GENERALIZED TOWERS AND PRODUCTS

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Let X be a set of reals and $C_p(X)$ be the set of all continuous real-valued functions on X with the pointwise convergence topology. By the result of Gerlits and Nagy [1] the space $C_p(X)$ has the Fréchet–Urysohn property (a generalization of first-countability) if and only if the set X is a γ-set, i.e., has a combinatorial covering property. The existence of uncountable γ-sets of reals is independent of ZFC. Tsaban proved [3] that sets with some special combinatorial structure are γ-sets. We generalize this class of sets and prove that their products have the property γ. We also show that for every set X from our class and every gamma set Y, the product space $X \times Y$ have a strong property weaker than the property γ. These investigations are motivated by the result of Miller, Tsaban and Zdomskyy [2] that under CH, there are two γ-sets whose product space is not even Menger (in particular it is not γ).

References

[1] J. Gerlits, Zs. Nagy, *Some Properties of* $C_p(X)$, *I*, Topology and its Applications **14** (1982), 151–161.

[2] A. Miller, B. Tsaban, L. Zdomskyy, *Selective covering properties of product spaces, II: γ spaces*, Transactions of the American Mathematical Society **368** (2016), 2865–2889.

[3] T. Orenshtein, B. Tsaban, *Linear σ-additivity and some applications*, Transactions of the American Mathematical Society **363** (2011), 3621–3637.

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