C&C

HIGH TATRAS, SLOVAKIA

**Igor Fabrici, Mirko Horňák, Stanislav Jendroľ ed.**

# Workshop

# Cycles and Colourings 2008

**September 07–12, 2008** **Tatranská Štrba**

http://umv.science.upjs.sk/c&c

Dear Participant,

welcome to the Seventeenth Workshop Cycles and Colourings. Except for the first workshop in the Slovak Paradise (Čingov 1992), the remaining fifteen workshops took place in the High Tatras (Nový Smokovec 1993, Stará Lesná 1994–2003, Tatranská Štrba 2004–2007).

The series of C&C workshops is organised by combinatorial groups of Košice and Ilmenau. Apart of dozens of excellent invited lectures and hundreds of contributed talks the scientific outcome of our meetings is represented also by special issues of journals Tatra Mountains Mathematical Publications and Discrete Mathematics (TMMP 1994, 1997, DM 1999, 2001, 2003). The editorial work on the special issue of DM from C&C 2006 is over and all twelve papers, that have been accepted for publication, are available online through ScienceDirect at URL http://www.sciencedirect.com/science/journal/0012365X.

We are ready to prepare a special issue of DM devoted to the forthcoming workshop. For the first time our editorial work will be done in the frame of Elsevier Editorial System. Authors of papers intended for a possible publication in the special issue are therefore asked to wait until the option of choosing special issue from C&C 2008 will be available in EES. (You will be notified about that.) The deadline for submitting papers to the special issue is October 31, 2008.

The scientific programme of the workshop consists of 50 minute lectures of invited speakers and of 20 minute contributed talks. This booklet contains abstracts as were sent to us by the authors.

**Invited speakers:**

| | |
|---|---|
| Jørgen Bang-Jensen | University of Southern Denmark, Odense, Denmark |
| Hajo Broersma | University of Durham, Durham, United Kingdom |
| Jarosław Grytczuk | Jagiellonian University, Kraków, Poland |
| Frédéric Havet | INRIA Sophia-Antipolis, Sophia-Antipolis, France |
| Zdeněk Ryjáček | University of West Bohemia, Pilsen, Czech Republic |
| Éric Sopena* | University of Bordeaux, Bordeaux, France |
| Jacques Verstraete* | University of California, San Diego, CA, USA |

Have a pleasant and successfull stay in Tatranská Štrba.

**Organising committee:**

Igor Fabrici
Jochen Harant
Erhard Hexel
Mirko Horňák
Stanislav Jendroľ (chair)
Štefan Schrötter

*Participation cancelled in August 2008

# Contents

# Erdős-type condition for long cycles in bipartite graphs

**Lech Adamus**

Let $G$ be a graph of order $n$ with the minimal degree $\delta(G) \geq r$. In 1962, Erdős found the minimal number of edges of $G$, $f(n,r)$, which guarantees the existence of a hamiltonian cycle in $G$. Woodall generalized this result in 1972 to long cycles. He gave a complete list of conditions on size of $G$ ensuring the existence of a cycle of length $n-k$ in $G$ (for any $0 \leq k \leq \frac{n-3}{2}$). His result is best possible.

We are interested in finding the bipartite counterpart of Woodalls result. We conjecture Erdős-type criterion (conditions on size together with conditions on the minimal degree) for a balanced bipartite graph of order $2n$ to contain a long cycle $C_{2n-2k}$, where $0 \leq k < \frac{n}{2}$. For $k=0$, this is the classical hamiltonicity criterion of Moon and Moser. We will show that the conjecture is true also for $k=1$.

# New near-factorizations of finite groups

**Gábor Bacsó**

The attempts to construct a counterexample to the Strong Perfect Graph Conjecture yielded the notion of partitionable graphs. The reason is that every minimal imperfect graph is partitionable. Then near-factorizations of finite groups gained interest since from any near-factorization some partitionable graphs can be defined in a natural way. Recently, the SPGC has been proved by Chudnovsky, Robertson, Seymour and Thomas, but near-factorizations and "the world around perfect graphs" remain interesting on their own rights because
(i) near-factorizations are rare objects being "close" to factorizations of groups;
(ii) as an application of perfect graph theory, one can construct new near-factorizations of finite groups.

# Paths, cycles and strong subdigraphs in directed graphs

**Jørgen Bang-Jensen**

We survey a number of recent results concerning paths, cycles or strong subdigraphs of digraphs. We illustrate the richness of the topic by a number of results and open problems in the area. The talk is based on the second edition of the book "Digraphs: Theory, Algorithms and Applications", Bang-Jensen and Gutin, to be published by Springer in late 2008.

# Ramsey numbers for some graphs

**Halina Bielak**

Let $G, H, F$ be simple graphs with at least two vertices. The Ramsey number $R(G, H)$ is the smallest integer $n$ such that every graph $F$ of order $n$ contains a subgraph isomorphic to $G$ or $\overline{F}$ contains a subgraph isomorphic to $H$, where $\overline{F}$ is the complement of $F$.

The graph $H$ is $G$-good if $R(H, G) = (\chi(G) - 1)(|V(H)| - 1) + s(G)$, where $s(G)$ is the chromatic surplus of $G$, i.e. the minimum cardinality of colour classes over all chromatic colourings of $V(G)$.

We give the Ramsey number for a disjoint union of some $G$-good graphs versus a graph $G$ with $s(G) > 1$ generalizing the results of Stahl [3], Baskoro et al. [1] and the previous result of the author [2] for $s(G) = 1$.

### References


[1] E. T. Baskoro, Hasmawati, H. Assiyatun, Note. The Ramsey Number for disjoint unions of trees, Discrete Math. 306 (2006), 3297–3301.

[2] H. Bielak, Ramsey numbers for a disjoint union of some graphs, Appl. Math. Lett. (to appear).

[3] S. Stahl, On the Ramsey Number $r(F, K_m)$ where $F$ is a forest, Canad. J. Math. 27 (1975), 585–589.


# Degree sequences conditions for graph properties

**Hajo Broersma**

(joint work with Douglas Bauer, Jan van den Heuvel, Nathan Kahl, and Edward F. Schmeichel)

We discuss recent work on graphical degree sequences, i.e., sequences of integers that correspond to the degrees of the vertices of a graph. Historically, such degree sequences have been used to provide sufficient conditions for graphs to have a certain property, such as being $k$-connected or hamiltonian. For hamiltonicity, this research has culminated in a beautiful theorem due to Chvátal (1972). This theorem gives a sufficient condition for a graphical degree sequence to be forcibly hamiltonian, i.e., such that every graph with this degree sequence is hamiltonian. Moreover, the theorem is the strongest of an entire class of theorems in the following sense: if the theorem does not guarantee that a sequence $\pi$ is forcibly hamiltonian, then there exists a nonhamiltonian graph with a degree sequence that majorizes $\pi$. Very recently, Kriesell solved an open problem due to Bauer et al. by establishing similar conditions for $k$-edge connectivity for any fixed $k$. We will introduce a general framework for such conditions and discuss recent progress and open problems on similar sufficient conditions for toughness and for the existence of $k$-factors.

# Parity vertex colouring of graphs

**Kristína Budajová**
(joint work with Stanislav Jendroľ and Stanislav Krajči)

A parity path in a vertex colouring of a graph is a path along which each colour is used an even number of times. Let $\chi_p(G)$ be the least number of colours in a vertex colouring of $G$ having no parity path. It is proved that for any graph $G$ there is

$$\chi(G) \leq \chi_p(G) \leq |V(G)| - \alpha(G) + 1$$

where $\chi(G)$ and $\alpha(G)$ is the chromatic number and the independence number of $G$, respectively. The bounds are tight. This result is improved for trees. Namely, if $T$ is a tree with diameter $\mathrm{diam}(T)$ and radius $\mathrm{rad}(T)$, then

$$\left\lceil \log_2 \big(2 + \mathrm{diam}(T)\big) \right\rceil \leq \chi_p(T) \leq 1 + \mathrm{rad}(T).$$

The bounds are tight.

# Colorings of stably bounded hypergraphs

**Csilla Bujtás**
(joint work with Zsolt Tuza)

A hypergraph $\mathcal{H} = (X, \mathcal{E})$ is a set system over the vertex set $X$. There exist several new types of coloring constraints imposed on the (hyper)edges while coloring the vertices. In the talk we consider two general models, which offer common frame to express many variants of hypergraph colorings and non-classical graph colorings.

In a color-bounded hypergraph every edge $E_i \in \mathcal{E}$ is associated with a lower color-bound $s_i$ and with an upper color-bound $t_i$. This means that in a proper coloring each edge $E_i$ has to receive at least $s_i$ and at most $t_i$ different colors. In the more general model of stably bounded hypergraphs, beside $s_i$ and $t_i$ we have two further color-bounds $a_i$ and $b_i$, which prescribe lower and upper bounds on the cardinality of the largest monochromatic subset of each edge $E_i$.

It is also worth studying and comparing the cases where only a given subset of these four color-bound functions are used. In the talk we concentrate on interval hypergraphs (where every hyperedge is a subpath of a given host path) and hypertrees (where every hyperedge corresponds to a subtree of the host tree graph). We prove bounds on the minimum and maximum number of colors that can be used in a proper coloring, and consider the time complexity of deciding colorability. Moreover, we study the $r$-uniform hypertrees of different models and characterize the possible feasible sets $\Phi(\mathcal{H}) = \{k : \mathcal{H}$ has a proper coloring with exactly $k$ colors$\}$.

# Super edge-graceful paths and cycles

**Sylwia Cichacz**

A graph $G(V, E)$ of order $|V| = p$ and size $|E| = q$ is called super edge-graceful if there is a bijection $f$ from $E$ to $\{0, \pm 1, \pm 2, \ldots, \pm \frac{q-1}{2}\}$ when $q$ is odd and from $E$ to $\{\pm 1, \pm 2, \ldots, \pm \frac{q}{2}\}$ when $q$ is even such that the induced vertex labeling $f^*$ defined by $f^*(x) = \sum_{xy \in E(G)} f(xy)$ over all edges $xy$ is a bijection from $V$ to $\{0, \pm 1, \pm 2 \ldots, \pm \frac{p-1}{2}\}$ when $p$ is odd and from $V$ to $\{\pm 1, \pm 2, \ldots, \pm \frac{p}{2}\}$ when $p$ is even.

We prove that all paths $P_n$ except $P_2$ and $P_4$ and all cycles $C_n$ except $C_4$ and $C_6$ are super edge-graceful.

# Weak and strong parity vertex colourings

**Július Czap**

(joint work with Stanislav Jendroľ)

Consider a vertex colouring of a connected plane graph $G$. A colour $c$ is used $k$ times by a face $\alpha$ of $G$ if it appears $k$ times along the facial walk of $\alpha$. There are two problems.

**Problem 1.** A vertex colouring $\varphi$ is a *weak parity vertex colouring* of a connected plane graph $G$ if each face of $G$ uses at least one colour an odd number of times. The problem is to determine the minimum number $\chi_w(G)$ of colours used in a weak parity vertex colouring of $G$.

In [1] it is proved that $\chi_w(G) \leq 4$ for every connected plane graph $G$ with minimum degree at least 3. We stated the following conjecture.

**Conjecture.** Let $G$ be a connected plane graph of minimum face degree at least 3. Then $\chi_w(G) \leq 3$.

This conjecture is true for 2-connected cubic plane graphs, see [1].

**Problem 2.** A vertex colouring $\varphi$ is a *strong parity vertex colouring* of a 2-connected plane graph $G$ if for each face $\alpha$ and each colour $c$ the face $\alpha$ uses the colour $c$ an odd number of times or does not use it at all. The problem is to find the minimum number $\chi_s(G)$ of colours used in a strong parity vertex colouring of $G$. We believe that the following is true.

**Conjecture.** There is a constant $K$ such that, for every 2-connected plane graph $G$, $\chi_s(G) \leq K$.

We do not know any 2-connected plane graph $H$ with $\chi_s(H) \geq 7$. Hence, we believe that $K = 6$ in the above conjecture.

References

[1] J. Czap and S. Jendroľ, Colouring vertices of plane graphs under restrictions given by faces, Discuss. Math. Graph Theory (accepted).


# On the crossing numbers of products with stars


**Emília Draženská**


The *crossing number*, $\mathrm{cr}(G)$, of a graph $G$ is the minimum number of pairwise intersections of edges in a drawing of $G$ in the plane. Computing the crossing number of a given graph is in general a difficult problem. Crossing numbers of few families of graphs are known. Most of them are Cartesian products of special graphs.

The crossing numbers of the Cartesian products of all graphs on four vertices with stars were determined. There are known the crossing numbers of the Cartesian products of stars with some graphs of order five.

We extend the previous results for graphs of order six. Hanfei Mei and Yuanqiu Huang (2007) proved that the crossing number of the Cartesian product of the graph $K_{1,5}$ with the star $K_{1,n}$ is $6\left\lfloor \frac{n}{2} \right\rfloor \left\lfloor \frac{n-1}{2} \right\rfloor + 4\left\lfloor \frac{n}{2} \right\rfloor$. In the talk we give the exact values of the crossing numbers of the Cartesian products of another six vertex graphs with stars. For some Cartesian products of six vertex graphs with stars we give only lower or upper bounds.

# Graph primality


**Ewa Drgas-Burchardt**


A set $W \subseteq V(H)$ is a module in a graph $H$ if for each two vertices $x, y \in W$, $N_H(x) \setminus W = N_H(y) \setminus W$. The trivial modules in $H$ are $V(H)$, $\emptyset$ and singletons. A graph having only trivial modules is called prime.

Gallai [1] proved modular decomposition theorem, which states that for each graph $H$ with at least two vertices exactly one of the following conditions holds:

1. $H$ is not connected, and it can be decomposed into its connected components;
2. $\bar{H}$ is not connected, and $H$ can be decomposed into the complements of connected components of $\bar{H}$;
3. $H$ and $\bar{H}$ are connected. There is some $U \subseteq V(H)$ and a unique partition $\phi = \{V_1, \ldots, V_n\}$ of $V(H)$ satisfying that

(a) $|U| > 3$;
(b) $H[U]$ is a maximal prime subgraph of $H$;
(c) $V_i$ is a module in $H$ and $V_i \cap U$ is one-element set for each $i \in [n]$.

We propose a new insight into the modular decomposition of a graph. This approach offers the possibility to define new classes of graphs and describes known classes of graphs in a new way.

References

[1] T. Gallai, Transitiv orientierbare Graphen, Acta Mathematica Academiae Scientiarum Hungaricae 18 (1967) 25–66.

# Three-coloring triangle-free planar graphs in linear time

**Zdeněk Dvořák**
(joint work with Ken-ichi Kawarabayashi and Robin Thomas)

Grötzsch's theorem states that every triangle-free planar graph is 3-colorable, and several relatively simple proofs of this fact were provided by Thomassen and other authors. It is easy to convert these proofs into quadratic-time algorithms to find a 3-coloring, but it is not clear how to find such a coloring in linear time (Kowalik used a nontrivial data structure to construct an $O(n \log n)$ algorithm).

We design a linear-time algorithm to find a 3-coloring of a given triangle-free planar graph. The algorithm avoids using any complex data structures, which makes it easy to implement. As a by-product we give another simple proof of Grötzsch's theorem.

# Altitude of wheels and wheel-like graphs

**Tomasz Dzido**
(joint work with Hanna Furmańczyk)

An *edge-ordering* of a graph $G = (V, E)$ is a one-to-one function $f \colon E(G) \to \{1, 2, \ldots, |E(G)|\}$. A path of length $k$ in $G$ is called a $(k, f)$-*ascent* if $f$ increases along the successive edges forming the path. The *altitude* $\alpha(G)$ of $G$ is the greatest integer $k$ such that for all edge-orderings $f$, $G$ has a $(k, f)$-ascent.

In our talk we give exact values of $\alpha(G)$ for all wheels. Furthermore, we use our result to obtain altitude for graphs being subgraphs or supergraphs of wheels.

## References


[1] A. Brandstdt, V. B. Le, J. P. Spinrad, Graph classes: A survey, Philadelphia, PA: SIAM (1987).

[2] A. P. Burger, E. J. Cockayne, C. M. Mynhardt, Altitude of small complete and complete bipartite graphs, Australas. J. Combin. 31 (2005), 167–177.

[3] A. P. Burger , C. M. Mynhardt, T. C. Clark, B. Falvai, N. D. R. Henderson, Altitude of regular graphs with girth at least five, Discrete Math. 294 (2005), 241–257.

[4] V. Chvátal, J. Komlós, Some combinatorial theorems on monotonicity, Canad. Math. Bull. 14 (1971), 151–157.

[5] E. J. Cockayne, C. M. Mynhardt, Altitude of $K_{3,n}$, J. Combin. Math. Combin. Comp. 52 (2005), 143–157.


# Some new results on edge-antimagic graphs


**Andrea Feňovčíková**

(joint work with Martin Bača and Muhammad Kashif Shafiq)


A $(p, q)$-graph $G$ is said to be graceful if it admits a labelling of the vertices by the integers $\{1, 2, \ldots, q + 1\}$ such that the absolute value of the difference in vertex labels between adjacent vertices generate the set $\{1, 2, \ldots, q\}$. An $(a, d)$-edge-antimagic total labeling of a $(p, q)$-graph is defined as a one-to-one map taking the vertices and the edges onto the integers $1, 2, \ldots, p + q$ with the property that the edge-weights (the sums of the endpoint labels and the edge label) form an arithmetic sequence starting from $a$ and having a common difference $d$.

In the talk we use the connection between graceful labelings and edge-antimagic labelings for generating large classes of edge-antimagic total trees from smaller graceful trees.

# Acyclic edge colouring of graphs


**Anna Fiedorowicz**

(joint work with Mieczysław Borowiecki)


An *acyclic edge $k$-colouring* of a graph $G$ is a proper edge $k$-colouring of $G$ such that there are no bichromatic cycles. In other words, for every two distinct colours $i$ and $j$, the subgraph induced in $G$ by all the edges which have either colour $i$ or $j$ is acyclic. The *acyclic chromatic index* of $G$ is the minimum $k$ such that $G$ has an acyclic edge $k$-colouring and is denoted by $\chi'_a(G)$.

It was conjectured in [1] that for any graph $\chi'_a(G) \leq \Delta(G) + 2$, where $\Delta(G)$ denotes the maximum degree of $G$. We prove the conjecture for planar graphs without cycles of length 3 and 4. We also present new upper bounds for the acyclic chromatic index for some other classes of planar graphs. Moreover, we obtain a general upper bound for $\chi'_a(G)$ for any graph $G$ which satisfies $|E(G)| < t|V(G)|$, where $t$ is a given positive integer, as well as the linear algorithm for finding such a colouring.

### References


[1] N. Alon, B. Sudakov, A. Zaks, Acyclic edge colorings of graphs, J. Graph Theory 37 (2001), 277–288.


# Mixed graph edge coloring


**Hanna Furmańczyk**

(joint work with Adrian Kosowski, Bernard Ries, and Paweł Żyliński)


We are interested in coloring the edges of a mixed graph, i.e., a graph containing unoriented and oriented edges. This problem is related to a network communication problem. We give general bounds on the number of required colors and analyze the complexity status of this problem. In particular, we show that it is NP-complete even in 3-regular planar bipartite graphs. For trees we develop a polynomial time algorithm based on dynamic programming.

# Rainbow numbers for disjoint copies of graphs


**Izolda Gorgol**


A subgraph of an edge-coloured graph is *rainbow* if all of its edges have different colours. For a graph $H$ and a positive integer $n$, the *anti-Ramsey number* $f(n, H)$ is the maximum number of colours in an edge-colouring of $K_n$ with no rainbow copy of $H$. The *rainbow number* $rb(n, H)$ is the minimum number of colours such that any edge-colouring of $K_n$ with $rb(n, H)$ number of colours contains a rainbow copy of $H$. Certainly $rb(n, H) = f(n, H) + 1$. Anti-Ramsey numbers were introduced by Erdős et al. [1] and studied in numerous papers.

We discuss the rainbow numbers for disjoint copies of some graphs.

### References


[1] P. Erdős, M. Simonovits, V. Sós, Anti-Ramsey theorems, Infinite and finite sets (A. Hajnal, R. Rado, and V. Sós, eds.), Colloq. Math. Soc. J. Bolyai, North-Holland, 1973, 633–643.

# Thue games

**Jarosław Grytczuk**

Thue games can be played on various combinatorial structures: graphs, hypergraphs, words, Euclidean spaces, etc. Basic idea goes back to Thue, who proved that the integers can be 3-colored so that adjacent intervals have different color patterns. This striking result has been generalized in many directions, leading to a variety of peculiar coloring problems. Is it true, for instance, that planar graphs can be colored with a bounded number of colors so that no path contains two adjacent segments with the same color pattern? Using techniques form graph coloring games one can show that the answer is positive if we restrict the condition to segments of bounded length. Actually, the tool is strong enough to guarantee a uniquely colored vertex on every path of bounded length, even if colors are chosen from sufficiently large preassigned lists.

New aspects appear if a coloring is created by two players, Ann and Ben, say, who tend to minimize and maximize the number of colors used during the play, respectively. In one of possible versions they alternately color consecutive positive integers so that no two adjacent segments look the same (after each round), and a new color may be introduced only if necessary. Will Ann be able to keep the number of colors bounded? In another version Ben subdivides a chosen edge of a path with a new vertex, while Ann colors it immediately. Can Ann play arbitrarily long with bounded number of colors?

Similar questions can be considered for other Thue type colorings in various settings. In general, the task is to color a given structure so that adjacent substructures get different color patterns, or different multisets of colors, or even different sums, if we are using the integers as colors. This leads to some unexpected connections to other topics, like the necklace splitting problem, or graph colorings by edge weightings. I will discuss these connections, some recent results, and several open problems in this area.

# Hamiltonian cycles through prescribed edges of 4-connected maximal planar graphs

**Jochen Harant**
(joint work with Frank Göring)

In 1956, W. T. Tutte proved that a 4-connected planar graph is hamiltonian. Moreover, in 1997, D. P. Sanders extended this to the result that a 4-connected planar graph contains a hamiltonian cycle through any two of its edges. It is shown that Sander's result is best possible by constructing 4-connected maximal planar graphs with three edges of large distance such that any hamiltonian cycle misses one of them. If the maximal planar graph is 5-connected then such a construction is impossible.

## Grundy number and lexicographic product of graphs

**Frédéric Havet**

(joint work with Marie Asté and Claudia Linhares-Sales)

The *Grundy number* of a graph $G$, denoted by $\Gamma(G)$, is the largest $k$ such that $G$ has a *greedy* $k$-colouring, that is a colouring with $k$ colours obtained by applying the greedy algorithm according to some ordering of the vertices of $G$. In this talk, we study the Grundy number of the lexicographic, cartesian and direct products of two graphs in terms of the Grundy numbers of these graphs.

Regarding the lexicographic product, we show that $\Gamma(G) \times \Gamma(H) \leq \Gamma(G[H]) \leq 2^{\Gamma(G)-1}(\Gamma(H) - 1) + \Gamma(G) - 1$. In addition, we show that if $G$ is a tree or $\Gamma(G) = \Delta(G) + 1$, then $\Gamma(G[H]) = \Gamma(G) \times \Gamma(H)$. We then deduce that for every fixed $c \leq 1$, given a graph $G$, it is CoNP-Complete to decide if $\Gamma(G) \leq c \times \chi(G)$ and it is CoNP-Complete to decide if $\Gamma(G) \leq c \times \omega(G)$.

Regarding the cartesian product, we show that there is no upper bound of $\Gamma(G \Box H)$ as a function of $\Gamma(G)$ and $\Gamma(H)$. Nevertheless, we prove that for any fixed graph $G$, there is a function $h_G$ such that, for any graph $H$, $\Gamma(G \Box H) \leq h_G(\Gamma(H))$.

Regarding the direct product, we show that $\Gamma(G \times H) \geq \Gamma(G) + \Gamma(H) - 2$ and construct for any $k$ some graph $G_k$ such that $\Gamma(G_k) = 2k + 1$ and $\Gamma(G_k \times K_2) = 3k + 1$.

## A new characterization of $P_6$-free graphs

**Pim van 't Hof**

(joint work with Daniël Paulusma)

We study $P_6$-free graphs, i.e., graphs that do not contain an induced path on six vertices. Our main result is a new characterization of this graph class: a graph $G$ is $P_6$-free if and only if each connected induced subgraph of $G$ on more than one vertex contains a dominating induced cycle on six vertices or a dominating (not necessarily induced) complete bipartite subgraph. This characterization is minimal in the sense that there exists an infinite family of $P_6$-free graphs for which a smallest connected dominating subgraph is a (not induced) complete bipartite graph. Our characterization of $P_6$-free graphs strengthens results of Liu and Zhou, and of Liu, Peng and Zhao. Our proof has the extra advantage of being constructive: we present an algorithm that finds such a dominating subgraph of a connected $P_6$-free graph in polynomial time. This enables us to solve the HYPERGRAPH 2-COLORABILITY problem in polynomial time for the class of hypergraphs with $P_6$-free incidence graphs.

# Tilings of $\mathbb{R}^n$ by cubes

**Peter Horák**

A tiling $\mathcal{T}$ of $\mathbb{R}^n$ by cubes is lattice-like if the centers of cubes in $\mathcal{T}$ form a group under vector addition. Interest in tilings of $\mathbb{R}^n$ by cubes and by sets of cubes goes back to the famous conjecture of Minkowski. In 1907 he asked whether in a lattice-like tiling of $\mathbb{R}^n$ by cubes there must be a pair of cubes that share a complete $(n-1)$-dimensional face. The conjecture has been settled in 1941 by Hajós. In this talk we discuss several variations of the conjecture and show how these problems are related to the Golomb-Welch conjecture and to the topic of the conference.

# Finding paths between 3-colourings

**Matthew Johnson**

Given a graph $G$ and a positive integer $k$ that is at least the chromatic number of $G$, the *k-colour graph of* $G$ has as its vertex set the set of all $k$-colourings of $G$ and a pair of colourings are joined by an edge in the colour graph if they differ on just one vertex.

We are interested in various properties of colour graphs such as, for example, the complexity of deciding whether the colour graph (for given $G$ and $k$) is connected or whether a given pair of colourings belong to the same component of the colour graph. In particular, we describe an algorithm that finds a "path" between a pair of 3-colourings or finds an "obstruction" that certifies that no such path can be found.

# Perfect matchings in fullerene graphs

**František Kardoš**

(joint work with Daniel Kráľ, Jozef Miškuf, and Jean-Sébastien Sereni)

A fullerene graph is a planar cubic 3-connected graph with only pentagonal and hexagonal faces. We show that fullerene graphs have exponentially many perfect matchings.

# Progress on the Traceability conjecture for oriented graphs

**Peter Katrenič**

(joint work with Susan van Aardt, Marietjie Frick, and Morten Hegner Nielsen)

A digraph is $k$-traceable if each of its induced subdigraphs of order $k$ is traceable. The Traceability Conjecture is that for $k \geq 2$ every $k$-traceable oriented graph of order at least $2k-1$ is traceable. The conjecture has been proved for $k \leq 5$. We prove that it also holds for $k = 6$ and we obtain here a lower bound of $n-k+5$ on the length of a longest path in any $k$-traceable oriented graph of order $n \geq k \geq 7$.

Further, we prove that if the order of the oriented graph is restricted to being exactly $2k-1$, then the conjecture holds also for $k = 7, 8, 9, 10$.

Finally, we discuss some open questions and point out the importance of hypo-traceable oriented graphs in relation to this conjecture.

# $d$-strong edge colorings of graphs

**Arnfried Kemnitz**

If $c : E \to \{1, 2, \ldots, k\}$ is a proper edge coloring of a graph $G = (V, E)$ then the palette $S(v)$ of a vertex $v \in V$ is the set of colors of the incident edges: $S(v) = \{c(e) : e = vw \in E\}$. An edge coloring $c$ distinguishes vertices $u$ and $v$ if $S(u) \neq S(v)$. A $d$-strong edge coloring of $G$ is a proper edge coloring that distinguishes all pairs of vertices $u$ and $v$ with distance $d(u, v) \leq d$. The minimum number of colors of a $d$-strong edge coloring is called $d$-strong chromatic index $\chi'_s(G, d)$ of $G$.

We will present some basic general results on $d$-strong edge colorings as well as some results for specific graph classes.

# The join of stars and crossing numbers

**Marián Klešč**

The *crossing number* $cr(G)$ of a graph $G$ is the minimum possible number of edge crossings in a drawing of $G$ in the plane. The investigation on the crossing number of graphs is a classical and however very difficult problem. Garey and Johnson have proved that the problem to determine the crossing number of graphs is NP-complete. The crossing numbers of some classes of graphs have been obtained. Such as the complete graph $K_n$ with small number of vertices, some Cartesian products of special graphs, some line graphs. The crossing number of complete bipartite graph $K_{m,n}$ were computed by D. J. Kleitman for all $m \leq 6$ and all $n$. There are known few results concerning crossing numbers of the complete tripartite graphs.

Let $G$ and $H$ be two disjoint graphs. The *join product* of $G$ and $H$, denoted by $G+H$, is obtained from vertex-disjoint copies of $G$ and $H$ by adding all possible edges between $V(G)$ and $V(H)$. For $|V(G)| = m$ and $|V(H)| = n$, the edge set of $G+H$ is the union of disjoint edge sets of the graphs $G$, $H$, and the complete bipartite graph $K_{m,n}$. For stars, Jiangen Zhang, Rongxia Hao and Jingbo Liu (2008) proved that the crossing number of $S_3 + S_n$ is $n^2 - \lfloor \frac{n}{2} \rfloor$. We extend this result and give exact values of crossing numbers for join products of stars and special graphs of order five and six.

# Unicyclic radially-maximal graphs on the minimum number of vertices

**Martin Knor**

A graph is radially-maximal if its radius decreases after adding of any edge from its complement. One can expect that such graphs are either very dense, or "balanced" (symmetric) ones. Therefore it is interesting that there exist radially-maximal graphs which are unicyclic and non-selfcentric (i.e., such that their radius is strictly smaller than their diameter). We characterize unicyclic, non-selfcentric, radially-maximal graphs on the minimum number of vertices. Such graphs must have radius $r \geq 5$, and we prove that the number of these graphs is $\frac{1}{48}r^3 + O(r^2)$.

# Bounds for the minimum order of a rainbow subgraph

**Maria Koch and Stephan Matos Camacho**

(joint work with Ingo Schiermeyer)

Our research was motivated by the pure parsimony haplotyping problem: Given a set $\mathcal{G}$ of genotypes, the haplotyping problem consists in finding a set $\mathcal{H}$ of haplotypes that explains $\mathcal{G}$. In the pure parsimony haplotyping problem (PPH) we are interested in finding a set $\mathcal{H}$ of smallest possible cardinality.

The pure parsimony haplotyping problem can be described as a graph colouring problem as follows:

**The minimum rainbow subgraph problem.** Given a graph $G$, whose edges are coloured with $p$ colours. Find a subgraph $F \subseteq G$ of $G$ of minimum order $r^*(G)$ with $|E(F)| = p$ such that each colour occurs exactly once.

If $G$ is a graph with maximum degree $\Delta(G)$, then

$$\frac{2p}{\Delta(G)} \leq r^*(G) \leq 2p.$$

In this talk we will present improved lower and upper bounds for the minimum order $r^*(G)$ of a rainbow subgraph of $G$.

# Number of perfect matchings in cubic bridgeless graphs

**Daniel Kráľ**

(joint work with Jean-Sébastien Sereni and Michael Stiebitz)

We study the number of perfect matchings in a cubic bridgeless graph. A conjecture of Lovász and Plummer from 1970s asserts that every cubic bridgeless graph has an exponential number of perfect matchings. The conjecture has been recently verified for planar cubic bridgeless graphs by Chudnovsky and Seymour (2008). Using the brick and brace decompositions, we improve the general lower bound of $\frac{n}{4} + 2$ due to Edmonds, Lovasz and Pulleyblank (1982) and Naddef (1982) to $\frac{n}{2}$ and list all cubic bridgeless graphs with exactly $\frac{n}{2}$ and $\frac{n}{2} + 1$ perfect matchings.

# Crossing numbers for products of power graphs

**Daniela Kravecová**

The *crossing number* $\mathrm{cr}(G)$ of a graph $G$ is the minimum possible number of edge crossings in a drawing of $G$ in the plane. Computing the crossing number of a given graph is in general an elusive problem. Garey and Johnson have proved that this problem is NP-complete. The exact values of the crossing numbers are known only for some families of graphs. The structure of Cartesian products of graphs makes Cartesian products of special graphs one of few graph classes, for which exact crossing number results are known.

In 1973, Harary, Kainen, and Schwenk established the crossing number of $C_3 \times C_3$ and conjectured that $\mathrm{cr}(C_m \times C_n) = m(n-2)$ for $3 \leq m \leq n$. Recently has been proved by Glebsky and Salazar that for any fixed $m \geq 3$, the conjecture holds for all $n \geq m(m+1)$. Besides the Cartesian products of two cycles, there are several other exact results. In 2006, Bokal proved the conjecture given by Jendroľ and Ščerbová that $\mathrm{cr}(K_{1,n} \times P_m) = (m-1)\lfloor\frac{n}{2}\rfloor\lfloor\frac{n-1}{2}\rfloor$ for the path $P_m$ of length $m$.

We extend the previous results concerning crossing numbers for the Cartesian products of power of some graphs with paths and cycles. In addition, we give some upper and lower bounds for other products.

# Paths in universal chess; hyper-knight's moves

**Jorma Kyppö**

Strangeness of Knight's move and its numerical symbolism has been one of the main factors of hypothesis connecting the origins and structure of chess with secret magical and religious rituals of ancient India. Several theories are made during the years about the birth of chess and the structure of it. In addition to rectangular chessboard it is also possible to form two dimensional triangular or hexagonal chessboards and higher dimensional hypercube chessboards with defined knight's moves. Not only the chessboard, but also the knight's move can be generalized. Knight's tour and knight's path are special cases of Hamiltonian cycles and Hamiltonian paths in graph theory. This paper deals with artificial generalized chessboards and artificial "Knights".

# Rainbow numbers for certain sunflower hypergraphs

**Ewa Łazuka**

(joint work with Izolda Gorgol)

A subgraph of an edge-colored hypergraph is called *rainbow* if all of its edges have different colors. For an $h$-uniform hypergraph $\mathcal{H}$ and a positive integer $n$, the *anti-Ramsey number* $f(n, \mathcal{H})$ is the maximum number of colors in an edge-coloring of a complete $h$-uniform hypergraph on $n$ vertices with no rainbow copy of $\mathcal{H}$. The *rainbow number* $rb(n, \mathcal{H})$ is the minimum number of colors such that any edge-coloring of a complete $h$-uniform hypergraph on $n$ vertices with $rb(n, \mathcal{H})$ number of colors contains a rainbow copy of $\mathcal{H}$. Certainly $rb(n, \mathcal{H}) = f(n, \mathcal{H}) + 1$. Anti-Ramsey numbers were introduced both for graphs and hypergraphs by Erdős et al. in [1].

Let $t, p, k \geq 1$. A *sunflower hypergraph* $\mathcal{S}(t, p, k)$ is a $(t + p)$-uniform hypergraph of order $n = t + kp$ and size $k$ in which any two of its edges (or *petals*) overlap in a fixed $t$-subset of vertices called a *middle*.

We present some new results concernig the rainbow numbers for certain sunflower hypergraphs.

## References

[1] P. Erdős, M. Simonovits, V. Sós, Anti-Ramsey theorems, Infinite and finite sets (A. Hajnal, R. Rado, and V. Sós, eds.), Colloq. Math. Soc. J. Bolyai, North-Holland, 1973, 633–643.

# Short cycle covers of graphs with minimum degree three

**Bernard Lidický**
(joint work with Tomáš Kaiser, Daniel Kráľ, and Pavel Nejedlý)

The Shortest Cycle Cover Conjecture asserts that the edges of every bridgeless graph with $m$ edges can be covered by cycles of total length at most $7m/5 = 1.4m$. Due to Fan it is known that every bridgeless cubic graph has a cycle cover comprised of three cycles of total length at most $44m/27 = 1.6296m$. We extend this result from cubic graphs to all graphs with minimum degree three.

# Snarks with given real flow number

**Robert Lukoťka**

Pan and Zhu [1] construct infinitely many graphs with real flow number $r$ for every rational number $r$, $4 < r \leq 5$. Moreover these graphs are 4-cyclically edge connected, so they can be used to construct snarks by replacing every vertex of degree higher than four with a cubic network. We show how to make this construction in such a way that we preserve both the cyclic connectivity and the real flow number. This construction gives infinitely many snarks with real flow number $r$ for every rational number $r$, $4 < r < 5$.

References

[1] Z. Pan, X. Zhu, Construction of graphs with given circular flow numbers, J. Graph Theory 43 (2003), 304–318.

# Acyclic edge colouring of regular graphs

**Edita Máčajová**
(joint work with Ján Mazák)

An *acyclic edge colouring* is a proper edge colouring such that the subgraph induced by the edges of any two colours is acyclic. The *acyclic edge chromatic number* of a graph $G$ is the smallest number of colours in an acyclic colouring of $G$. In the talk we will present new upper and lower bounds for the acyclic edge chromatic number of regular graphs.

# On doubly light graphs

**Tomáš Madaras**

A connected graph $H$ is *doubly light* in a family $\mathcal{H}$ of plane graphs if there exists a pair $\Phi(H, \mathcal{H}) = (a, b)$ of finite integers such that each graph $G \in \mathcal{H}$ containing $H$ contains also a subgraph $K \cong H$ such that $\deg_G(x) \leq a$ and $\deg_G(\alpha) \leq b$ for each $x \in V(K)$ and each face $\alpha \in F(G)$ incident with $x$. Further, $H$ is called *weakly doubly light* in $\mathcal{H}$ if there exists a pair $\Phi'(H, \mathcal{H}) = (a', b')$ of finite integers such that each graph $G \in \mathcal{H}$ containing $H$ contains also a subgraph $K \cong H$ such that $\deg_G(x) \leq a'$ and $\deg_G(\alpha) \leq b'$ for each $x \in V(K)$ and each face $\alpha \in F(G)$ incident with an edge of $K$. We exhibit several introductory results on doubly and weakly doubly light graphs in families of plane graphs with restricted minimum degree and face size.

# Lower bound on the circular chromatic index of a snark of given order

**Ján Mazák**
(joint work with Martin Mačaj)

We introduce circular edge-colourings and summarize known results about circular chromatic index of snarks. We list two recent conjectures and discuss them in the light of the known results. As a main point we derive a lower bound on the circular chromatic index of a snark of given order.

# Long cycles in hypergraphs

**Paweł Naroski**
(joint work with Zbigniew Lonc)

By a cycle of length $k$ in a hypergraph $H$ we mean an alternating sequence $(v_0, e_1, v_1, e_2, v_2, ..., v_{k-1}, e_k, v_k)$ of vertices and edges of this hypergraph satisfying the following conditions:

(i) $e_i = e_j \Rightarrow i = j$ for $i, j = 1, ..., k$;
(ii) $\{v_{i-1}, v_i\} \subseteq e_i$ for $i = 1, ..., k$;
(iii) $v_{i-1} \neq v_i$ for $i = 1, ..., k$;
(iv) $v_0 = v_k$.

We consider cycles containing all edges of a hypergraph (long cycles) and provide a necessary and sufficient condition for existence of such a cycle in some special

3-uniform hypergraphs that we call strongly connected. We discuss existence of long cycles in some broader classes of hypergraphs as well.

Problems of this kind arise in computer graphics and geographic information systems.

# Short cycle covers of cubic bridgeless graphs

**Pavel Nejedlý**

The Shortest Cycle Cover Conjecture asserts that the edges of every bridgeless graph with $m$ edges can be covered by cycles of total length at most $7m/5 = 1.4m$. We show that every cubic bridgeless graph has a cycle cover of total length at most $34m/21 \approx 1.619m$, improving the previous bound of $44m/27 \approx 1.630m$ obtained by Fan [1].

## References

[1] G. Fan, Shortest cycle covers of cubic graphs, J. Graph Theory 18 (1994), 131–141.

# Path factors and parallel knock-out schemes of almost claw-free graphs

**Daniël Paulusma**

(joint work with Matthew Johnson and Chantal Wood)

An $H_1, \{H_2\}$-factor of a graph $G$ is a spanning subgraph of $G$ with exactly one component isomorphic to the graph $H_1$ and all other components (if there are any) isomorphic to the graph $H_2$. We completely characterise the class of connected almost claw-free graphs that have a $P_7, \{P_2\}$-factor, where $P_7$ and $P_2$ denote the paths on seven and two vertices, respectively. We apply this result to parallel knock-out schemes for almost claw-free graphs. These schemes proceed in rounds in each of which each surviving vertex simultaneously eliminates one of its surviving neighbours. A graph is reducible if such a scheme eliminates every vertex in the graph. Using our characterisation we are able to classify all reducible almost claw-free graphs, and we can show that every reducible almost claw-free graph is reducible in at most two rounds. This leads to a quadratic time algorithm for determining if an almost claw-free graph is reducible (which is a generalisation and improvement upon the previous strongest result that showed that there was a $O(n^{5.376})$ time algorithm for $n$-vertex claw-free graphs).

# Strong isometric dimension of trees and $k$-trees

**Iztok Peterin**

"Every connected graph is isometrically embeddable into a strong product of paths" is a result that goes back to Shönberg (1938). Thus a strong isometric dimension of a graph $G$, $\mathrm{idim}(G)$, that is the least number $k$ of paths such that $G$ is isometrically embeddable in the strong product of $k$ paths, is well defined. Fitzpatrick and Nowakowski (2000) set the basic stone for strong isometric dimension by showing that $\mathrm{idim}(C_n) = \lceil n/2 \rceil$, $\mathrm{idim}(Q_k) = 2^{k-1}$, and $\lceil \log_2 \ell \rceil \leq \mathrm{idim}(T) \leq 2 \lceil \log_2 \ell \rceil$ for every tree $T$ with $\ell$ leaves. We will present a constructive embedding for every tree $T$ into a strong product of $\lceil \log_2 \ell \rceil$ paths. Furthermore, this embedding works also for $s$-trees, where a $s$-tree $S$ with $\ell$ leaves (vertices of degree $s$) can be embedded into a strong product of $\lceil \log_2 \ell \rceil + s - 1$ paths.

# A note on packing of two copies of a hypergraph

**Monika Pilśniak**

A *2-packing* of a hypergraph $\mathcal{H}$ is a permutation $\sigma$ on $V(\mathcal{H})$ such that if an edge $e$ belongs to $\mathcal{E}(\mathcal{H})$, then $\sigma(e)$ does not belong to $\mathcal{E}(\mathcal{H})$.

It is known that a hypergraph which does not contain neither empty edge $\emptyset$ nor complete edge $V(\mathcal{H})$ and has at most $\frac{1}{2}n$ edges is 2-packable.

Let $\mathcal{H}$ be a hypergraph of order $n$ which contains edges of cardinality at least 2 and at most $n-2$. We prove that if $\mathcal{H}$ has at most $n-2$ edges then is 2-packable. 2-uniform hypergraphs of order $n$ with more than $n-2$ edges are not 2-packable in general. It shows that this result cannot be improved by increasing the size of $\mathcal{H}$.

# Linear bound on the irregularity strength and the total vertex irregularity strength of graphs

**Jakub Przybyło**

Let $G$ be a simple graph of order $n$ with no isolated edges and at most one isolated vertex. For a positive integer $w$, a $w$-weighting of $G$ is a map $f : E(G) \to \{1, 2, \ldots, w\}$. An irregularity strength of $G$, $s(G)$, is the smallest $w$ such that there is a $w$-weighting of $G$ for which $\sum_{e:u\in e} f(e) \neq \sum_{e:v\in e} f(e)$ for all pairs of different vertices $u, v \in V(G)$. A tight result by Nierhoff says that $s(G) \leqslant n - 1$. We will discuss our new general upper bound, which is linear in $n/\delta$, hence better starting from a given $\delta$ upwards. In the case of the $d$-regular graphs, we show a better linear function of $n/d$ as an upper bound on $s(G)$, which corresponds with the conjecture by Faudree and Lehel that $s(G) \leqslant n/d + c$ for some absolute constant $c$. The recently introduced by Bača, Jendroľ, Miller and Ryan total version of the problem is also discussed and supported by a number of new bounds, also linear in $n/\delta$.

# Cycle length parities and the chromatic number

**Dieter Rautenbach**

(joint work with Christian Löwenstein and Ingo Schiermeyer)

For a simple, finite and undirected graph $G = (V, E)$ let $\mathcal{L}(G)$ denote the set of cycle lengths of $G$.

In this talk we study the influence of the cycle lengths $\mathcal{L}(G)$ of a graph $G$ on its chromatic number $\chi(G)$. This research was essentially initiated by Erdős and Hajnal in 1966 who proved that, if $l$ is the maximum odd element in $\mathcal{L}(G)$, then $\chi(G) \leq l+1$. Extending this result, Gyárfás proved in 1992 that, if $\mathcal{L}(G)$ contains $k$ odd elements, then $\chi(G) \leq 2k+2$ which was originally conjectured by Bollobás and Erdős.

We present bounds on the chromatic number of graphs

- whose cycle lengths are either small or have a fixed parity modulo some odd prime $p$,
- whose cycle lengths are either small or have a parity different from 2 modulo $p$,
- whose cycle lengths are either small or have a parity modulo $p$ belonging to some set of allowed parities, or
- that have a bounded number of cycle lengths which do not have a parity modulo $p$ belonging to some set of allowed parities.

Our main result is the following.

**Theorem.** *Let $p$ be an odd prime, $k \in \mathbb{N}$ and $I \subseteq \{0, 1, \ldots, p-1\}$ with $|I| \leq p-1$. If $G$ is a graph such that $\mathcal{L}(G)$ contains at most $k$ elements which are not in $I$ modulo $p$, then*

$$\chi(G) \leq \left(1 + \frac{|I|}{p - |I|}\right) k + p(p-1)(r(2p, 2p) + 1) + 1$$

*where $r(p, q)$ denotes the ordinary Ramsey number.*

# On a relation between the cycle packing number and the cyclomatic number of a graph

**Peter Recht**

Let $G = (V, E)$ be a graph. A cycle packing $\mathcal{Z} = \{C_1, \ldots, C_l\}$ of $G$ is a collection of pairwise edge-disjoint cycles $C_i$ of $G$ $(i = 1, \ldots, l)$.

This talk deals with $\nu(G)$, the maximum cardinality of a cycle packing $\mathcal{Z}$. Bounds on $\nu(G)$ are given, if $G$ is Eulerian. Moreover this number is related to the cyclomatic number $\gamma(G)$, the maximum number of independent cycles in $G$. A characterization of such graphs is given for which $\gamma(G) - \nu(G) = 1$ holds.

# Almost resolvable even-cycle systems

**Alexander Rosa**

(joint work with Curt Lindner and Mariusz Meszka)

A $k$-cycle system of order $n$ is almost resolvable if its $k$-cycles can be partitioned into $\frac{n-1}{2}$ almost parallel classes and one half-parallel class. We settle completely the existence problem for almost resolvable $k$-cycle systems when $k = 6$, $k = 10$, and $k = 14$ (modulo one possible exception in the latter case). We also indicate an approach to settle the general existence problem for almost resolvable even-cycle systems.

# Hamiltonian properties and closure concepts

**Zdeněk Ryjáček**

For a claw-free graph $G$, $\mathrm{cl}(G)$ denotes its closure (obtained by local completions at locally connected vertices). More generally, for $k \geq 1$, the $k$-closure $\mathrm{cl}_k(G)$ of $G$ is obtained from $G$ by local completions at locally $k$-connected vertices. A class of graphs $\mathcal{C}$ is said to be *$k$-stable* (under the $k$-closure) if $G \in \mathcal{C} \Rightarrow \mathrm{cl}_k(G) \in \mathcal{C}$. A property $\mathcal{P}$ is said to be *$k$-stable* in a $k$-stable class $\mathcal{C}$ if, for any $G \in \mathcal{C}$, $G$ has $\mathcal{P} \Leftrightarrow \mathrm{cl}_k(G)$ has $\mathcal{P}$. Similarly, a graph invariant $\pi$ is said to be *$k$-stable* in $\mathcal{C}$ if, for any $G \in \mathcal{C}$, $\pi(G) = \pi(\mathrm{cl}_k(G))$.

In the talk we mention some known results on stability of graph properties under $\mathrm{cl}_k(G)$ and some applications. As a recent application it is shown (joint work with Petr Vrána, Plzeň) that every 7-connected claw-free graph is Hamilton-connected.

# Approximation algorithms for the minimum rainbow subgraph problem

**Ingo Schiermeyer**
(joint work with Stephan Matos Camacho and Zsolt Tuza)

Our research was motivated by the pure parsimony haplotyping problem: Given a set $\mathcal{G}$ of genotypes, the haplotyping problem consists in finding a set $\mathcal{H}$ of haplotypes that explains $\mathcal{G}$. In the pure parsimony haplotyping problem (PPH) we are interested in finding a set $\mathcal{H}$ of smallest possible cardinality.

The pure parsimony haplotyping problem can be described as a graph colouring problem as follows:

**The minimum rainbow subgraph problem.** Given a graph $G$, whose edges are coloured with $p$ colours. Find a subgraph $F \subseteq G$ of $G$ of minimum order with $|E(F)| = p$ such that each colour occurs exactly once.

In this talk we will present polynomial time approximtaion algorithms for the minimum rainbow subgraph problem:

- Applying the greedy algorithm we obtain an approximation algorithm with an approximation ratio of $\Delta(G)$ for graphs with maximum degree $\Delta(G)$.
- Based on matching techniques we present an approximation algorithm with an approximation ratio of $\frac{5}{3}$ for graphs with maximum degree 2.

# Local computability of cycles and colourings

**Jens Schreyer**
(joint work with Thomas Böhme)

In distributed computing, many people look for algorithms that compute global graph parameters locally. That means, the graph is not given explicitly but as an existing network, where each vertex corresponds to an agent who only knows and communicates with its immediate neighbours. The task is to compute graph parameters or at least approximations using only this local information. In the general setting every vertex is associated with an agent with unlimited computation capability and the limiting factor is the time that is needed to gather enough information about the neighborhood. But also limited space capacities and computation abilities have already been investigated in the past.

We investigate, which problems can be solved under the assumption that only very little information can be exchanged between the agents. In the ideal case only the actions of the agents or just their implications can be observed. Moreover, the memory and computation capability of each agent is considered to be very limited.

We use some results on repeated games to show, that a colouring can almost surely be computed by a learning process without any information transmission. Moreover, with very little exchanged information, a Hamilton cycle can be found as well.

# Cayley and vertex-transitive near-cages of girth 5 and 6

**Jana Šiagiová**
(joint work with Eyal Loz, Mirka Miller, Jozef Širáň, and Jana Tomanová)

We re-visit the known constructions of near-cages (that is, graphs of order "close" to the order of cages) of a given degree and girth 6 and 5 and we present a unified construction for all cases, based on graph coverings. We show that in the case of girth 6 all the known near-cages are Cayley graphs and in the case of girth 5 we identify the near-cages which are Cayley and vertex-transitive non-Cayley.

# Some properties of marking games

**Elżbieta Sidorowicz**

A *marking game* is played on a graph $G$ by two players Alice and Bob with Alice having the first move. At the start of the game, all vertices of $G$ are unmarked. A move consists of marking an unmarked vertex. The game ends when all vertices have been marked. For any $t \in \{1, ..., |V|\}$, let $M^t$ denotes the set of marked vertices after $t$ moves and $U^t = V - M^t$ denotes the set of unmarked vertices after $t$ moves. For an unmarked vertex $u$, let $S^t(u) = N(u) \cap M^t$. The *score of marking games* is $\max\{|S^t(u)| : 1 \leq t \leq |V| \wedge u \in U^t\}$.

In the marking game Alice's goal is to minimize the score while Bob tries to maximize the score. The *game colouring number* denoted by $\mathrm{col}_g(G)$ is the least $s$ such that Alice has a strategy to ensure that the resulting score of the marking game is less than $s$.

The set of graphs with the game colouring number less or equal to $s$ is a hereditary property so it can be characterized by the set of minimal forbidden subgraphs. We give the construction of such minimal forbidden subgraphs and describe some properties of the marking game. We also investigate various versions of the marking games.

# *k*-list critical graphs

**Margit Voigt**

(joint work with Michael Stiebitz and Zsolt Tuza)

Some basic properties of $k$-list critical graphs are discussed in the talk. A graph $G$ is *$k$-list critical* if there exists a list assignment $L$ for $G$ with $|L(v)| = k - 1$ for all vertices $v$ of $G$ such that every proper subgraph of $G$ is $L$-colorable, but $G$ itself is not $L$-colorable. This generalizes the usual definition of a $k$-chromatic critical graph, where $L(v) = \{1, \ldots, k-1\}$ for all vertices $v$ of $G$. While the investigation of $k$-critical graphs is a well established part of coloring theory, not much is known about $k$-list critical graphs. Several unexpected phenomena occur, for instance a $k$-list critical graph may contain another one as a proper induced subgraph, with the same value of $k$.

# On generalized Pell numbers in graphs

**Iwona Włoch**

Let $k$ be a fixed integer, $k \geq 2$. A subset $S \subseteq V(G)$ is a $k$ independent set of $G$ if for each two distinct vertices $x, y \in S$, $d_G(x, y) \geq k$.

The *Pell numbers* are defined by the recurrence relation $P_0 = 0$, $P_1 = 1$ and $P_n = 2P_{n-1} + P_{n-2}$, for $n \geq 2$. The *Pell-Lucas numbers* (or the *companion Pell numbers*) are defined by the recurrence relation $Q_0 = Q_1 = 2$ and $Q_n = 2Q_{n-1} + Q_{n-2}$, for $n \geq 2$. The Pell-Lucas number can be also expressed by $Q_n = 2P_{n-1} + 2P_n$.

We give a generalization of the Pell numbers and the Pell-Lucas numbers and next we apply this concept for their graph representations. We shall show that the generalized Pell numbers and the Pell-Lucas numbers are equal to the total number of $k$-independent sets in special graphs.

## References

[1] M. Kwaśnik, I. Włoch, The total number of generalized stable sets and kernels of graphs, Ars Combin., 55 (2000), 139–146.

[2] I. Włoch, On generalized Pell numbers in graphs, submitted.

# Large digraph of given diameter and degree from graph coverings

**Mária Ždímalová**

Interest in the design of large vertex-symmetric graphs and digraphs is motivated by their use in modelling interconnection networks. Such networks are often subject to two fundamental restrictions: the number of connections that can be attached at any one node is limited, as is the number of intermediate nodes on communications paths between nodes.

This gives rise to the well known *degree-diameter problem*, which is to construct largest possible graphs and digraphs with given maximum degree and given diameter.

In our contribution we will show that some of the infinite families of large vertex-symmetric digraphs of given degree and diameter discovered earlier can also be described and analyzed in terms of graphs coverings.

# A signed star domination sequence of a graph

**Maciej Zwierzchowski**

All graphs considered here are finite, undirected without loops and multiple edges. For a vertex $v \in V(G)$, $E_G(v) = \{uv : u \in V(G)\}$ is called the *edge-neighbourhood* of $v$ in $G$ and $\deg_G(v) = |E_G(v)|$ is called a *degree* of $v$ in $G$. A vertex $v \in V(G)$ is an *isolated* vertex of $G$ if $\deg_G(v) = 0$. Additionally we assume that $G$ has no isolated vertex.

A function $f : E(G) \to \{-1, 1\}$ is a *signed star domination function (SSDF)* of $G$ if $\sum_{e \in E_G(v)} f(e) \geq 1$ for every $v \in V(G)$.

Let $E_f^+(G) = \{e \in E(G) : f(e) = 1\}$. A *SSDF* $f$ is called a *minimal signed star domination function (mSSDF)* of $G$ if for every edge $e \in E_f^+(G)$, a function $g : E(G) \to \{-1, 1\}$ such that $g(e') = \begin{cases} f(e') & \text{if } e' \neq e \\ -1 & \text{if } e' = e \end{cases}$ is not a SSDF of $G$.

For a vertex $v \in V(G)$ by a symbol $f(v)$ we mean $\sum_{e \in E_G(v)} f(e)$. A sequence $\pi = (w_1, ..., w_n)$ of positive integers is called a *signed star domination sequence* if there exists a graph $G$ with $V(G) = \{v_1, ..., v_n\}$ and a minimal signed star domination function $f$ of $G$ such that $f(v_i) = w_i$. We present a necessary and sufficient conditions for a sequence $\pi$ to be a signed star domination sequence.

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# Programme of the Conference

| Sunday | | |
|---|---|---|
| M | 16:00 - 22:00 | Registration |
| M | 18:00 - 21:00 | Dinner |

| Monday | | | |
|---|---|---|---|
| M/R | 07:30 - 08:30 | Breakfast | |
| R | 08:40 - 08:45 | Opening | |
| R | 08:45 - 09:35 | Bang-Jensen J. | Paths, cycles and strong subdigraphs in directed graphs |
| R | 09:40 - 10:00 | Rautenbach D. | Cycle length parities and the chromatic number |
| R | 10:05 - 10:30 | Coffee break | |
| R | 10:30 - 11:20 | Havet F. | Grundy number and lexicographic product of graphs |
| R | 11:25 - 11:45 | Koch M., Matos Camacho S. | Bounds for the minimum order |
| R | 11:50 - 12:10 | Schiermeyer I. | Approximation algorithms for the minimum rainbow subgraph problem |
| M | 12:30 - 13:15 | Lunch | |
| M-A | 15:00 - 15:20 | Gorgol I. | Rainbow numbers for disjoint copies of graphs |
| M-B | | Pilśniak M. | A note on packing of two copies of a hypergraph |
| M-A | 15:25 - 15:45 | Łazuka E. | Rainbow numbers for certain sunflower hypergraphs |
| M-B | | Adamus L. | Erdős-type condition for long cycles in bipartite graphs |
| M-A | 15:50 - 16:10 | Madaras T. | On doubly light graphs |
| M-A | | Naroski P. | Long cycles in hypergraphs |
| M | 16:15 - 16:40 | Coffee break | |
| M-A | 16:40 - 17:00 | Recht P. | On a relation between the cycle packing number |
| M-B | | Kyppö J. | Paths in universal chess; hyper-knight's moves |
| M-A | 17:05 - 17:25 | Bielak H. | Ramsey numbers for some graphs |
| M-B | | Lukoťka R. | Snarks with given real flow number |
| M-A | 17:30 - 17:50 | Drgas-Burchardt E. | Graph primality |
| M-B | | Ždímalová M. | Large digraph of given diameter and degree from graph coverings |
| M-A | 17:55 - 18:15 | Dvořák Z. | Three-coloring triangle-free planar graphs |
| M | 18:30 - 19:15 | Dinner | |
| M | 20:00 - | Welcome party | |

M . . . Hotel Meander, M-A . . . Meander, Lecture room A, M-B . . . Meander, Lecture room B, R . . . Hotel Rysy

| Tuesday | | | |
|---|---|---|---|
| M/R | 07:30 - 08:30 | Breakfast | |
| R | 08:45 - 09:35 | Broersma H. | Degree sequences conditions for graph properties |
| R | 09:40 - 10:00 | Harant J. | Hamiltonian cycles through prescribed edges of 4-connected maximal planar graphs |
| R | 10:05 - 10:30 | Coffee break | |
| R | 10:30 - 10:50 | Rosa A. | Almost resolvable even-cycle systems |
| R | 10:55 - 11:15 | Horák P. | Tilings of $\mathbb{R}^n$ by cubes |
| R | 11:20 - 11:40 | Bacsó G. | New near-factorizations of finite groups |
| R | 11:45 - 12:05 | Knor M. | Unicyclic radially-maximal graphs on the minimum number of vertices |
| M | 12:30 - 13:15 | Lunch | |
| M-A | 15:00 - 15:20 | Schreyer J. | Local computability of cycles and colourings |
| M-B | | Cichacz S. | Super edge-graceful paths and cycles |
| M-A | 15:25 - 15:45 | Johnson M. | Finding paths between 3-colourings |
| M-B | | Fiedorowicz A. | Acyclic edge colouring of graphs |
| M-A | 15:50 - 16:10 | Włoch I. | On generalized Pell numbers in graphs |
| M-A | | Mazák J. | Lower bound on the circular chromatic index of a snark of given order |
| M | 16:15 - 16:40 | Coffee break | |
| M-A | 16:40 - 17:00 | Peterin I. | Strong isometric dimension of trees and $k$-trees |
| M-B | | Katrenič P. | Progress on the Traceability conjecture for oriented graphs |
| M-A | 17:05 - 17:25 | Sidorowicz E. | Some properties of marking games |
| M-B | | Kravecová D. | Crossing numbers for products of power graphs |
| M-A | 17:30 - 17:50 | Feňovčíková A. | Some new results on edge-antimagic graphs |
| M-B | | Draženská E. | On the crossing numbers of products with stars |
| M-A | 17:55 - 18:15 | Kardoš F. | Perfect matchings in fullerene graphs |
| M | 18:30 - 19:15 | Dinner | |
| M | 20:00 - 21:20 | Videopresentation C&C 2007 | |

| Wednesday | | |
|---|---|---|
| M/R | 07:30 - 08:30 | Breakfast |
| | 08:30 - 16:00 | Trip |
| M | 19:00 - 20:00 | Dinner |

| Thursday | | | |
|---|---|---|---|
| M/R | 07:30 - 08:30 | Breakfast | |
| R | 08:45 - 09:35 | Ryjáček Z. | Hamiltonian properties and closure concepts |
| R | 09:40 - 10:00 | Voigt M. | $k$-list critical graphs |
| R | 10:05 - 10:30 | Coffee break | |
| R | 10:30 - 11:20 | Grytczuk J. | Thue games |
| R | 11:25 - 11:45 | Kráľ D. | Number of perfect matchings in cubic bridgeless graphs |
| R | 11:50 - 12:10 | Kemnitz A. | $d$-strong edge colorings of graphs |
| M | 12:30 - 13:15 | Lunch | |
| M-A | 15:00 - 15:20 | Zwierzchowski M. | A signed star domination sequence of a graph |
| M-B | | Šiagiová J. | Cayley and vertex-transitive near-cages of girth 5 and 6 |
| M-A | 15:25 - 15:45 | van 't Hof P. | A new characterization of $P_6$-free graphs |
| M-B | | Furmańczyk H. | Mixed graph edge coloring |
| M-A | 15:50 - 16:10 | Przybyło J. | Linear bound on the irregularity strength and the total vertex irregularity strength of graphs |
| M-A | | Dzido T. | Altitude of wheels and wheel-like graphs |
| M | 16:15 - 16:40 | Coffee break | |
| M-A | 16:40 - 17:00 | Máčajová E. | Acyclic edge colouring of regular graphs |
| M-A | 17:05 - 17:25 | Bujtás Cs. | Colorings of stably bounded hypergraphs |
| M-A | 17:30 - 17:50 | Paulusma D. | Path factors and parallel knock-out schemes of almost claw-free graphs |
| M-A | 17:55 - 18:15 | Klešč M. | The join of stars and crossing numbers |
| M | 19:00 - | Farewell party | |

| Friday | | | |
|---|---|---|---|
| M/R | 07:30 - 08:30 | Breakfast | |
| M-A | 09:10 - 09:30 | Nejedlý P. | Short cycle covers of cubic bridgeless graphs |
| M-A | 09:35 - 09:55 | Lidický B. | Short cycle covers of graphs with minimum degree three |
| M-A | 10:00 - 10:30 | Coffee break | |
| M-A | 10:30 - 10:50 | Budajová K. | Parity vertex colouring of graphs |
| M-A | 10:55 - 11:15 | Czap J. | Weak and strong parity vertex colourings |
| M | 12:00 - 13:00 | Lunch | |