromatic number $\chi_\rho(G)$ of a graph G is the smallest integer k such that the vertex set of G can be partitioned into packings with pairwise different widths. There is a general question what is the packing chromatic number for infinite planar lattices. It is known that χ_ρ is finite for square and hexagonal lattices. We show that χ_ρ is unbounded for triangular lattice and we exhibit an optimal solution for hexagonal lattice.

Real flow number and the cycle rank of a graph

Robert Lukoťka

(joint work with Martin Škoviera)

We establish a relationship between the real (circular) flow number of a graph and its cycle rank. We show that a connected graph with real flow number

$p/q + 1$ where p and q are two relatively prime numbers must have cycle rank at least $p + q - 1$. A special case of this result yields that the real flow number of a 2-connected cubic graph with chromatic index 4 and order at most $8k + 4$ is bounded from below by $4 + 1/k$. Using this bound we prove that the real flow number of the Isaacs snark I_{2k+1} equals $4+1/k$, completing the upper bound due to Steffen [1].

References

[1] E. Steffen, Circular flow numbers of regular multigraphs, J. Graph Theory 36 (2001), 24–34.

Hypohamiltonian cubic graphs of girth 7

Edita Máčajová

(joint work with Martin Škoviera)

To decide whether a given graph is hamiltonian is a well known NP-complete problem and it is NP-complete even within the family of cubic graphs. *Hypohamiltonian graphs* – graphs for which the removal of an arbitrary vertex leads to a hamiltonian graph – lie on the border between the class hamiltonian graphs and its complement. Better understanding of hypohamiltonian graphs can therefore lead to demarcation line for hamiltonian graphs.

Roughly speaking, the higher girth and cyclic connectivity, the harder to find a hypohamiltonian graph with these parameters. Moreover, Thomassen conjectured, that there exists a constant k (possibly $k = 8$) such that every cyclically k-edge-connected cubic graph is hamiltonian. Interesting in this sense is the Coxeter graph, having both girth and cyclic connectivity 7, for it is the only highly connected graph of girth at least 7 that is known to be hypohamiltonian.

We provide a construction, which uses as building blocks a graph derived from the Coxeter graph, giving rise to an infinite family of cubic hypohamiltonian graphs of girth 7, with cyclic connectivity 4, 5, or 6.

On the light graphs in families of polyhedral graphs with prescribed edge and dual edge weight

Tomáš Madaras

(joint work with Barbora Ferencová)

A graph H is defined to be light in a family $\mathcal{H}$ of graphs if there exists a finite number $\varphi(H, \mathcal{H})$ such that each $G \in \mathcal{H}$ which contains H as a subgraph, contains also a subgraph $K \cong H$ such that the $\Delta_G(K) \leq \varphi(H, \mathcal{H})$.

We study global structural properties and light graphs in families of polyhedral graphs with prescribed minimum vertex degree δ, minimum face degree ρ, minimum edge weight w and dual edge weight w^*. In particular, there are 35 quadruples (δ, ρ, w, w^*) for which the asociated family is nonempty; among them, 9 generate extremal families. For those families, we show that there exists a variety of light stars and cycles.

Map enumeration

Roman Nedela

We present how a new method of enumeration of conjugacy classes of subgroups of given index in a finitely generated group can be applied to enumerate maps and hypermaps. Relations to some classical problems of enumeration of subgroups in a free group will be shown as well.

Efficient dominating sets in directed tori

Ľudovít Niepel

Dominating set S of vertices in a directed graph $D = (V, A)$ is efficient if each vertex in $V - S$ is dominated by exactly one vertex from S. We show a connection of efficient dominating sets to Vizing's conjecture and give a complete characterization of efficient dominating sets in 3-dimensional directed infinite grids and directed tori.

Integer distance graphs and Lonely runners - tight sets and tight vectors

Janka Oravcová

(joint work with Roman Soták)

Let's suppose, D is a subset of positive integers. The integer distance graph $G(D)$ with distance set D is the graph with vertex set $\mathbb{Z}$ in which two vertices u, v are adjacent if and only if $|u - v| \in D$.

It is known, upper bound for chromatic number of graph $G(D)$ is $|D| + 1$. Also, there are known some classes of distance graphs, for which this bound is attained ([2], [3], [5]). Distance graphs (or their distance sets) satisfying this condition are said to be tight. The question is whether there are some additional tight sets to ones we already know.

Now suppose k runners having non-zero constant speeds running on circular track with unit length. The Lonely Runner Conjecture ([4]) states that there is a time at which all runners are at distance at least $1/(k+1)$ (the runners are 'lonely') from their starting point. Moreover, the assumed bound is the best possible, in other words, there is a set of runners (with such speed vector), that in all time, in which all runners are lonely, at least one runner is on the track at distance $1/(k+1)$ from starting point clockwise and at least one runner at same distance counterclockwise. Such speed vector is called as tight ([1]).

We say, set D is associated with vector v if elements of set D are entries of vector v. Analogue, vector v is associated with set D if elements of set D in increasing order, are entries of vector v.

We study relationship between tight sets and tight vectors. We show, that there are tight sets, those asssociated vectors are not tight as well as there are vectors, those asssociated sets are not tight. Additionally we find infinite class of tight vectors with this property.

References

[1] L. Goddyn, E. B. Wong, Tight instances of the lonely runner, Integers 6 (2006), A38, 14pp.

[2] A. Kemnitz, M. Marangio, Colorings and list colorings of integer distance graphs, Congr. Numer. 151 (2001), 75-84.

[3] R. Soták, On a problem concerning integer distance graphs, preprint.

[4] J. M. Wills, Zur simultanen homogenen diophantischen Approximation. I, Monatsh. Math. 72 (1968), 254-263.

[5] X. Zhu, Circular chromatic number of distance graphs with distance set of cardinality 3, J. Graph Theory 41 (2002), 195-207.

Relaxation of hamiltonicity in graphs on surfaces

Katsuhiro Ota

Tutte proved in 1956 that every 4-connected planar graph is hamiltonian. Beginning with this theorem, there are many results showing that 3-connected planar graphs are also close to be hamiltonian. In these results, the following spanning substructures are mainly considered; a *k-tree* is a spanning tree with maximum degree at most k; a *k-walk* is a spanning closed walk that visits every vertex at most k times; and a *k-covering* is a spanning 2-connected subgraph with maximum degree at most k. These are generalizations of hamiltonicity of graphs in a sense, because a 2-tree is a hamiltonian path and a 1-walk (a 2-covering) is a hamiltonian cycle. It is known that every 3-connected planar graph has a 2-walk, a 3-tree and a 6-covering. In this talk, we first survey such results concerning 3-connected planar graphs, together with the ones embedded on a surface. We also pose some related problems.

As a possible generalization of graphs on surfaces, we also consider a minor closed family of graphs. In particular, we consider a-connected $K_{a,t}$-minor-free graphs, and show that they have also a good spanning substructure.

Cyclability and a degree sum condition concerning the connectivity and the independence number

Kenta Ozeki

(joint work with Tomoki Yamashita)

Let G be a graph and $S \subset V(G)$. We denote by $\alpha(S)$ the maximum number of pairwise nonadjacent vertices in S. For $x, y \in V(G)$, the local connectivity $\kappa(x, y)$ is defined to be the maximum number of internally disjoint paths connecting x and y in G. We define $\kappa(S) = \min\{\kappa(x, y) : x, y \in S, x \neq y\}$.

In this talk, we show that if $\kappa(S) \geq 3$ and $\sum_{i=1}^{4} d_G(x_i) \geq |V(G)| + \kappa(S) + \alpha(S) - 1$ for every independent set $\{x_1, x_2, x_3, x_4\} \subset S$, then G contains a cycle passing through S. This degree condition is sharp. This gives a new degree sum condition for a 3-connected graph to be hamiltonian.

Coloring and distinguishing labellings

Dieter Rautenbach

(joint work with Stephan Brandt, Jozef Miškuf and Michael Stiebitz)

Mainly motivated by some beautiful and surprising problems and conjectures there has recently been considerable research about labellings of the vertices (the edges or both the vertices and edges) of a given graph for which appropriately defined weights of the vertices (edges) either are all different (*distinguishing*) or are different for adjacent / incident elements (*coloring*).

Karónski, Luczak and Thomason for instance conjectured that the edges of every connected graph of order at least 3 can be assigned labels from $\{1, 2, 3\}$, such that for all pairs of adjacent vertices the sums of the labels of the incident edges are different, i.e. such vertex weights define a proper vertex coloring. Additionally allowing labels on the vertices and considering as vertex weights the sums of their labels and the incident edge labels, Przybylo and Woźniak even conjecture that the labels 1 and 2 are sufficient for a proper vertex-coloring of the graph by these vertex weights.

Another intriguing problem in this context is due to Ivančo and Jendroľ who conjecture that for every graph with size m and maximum degree Δ it is possible to assign labels from $\left\{1, 2, ..., \max\left\{\left\lceil\frac{m+2}{3}\right\rceil, \left\lceil\frac{\Delta+1}{2}\right\rceil\right\}\right\}$ to the vertices and edges of a graph different from K_5 such that for all edges the sums of their own label and the labels of the two incident vertices are different.

We survey some recent results which have been obtained in this context and which often rely on a beautiful interplay between combinatorial / constructive and probabilistic methods. Furthermore, we confirm Ivančo and Jendrol's conjecture for graphs whose size is at least a constant times their maximum degree and prove that every graph of maximum degree Δ has a total labelling using $\Delta/2 + \mathcal{O}(\sqrt{\Delta \log \Delta})$ different labels which defines an edge-coloring.

A dynamic variant of independence

Friedrich Regen

Consider the following infinite two-player game according to Rautenbach: Let G be a graph and a "guard set" $X \subseteq V(G)$ be a subset of its vertices which we interpret as the positions of some guards. In each turn of the game, the first player ("attacker") chooses some vertex $a \in V(G)$, and the second player ("defender") responds by evacuating each guard $v \in N(a)$ in its neighbourhood to a vertex $w \in N(v) \setminus N(a)$, such that the targets of all guards are different and the resulting guard set is independent. If no such move exists, the defender loses. X is *securely independent*, if some strategy enables the defender to play an arbitrary number of turns regardless of the choice of the attack vertices. The *secure independence number* α_∞ is the maximum cardinality of a securely independent set.

For each tree, α_∞ coincides with the domination number γ.

Maximum cycle decomposition in the Petersen graph $P(n,3)$

Peter Recht

Let $G = (V, E)$ be a graph. A decomposition $\mathcal{Z} = \{G_1, \ldots, G_m\}$ of G is a collection of subgraphs G_i of G $(i = 1, \ldots, m)$ such that all G_i are mutually edge-disjoint and $G = \bigcup_{i=1}^{m} G_i$. For an arbitrary decomposition $\mathcal{Z}$ the set $\mathcal{C}(\mathcal{Z}) \subseteq \mathcal{Z}$ denotes the family of circuits in $\mathcal{Z}$, i. e. $G_i \in \mathcal{C}(\mathcal{Z}) \iff G_i \in \mathcal{Z}$ and G_i is a circuit.

It seems natural to ask for the *cydec number*

$$\varkappa(G) = \max\{|\,\mathcal{C}(\mathcal{Z})| : \mathcal{Z} \text{ is a decomposition of } G\},$$

i.e. the maximum number of cycles G can decomposed into. In the literature the problem of determining a maximum cycle decomposition appears as the *cycle* *packing* problem and the MAX-ECD-Problem (*maximal Eulerian cycle decomposition*), respectively.

In the talk we give the proof, that the cydec number of the generalized Petersen graph $P(n, 3)$ is

$$\varkappa(P(n,3)) = \left\lfloor \frac{n}{4} \right\rfloor,$$

if $n > 27$.

Closures, cycles and paths

Ingo Schiermeyer

In 1960 Ore proved the following theorem: Let G be a graph of order n. If $d(u) + d(v) \geq n$ for every pair of nonadjacent vertices u and v, then G is hamiltonian. Since then for several other graph properties similar sufficient degree conditions have been obtained, so called "Ore-type degree conditions". In 2000 Faudree, Saito, Schelp and Schiermeyer strengthened Ore's theorem as follows: They determined the maximum number of pairs of nonadjacent vertices that can have degree sum less than n (i.e. violate Ore's condition) but still imply that the graph is hamiltonian. In this talk we will show that for some other graph properties the corresponding Ore-type degree conditions can be strengthened as well. These graph properties include traceable graphs, hamiltonian connected graphs, k-leaf connected graphs, pancyclic graphs and graphs having a 2-factor with two components. Graph closures are computed to show these results.

The long way home

Jens Schreyer

(joint work with Thomas Böhme and Armin Mikler)

We consider a walk of an agent on a finite connected graph G. Each vertex x of G is associated with a counter $count(x)$. The walk starts at a vertex x_0 which is called the *start vertex*. Initially, the counter of x_0 is put to 1, and the counters of all other vertices are put to 0. The walk of the agent is subject to the following rules.

(1) The next vertex x' of the walk is chosen from the neighborhood $N(x)$ of the current vertex x such that $count(x') = \min\{count(v) | v \in N(x)\}$.

(2) When the agents visits a vertex x, $count(x)$ is incremented by one.

(3) The walk stops, when the agent visits the start vertex for the second time.

The number of transitions of the agent until it finally comes back to the start vertex is called the *length* of the walk. Let $l(G, x_o)$ denote the maximal length of a walk in G starting at x_o and $l(G) = \max\{l(G, x_0) | x_0 \in V(G)\}$.

We investigate both numbers and show some upper and lower bounds. Moreover, the analogue problem, where the counter is associated with the edges of the graph is considered, too.

Strong generators of hom-properties

Gabriel Semanišin

(joint work with Peter Mihók and Jozef Miškuf)

For a simple graph H, $\to H$ denotes the class of all graphs that admit homomorphisms to H (such classes of graphs are called *hom-properties*). A *strong generator* of the class $\mathcal{P}$ of graphs is a graph G such that for every graph G^* belonging to the class $\mathcal{P}$ there exists a proper subgraph of G isomorphic to G^* (see e.g. [4]).

We investigate hom-properties from the point of view of the lattice of hereditary properties of graphs (see also [1], [2], [3]). In particular, we are interested in a characterization of the strong generators of $\to H$. We prove the existence of a strong generator of $\to H$, for any finite graph H. Moreover, we show that the structure of the strong generator of $\to H$ preserves the structure of the unique factorization of the hom-property $\to H = (\to H_1) \circ (\to H_2) \circ \cdots \circ (\to H_n)$, where $H_1, H_2, \ldots H_n$ are indecomposable graphs satisfying $H = H_1 + H_2 + \cdots + H_n$ (see [2]). Some generalisations for compositions of graphs are discussed as well.

References

[1] A. Bonato, A note on uniquely H-colorable graphs, Discuss. Math. Graph Theory 27(1) (2007), 49–44.

[2] J. Kratochvíl and P. Mihók, Hom-properties are uniquely factorizable into irreducible factors, Selected topics in discrete mathematics (Warsaw, 1996), Discrete Math. 213 (2000), 189–194.

[3] J. Kratochvíl, P. Mihók, G. Semanišin, Graphs maximal with respect to hom-properties, Discuss. Math. Graph Theory 18 (1997), 77–88.

[4] R. Rado, Universal graphs and universal functions, Acta Arith. 9 (1964), 331-340.

Metric dimensions of cycle related graphs

Rinovia Simanjuntak

For an ordered subset $W = \{w_1, w_2, \ldots, w_k\}$ of vertices in a connected graph G and a vertex v of G, the metric representation of v with respect to W is the ordered k-tuple $r(v|W) = (d(v, w_1), d(v, w_2), \ldots, d(v, w_k))$. The set W is called a resolving set for G if every two vertices of G have distinct representations. The metric dimension $\dim(G)$ of G is the minimum cardinality of a resolving set for G.

In this talk, we determine the metric dimensions of several cycle-related graphs.

Edge-colourings of cubic graphs and Steiner triple systems

Martin Škoviera

In 1985, Archdeacon asked if it is possible to colour the edges of a cubic graph by points of a given Steiner triple system $\mathcal{S}$ subject to the condition that any three colours that meet at a vertex are distinct and form a triple of $\mathcal{S}$. This talk will report on the research that has been initiated by this question, and, in particular, will point out interesting relationships between Steiner colourings and several well-known conjectures in graph theory such as the Cycle-Double Cover Conjecture or the Fulkerson Conjecture.

Rainbow faces in edge colored plane graphs

Erika Škrabuľáková

(joint work with Stanislav Jendroľ, Jozef Miškuf and Roman Soták)

A face of an edge colored plane graph is called *rainbow* if all its edges receive distinct colors. The maximum number of colors used in an edge coloring of a connected plane graph G with no rainbow face is called *the edge-rainbowness* of G.

We prove that the edge-rainbowness of G equals to the maximum number of edges of a connected bridge face factor H of G, where a *bridge face factor* H of a plane graph G is a spanning subgraph H of G in which every face is incident with a bridge and the interior of any one face $f \in F(G)$ is a subset of the interior of some face $f' \in F(H)$. We also show upper and lower bounds on the edge-rainbowness of graphs based on edge connectivity, girth of the dual G^* and other basic graph invariants. Moreover, we present infinite classes of graphs where these equalities are attained.

References

[1] N. Alon, T. Jiang, Z. Miller and D. Pritikin, Properly colored subgraphs and rainbow subgraphs in edge-colorings with local constraints, Random Struct. Algorithms 23 (2003), 409–433.
[2] S. Jendroľ, Rainbowness of cubic polyhedral graphs, Discrete Math. 306 (2006), 3321–3326.
[3] V. Jungič, D. Kráľ and R. Škrekovski, Coloring of plane graphs with no rainbow faces, Combinatorica 26(2) (2006), 169–182.
[4] P. Keevash, D. Mubayi, B. Sudakov and J. Verstraete, Rainbow Turán Problems, Combin., Probab. Comput., to appear.
[5] V. Rödl, Z. Tuza, Rainbow subgraphs in properly edge-colored graphs, Random Struct. Algorithms 3 (1992), 175–182.

From Kotzig's theorem to Light theory

Riste Škrekovski

It is well-known that every planar graph has a vertex of degree ≤ 5. This is a fast corollary of the Euler formula "$v - e + f = 2$" for planar graphs.

In 1955, Kotzig proved that every 3-connected planar graph has an edge whose sum of the degrees of its end-vertices is at most 13. Moreover, if such a graph is of minimum degree at least 4, then this bound can be reduced to 11. Nowdays, such an edge is called *light* and this result is refereed as Kotzig's Theorem. The metod applied in the proof of this theorem is nowdays known as the Discharging Method, which was introduced by Heesch. This is the same method by which Appel and Haken in 1976 established the Four Color Problem.

Since then Kotzig's Theorem has been subject to many extensions and generalizations. It served as a motivation for further search of light configurations in various classes of planar graphs, and some of these results have been applied for dealing with various graph coloring problems. As a consequence of this study arised a new topic in graph theory, named Light Graph Theory. In my talk, I will try to give an overview of this field by presenting some results of mine and of the other mathematicians.

Generalized Fibonacci helps counting Hamilton cycles in squared cycle

Zdzisław Skupień

Results due to Comtet (1970, 1974), which (in the language of graph theory) concern counting l-distance independent subsets on n-path and n-cycle are corrected and extended. In particular, a Fibonacci-like recurrence of order $l + 1$ is established. Its solution in case of the n-cycle and $l = 2$, say γ_n, gives the main (exponential) additive contribution to the number of Hamilton cycles in the square of the n-cycle, $n \geq 5$.

Cycles through specified vertices with forbidden subgraphs

Takeshi Sugiyama

A graph G is H-free if G contains no induced subgraph isomorphic to H. On forbidden subgraphs, sufficient condition for hamiltonian properties was considered in [1], [2], [3]. In this talk, we define some graphs. On the graph forbidden some induced subgraphs, we consider the existence of the cycle that contains specified vertices.

References

[1] P. Bedrossian, Forbidden subgraph and minimum degree conditions for hamiltonicity, Ph.D. Thesis, Memphis State University, 1991.

[2] D. Duffus, R. J. Gould, M. S. Jacobson, Forbidden subgraphs and the hamiltonian theme, in: Chartrand, Alavi, Goldsmith, Lesniak and Lick, eds., The Theory and Applications of Graphs, 1982, 297–316.

[3] R. Faudree, R. J. Gould, Characterizing forbidden pairs for hamiltonian proparties, Discrete Math. 173 (1997), 45–60.

Set systems crossing all k-partitions, I.

Zsolt Tuza

(joint work with Csilla Bujtás)

In this two-part talk we introduce the concept of partition-crossing, and point out its various connections to important areas of combinatorics and graph theory via the investigation of an extremal function.

We say that a subset F of a finite set X *crosses* a k-partition $X_1 \cup X_2 \cup \dots \cup X_k$ of X (where $X_i \neq \emptyset$ is assumed for all $1 \leq i \leq k$) if either every partition class contains at most one element of F (for $|F| \leq k$) or every partition class contains at least one element of F (for $|F| \geq k$); that is, F intersects exactly $\min(|F|, k)$ of the k partition classes.

It is quite natural to ask about the smallest possible number of r-element subsets of an n-element set X such that every k-partition of X is crossed by at least one of the selected subsets.

We denote this minimum by $f(n, k, r)$, and discuss its asymptotic behavior as $n \to \infty$. Considering specified values of k and r, or letting k and r get large with n, lower and upper bounds are obtained that yield tight or asymptotically tight estimates in many cases. The solutions lead us to intensively studied areas of combinatorics such as Balanced Incomplete Block Designs and Turán-type extremal problems on graphs and hypergraphs.

The function $f(n, k, r)$ has an equivalent description in terms of hypergraph coloring theory, too. From this point of view, every selected r-element set is considered as a hyperedge associated with an appropriately fixed upper bound ($r-1$ or $k-1$) on the number of colors occurring in it. Then $f(n, k, r)$ can be interpreted as the minimum number of hyperedges that can forbid all colorings of the hypergraph with more than $k-1$ colors. By this connection, our results give an asymptotic solution to an open problem of 'mixed hypergraph theory', too.

Arbitrary decompositions into open and closed trails

Mariusz Woźniak
(joint work with Sylwia Cichacz and Yoshimi Egawa)

The problem of arbitrary decomposition of a graph G into closed trails *i.e.* a decomposition into closed trails of prescribed lengths summing up to the size of the graph G, was first considered in the case of the complete graph $G = K_n$ with connexion to vertex-distinguishing colouring of the union of cycles.

Next, the same problem was investigated for other families of graphs.

In the talk we consider a more general problem: arbitrary decomposition of a graph into open and closed trails. Our results generalize all known results on decomposition of a graph into closed trails as well some results concerning decomposition of a graph into open trails.

On relative length of longest paths and cycles

Tomoki Yamashita
(joint work with Kenta Ozeki and Masao Tsugaki)

In this talk, we consider only finite graphs without loops or multiple edges. Let $\alpha(G)$ and $\delta(G)$ be the independence number and the minimum degree of a graph G, respectively. If $\alpha(G) \geq k$, let $\sigma_k(G)$ be the minimum degree sum of an independent set of k vertices of G; otherwise we let $\sigma_k(G) = +\infty$. For a graph G, we denote by $p(G)$ and $c(G)$ the orders of a longest path and a longest cycle in G, respectively. The main interest of this talk is the difference $\mathrm{diff}(G) := p(G) - c(G)$, which is called *relative length*. It is easy to see that a connected graph G is hamiltonian if and only if $\mathrm{diff}(G) = 0$. Ore gave a degree sum condition for the existence of a hamiltonian cycle.

Theorem 1 (Ore [3]) Let G be a graph of order $n \geq 3$. If $\sigma_2(G) \geq n$, then G is hamiltonian, that is, $\mathrm{diff}(G) = 0$.

Enomoto, van den Heuvel, Kaneko and Saito obtained a degree sum condition for a graph G to have $\mathrm{diff}(G) \leq 1$.

Theorem 2 (Enomoto et al. [2]) Let G be a 2-connected graph on n vertices. If $\sigma_3(G) \geq n + 2$, then $\mathrm{diff}(G) \leq 1$.

We investigate a σ_4 condition for $\mathrm{diff}(G) \leq 2$.

Theorem 3 Let G be a 3-connected graph on n vertices. If $\sigma_4(G) \geq n + 6$, then $\mathrm{diff}(G) \leq 2$.

We propose the following conjecture, which has been verified for $k = 1$ (Theorem 1), $k = 2$ (Theorem 2) and $k = 3$ (Theorem 3).

Conjecture 4 Let G be a k-connected graph on n vertices. If $\sigma_{k+1}(G) \geq n + k(k-1)$, then $\mathrm{diff}(G) \leq k - 1$.

This conjecture is a generalization of the famous conjecture due to Bondy.

Conjecture 5 (Bondy [1]) Let G be a k-connected graph on n vertices, and let C be a longest cycle. If $\sigma_{k+1}(G) \geq n + k(k-1)$, then $p(G \setminus C) \leq k - 1$.

Moreover, we will show that $\mathrm{diff}(G) \leq 2$ implies two cycle-related properties.

References

[1] J. A. Bondy, Longest paths and cycles in graphs with high degree, Research Report CORR 80-16, Department of Combinatorics and Optimization, University of Waterloo, 1980.

[2] H. Enomoto, J. van den Heuvel, A. Kaneko and A. Saito, Relative length of long paths and cycles in graphs with large degree sums, J. Graph Theory 20 (1995), 213–225.

[3] O. Ore, Note on Hamilton circuits, Amer. Math. Monthly 67 (1960), 55.

Colouring graphs with bounded generalized colouring number

Xuding Zhu

Given a graph G and a positive integer p, $\chi_p(G)$ is the minimum number of colours needed to colour the vertices of G so that for any $i \leq p$, any subgraph H of G of tree-depth i gets at least i colours. This talk presents an upper bound for $\chi_p(G)$ in terms of the k-colouring number $\mathrm{col}_k(G)$ of G for $k = 2^{p-2}$. Conversely, for each integer k, we also prove an upper bound for $\mathrm{col}_k(G)$ in terms of $\chi_{k+2}(G)$. As a consequence, for a class $\mathcal{K}$ of graphs, the following two statements are equivalent:

(a) For every positive integer p, $\chi_p(G)$ is bounded by a constant for all $G \in \mathcal{K}$.

(b) For every positive integer k, $\mathrm{col}_k(G)$ is bounded by a constant for all $G \in \mathcal{K}$.

It was proved by Nešetřil and Ossona de Mendez that (a) is equivalent to the following:

(c) For every positive integer q, $\nabla_q(G)$ (the grad, i.e., the greatest reduced average degree, of G with rank q) is bounded by a constant for all $G \in \mathcal{K}$.

This implies that (b) and (c) are also equivalent. We shall give a direct proof of this equivalence, by giving an upper bound for $\nabla_k(G)$ in terms of $\mathrm{col}_{4k+1}(G)$, and an upper bound for $\mathrm{col}_k(G)$ in terms of $\nabla_{\lfloor k/2 \rfloor}(G)$. This gives an alternate proof of the equivalence of (a) and (c).

History of cyclic chromatic number

Jana Zlámalová

The cyclic chromatic number $\chi_c(G)$ of a graph G embedded to a surface is the minimum number of colours in such a colouring of vertices of G that any two vertices of G sharing a common face receive different colours.

In this talk the history of cyclic chromatic number and some new facts concerning this invariant will be presented.

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Programme of the Conference

Sunday	
14:00 - 22:00	Registration
18:00 - 21:00	Dinner

Monday		
07:30 - 08:30	Breakfast	
08:45 - 09:35	Ota K.	Relaxation of hamiltonicity in graphs on surfaces
09:40 - 10:00	Göring F.	Hamiltonian cycles through prescribed edges of at least 4-connected maximal planar graphs
10:05 - 10:25	Schreyer J.	The long way home
10:30 - 10:55	Coffee break	
10:55 - 11:15	Tuza Zs.	Set systems crossing all k-partitions, I.
11:20 - 11:40	Bujtás Cs.	Set systems crossing all k-partitions, II.
11:45 - 12:05	Bacsó G.	Guard sets in hypergraphs
12:10 - 12:30	Lidický B.	Packing chromatic number for lattices
12:30 - 13:15	Lunch	
14:45 - 15:35	Škrekovski R.	From Kotzig's theorem to Light theory
15:40 - 16:00	Sugiyama T.	Cycles through specified vertices with forbidden subgraphs
16:05 - 16:25	Knor M.	Embeddings of tables of groups of order 8
16:30 - 16:55	Coffee break	
16:55 - 17:15	Katrenič P.	Graphs with no path-kernels
17:20 - 17:40	Kyppö J.	Cycles and colourings of sociosemantic graphs
17:45 - 18:05	Hałuszczak M.	On Ramsey minimal graphs
18:10 - 18:30	Kardoš F.	Cyclic edge-cuts in fullerene graphs
18:30 - 19:15	Dinner	
20:00 -	Welcome party	

Tuesday		
07:30 - 08:30	Breakfast	
08:45 - 09:35	RAUTENBACH D.	Coloring and distinguishing labellings
09:40 - 10:00	SCHIERMEYER I.	Closures, cycles and paths
10:05 - 10:25	NEDELA R.	Map enumeration
10:30 - 10:55	Coffee break	
10:55 - 11:15	SKUPIEŃ Z.	Generalized Fibonacci helps counting Hamilton cycles in squared cycle
11:20 - 11:40	KEMNITZ A.	Circular total colorings of graphs
11:45 - 12:05	RECHT P.	Maximum cycle decomposition in the Petersen graph $P(n, 3)$
12:10 - 12:30	MADARAS T.	On the light graphs in families of polyhedral graphs with prescribed edge and dual edge weight
12:30 - 13:15	Lunch	
14:45 - 15:35	FLANDRIN E.	Cycles in graphs: around the hamiltonian problem
15:40 - 16:00	ŁAZUKA E.	Chromaticity of certain hypergraphs
16:05 - 16:25	FUJITA S.	Recent progress on vertex-disjoint cycles in graphs
16:30 - 16:55	Coffee break	
16:55 - 17:15	REGEN F.	A dynamic variant of independence
17:20 - 17:40	LUKOŤKA R.	Real flow number and the cycle rank of a graph
17:45 - 18:05	ŠKRABUL'ÁKOVÁ E.	Rainbow faces in edge colored plane graphs
18:10 - 18:30	ZLÁMALOVÁ J.	History of cyclic chromatic number
18:30 - 19:15	Dinner	
20:00 - 21:20	Videopresentation C&C 2006	

Wednesday	
07:00 - 08:30	Breakfast
08:30 - 16:00	Trip
19:00 - 20:00	Dinner

Thursday		
07:30 - 08:30	Breakfast	
08:45 - 09:35	ZHU X.	Colouring graphs with bounded generalized colouring number
09:40 - 10:00	BRANDT S.	Triangle-free graphs whose independence number equals the degree
10:05 - 10:25	FRICK M.	A traceability conjecture for oriented graphs
10:30 - 10:55	Coffee break	
10:55 - 11:15	KRÁL' D.	6-critical graphs on the Klein bottle
11:20 - 11:40	HEXEL E.	On vertices enforcing a Hamilton cycle
11:45 - 12:05	WOŹNIAK M.	Arbitrary decompositions into open and closed trails
12:10 - 12:30	SEMANIŠIN G.	Strong generators of hom-properties
12:30 - 13:15	Lunch	
14:45 - 15:35	HUJTER M.	Precoloring extension: the first fifteen years
15:40 - 16:00	MÁČAJOVÁ E.	Hypohamiltonian cubic graphs of girth 7
16:05 - 16:25	YAMASHITA T.	On relative length of longest paths and cycles
16:30 - 16:55	Coffee break	
16:55 - 17:15	BIELAK H.	Ramsey numbers for disjoint union of some graphs
17:20 - 17:40	NIEPEL Ľ.	Efficient dominating sets in directed tori
17:45 - 18:05	SIMANJUNTAK R.	Metric dimensions of cycle related graphs
18:10 - 18:30	OZEKI K.	Cyclability and a degree sum condition concerning the connectivity and the independence number
19:00 -	Farewell party	

Friday		
07:30 - 08:30	Breakfast	
08:45 - 09:35	Škoviera M.	Edge-colourings of cubic graphs and Steiner triple systems
09:40 - 10:00	Drgas-Burchardt E.	Unique factorization theorems
10:05 - 10:25	Degenhardt J.	Maximum cycle decomposition in line-graphs
10:30 - 10:55	Coffee break	
10:55 - 11:15	Klešč M.	On the crossing numbers of join between stars and other graphs
11:20 - 11:40	Oravcová J.	Integer distance graphs and Lonely runners - tight sets and tight vectors
11:45 - 12:05	Hajduk R.	On the azulenoids with large ring of 5-gons
12:10 - 12:30	Nejedlý P.	tba
12:30 - 13:15	Lunch	