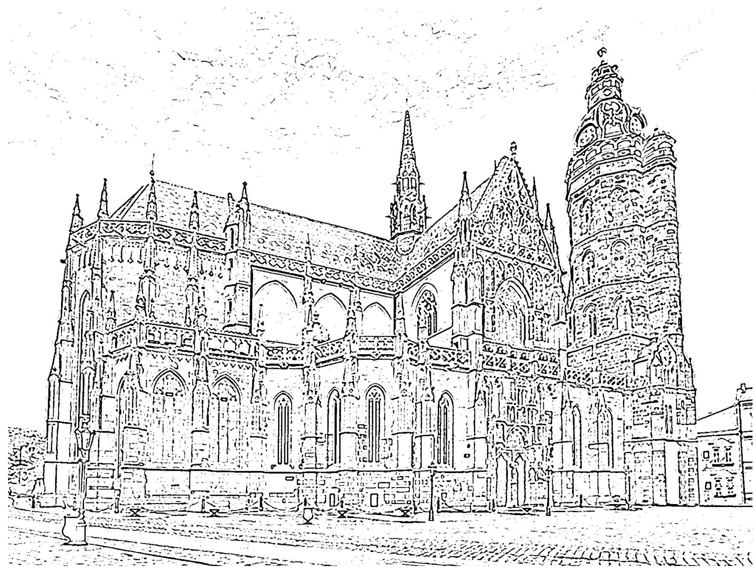
SET-THEORETIC METHODS IN TOPOLOGY AND REAL FUNCTIONS THEORY, DEDICATED TO 80TH BIRTHDAY OF LEV BUKOVSKÝ



A B S T R A C T S

Košice, Slovakia

September 9–13, 2019

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PREFACE

The conference **Set-theoretic methods in topology and real functions theory, dedicated to 80th birthday of Lev Bukovský**, is organized by the Faculty of Science of Pavol Jozef Šafárik University and the Mathematical Institute of the Slovak Academy of Sciences under the auspices of the rector of the university.

Professor Lev Bukovský was born on September 9th 1939 in Podkriváň—a village in central Slovakia. In 1961, he graduated from Comenius University in Bratislava. He spent some time at the Mathematical Institute of Czechoslovak Academy of Sciences in Prague as a member of Peter Vopěnka's seminar. His experience from the seminar decided his future career in set theory. In 1965, he got a position at the Faculty of Science of Pavol Jozef Šafárik University in Košice. He was a rector of the same university in the period 1991–1996.

Professor Bukovský is well-known for his result in recursive computation of cardinal exponentiation by the gimel function from 1965. In 1973, he proved that the intersection of finite iterations of ultraproducts of a model of ZFC with a measurable cardinal κ is a generic extension of their direct limit and in this extension κ has countable cofinality. In the same year he found a nice characterization of generic models via approximations of functions. He was the first to introduce 6 out of 10 cardinals of Cichoń's diagram. His PhD students are: Martin Gavalec, Eva Butkovičová, Eva Copláková-Hartová, Miroslav Repický, Peter Eliaš, Eva Trenklerová, Jozef Haleš, Michal Staš, Jaroslav Šupina, Zdenko Takáč.

This booklet includes the list of participants with titles of their conference talks and submitted abstracts. The abstracts are divided into two sections Invited Lectures and Contributed Talks and they are arranged in the alphabetical order according to the name of the first author.

We hope you will find the conference inspiring for your work.

Košice, September 2019 *Organizing Committee*

LIST OF PARTICIPANTS

1. **BARDYLA, Serhii**
   Institute of Mathematics, Kurt Gödel Research Center, Vienna, Austria
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   Talk: *Subspaces of countably compact topological spaces* (with *Taras Banakh and Alex Ravsky*)

2. **BARTOŠ, Adam**
   Faculty of Mathematics and Physics, Charles University and Institute of Mathematics of the Czech Academy of Sciences, Sokolovská 49/83, 186 75 Praha 8, Czech Republic
   E-mail: drekin@gmail.com
   Talk: *Compactifiable classes and their complexity* (with *J. Bobok, J. van Mill, P. Pyrih, and B. Vejnar*)

3. **BASILE, Fortunata Aurora**
   University of Messina, Via Stagno D'Alcontres n. 31, 98166 Messina, Italy
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   Talk: *On the quasicellularity of a space* (with *Nathan Carlson*)

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   Talk: *On $\varkappa$-metrisable and stratifiable spaces* (with *Andrzej Kucharski and Szymon Plewik*)

6. **BŁASZCZYK, Aleksander**
   Institute of Mathematics, University of Silesia, ul. Bankowa 14, 40-007 Katowice, Poland
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   Talk: *Topological constructions involving inverse limits*

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Talk: *New consistency results about cardinal invariants associated with the strong measure zero ideal*

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Talk: *The uniform boundedness principle for (ultra)filters* (with *Hans Vernaeve*)

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Talk: *If I were a rich density* (with *Jacek Tryba*)

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Talk: *σ-Lacunary Actions of Polish Groups*

18. **GUTIK, Oleg**
Ivan Franko National University of Lviv, Universytetska 1, Lviv, 79000, Ukraine
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Talk: *On a semitopological extended bicyclic semigroup with adjoined zero* (with *Kateryna Maksymyk*)

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Talk: *Small* $\mathfrak{u}(\kappa)$ *for a singular* κ *with compactness at* κ^{++}

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Talk: *Measures and Submeasures on Boolean Algebras*

26. **JUHÁSZ, István**
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Talk: *Spaces of small cellularity have nowhere constant continuous images of small weight* (with *L. Soukup and Z. Szentmiklóssy*)

27. **JURECZKO, Joanna**
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Talk: *Non "complete" case of Louveau–Simpson Theorem* (with *Ryszard Frankiewicz*)

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Talk: *On countably saturated linear orders and graphs*

31. **KRAWCZYK, Adam**
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Talk: *Games on finitely generated structures* (with *Wiesław Kubiś*)

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Institute of Mathematics, Faculty of Mathematics, Physics and Informatics, University of Gdańsk, ul. Wita Stwosza 57, 80-308 Gdańsk, Poland
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Talk: *Yet another ideal version of the bounding number* (with *Rafał Filipów*)

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Talk: *Patterns of Stationary Reflection* (with *Sy-David Friedman*)

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Talk: *On the sets which can be moved away from the sets of a certain family* (with *Grażyna Horbaczewska*)

37. **MAGIDOR, Menachem**
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Talk: *Inner models of set theory constructed by using generalized logics* (with *J. Kennedy and J. Vaananen*)

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Talk: *Rosenthal families and the cardinal invariants of Cichoń's diagram* (with *Piotr Koszmider*)

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Talk: *Mycielski among trees – category case* (with *Robert Rałowski and Szymon Żeberski*)

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Talk: *Topology expansions via specific ideals* (with *Aleksandar Pavlović*)

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Talk: *Topological games and selection properties of hyperspaces*

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Talk: *Properties of local closure functions in ideal topological spaces* (with *Anika Njamcul*)

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Talk: *Closed hereditary coreflective subcategories in certain categories of topological spaces*

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Talk: *Mycielski among trees – nonstandard proofs* (with *Marcin Michalski and Szymon Żeberski*)

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Talk: *On l-equivalence and the Menger property*

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Talk: *Divisibility in* βN *and* $^*N$

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Talk: *Convergence of measures on minimally generated Boolean algebras* (with *Lyubomyr Zdomskyy*)

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Talk: *Cardinal sequences and universal spaces*

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Talk: *Discrete sets, cellular families and the Lindelöf property* (with *Angelo Bella*)

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Talk: *Club stationary reflection and the tree property*

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Talk: *Size-based level measure as a standard level measure* (with *Lenka Halčinová*)

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Talk: *Generalized towers and products* (with *Magdalena Włudecka*)

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Talk: *Linearly sensitive properties* (with *Artur Bartoszewicz and Małgorzata Filipczak*)

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Talk: *Ideal convergence and matrix summability* (with *Rafał Filipów*)

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Talk: *Structure of the RK-order of P-points*

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Talk: *Polarised Partition Relations for Order Types* (with *Chris Lambie-Hanson, Shimon Garti, and William Chen*)

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Talk: *On $\mathcal{S}$-approximately continuous functions*

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Talk: *QN-spaces and covering properties of Hurewicz*

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Talk: *Nonmeasurable unions with respect to tree ideals* (with *Robert Rałowski*)

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DISCRETE SETS, CELLULAR FAMILIES AND THE LINDELÖF PROPERTY

Angelo Bella and Santi Spadaro

Presented by Santi Spadaro

Juhász, Tkachuk and Wilson [4] define a space to be *almost discretely Lindelöf* if every discrete set can be covered by a Lindelöf subspace. Inspired by Arhangel'skii's Theorem on the cardinality of Lindelöf Hausdorff first-countable spaces, they ask whether the cardinality of every Hausdorff first-countable almost discretely Lindelöf space is bounded by the continuum (see [4] and also [3]). We will give a consistent answer to their question. As a matter of fact, we will show that the cardinality of a sequential almost discretely Lindelöf space with points G_δ is bounded by the continuum under $2^{<\mathfrak{c}} = \mathfrak{c}$.

A space X is *cellular-Lindelöf* if for every family of pairwise disjoint non-empty open sets $\mathcal{U}$ there is a Lindelöf subspace $L \subset X$ which meets every member of $\mathcal{U}$. The class of cellular-Lindelöf spaces contains both almost discretely Lindelöf spaces and spaces with the countable chain condition. It's an open question whether the cardinality of a cellular-Lindelöf first-countable Hausdorff space is bounded by the continuum. A positive answer would lead to a common generalization of two seemingly unrelated fundamental results in the theory of cardinal functions: Arhangel'skii's Theorem and the Hajnal-Juhász bound on the cardinality of first-countable ccc spaces. We will show that the answer is positive for normal spaces under $2^{<\mathfrak{c}} = \mathfrak{c}$ and, time permitting, present various other results about the classes of almost discretely Lindelöf and cellular-Lindelöf spaces.

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Key words and phrases. Cellularity, Lindelof space, Arhangel'skii's Theorem, discrete set.
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Set-theoretic methods in topology and real functions theory, dedicated to 80th birthday of Lev Bukovský
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TOPOLOGICAL CONSTRUCTIONS INVOLVING INVERSE LIMITS

Aleksander Błaszczyk

Inverse limits appears in several branches of topology including theory of continua, general topology and set-theoretic topology. It also appears in the theory of Boolean algebras in the form of chains of subalgebras. I will first survey the most influential topological constructions using inverse limits. Then I will sketch new constructions using these methods.

Key words and phrases. Inverse limit.
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MEASURES AND SUBMEASURES ON BOOLEAN ALGEBRAS

Thomas Jech

We give a survey of various results, spanning the last 100 years, dealing with the existence of measures and submeasures on Boolean algebras. We give an algebraic characterization of the following Boolean algebras:

Complete Boolean algebras that carry a σ-additive measure.

Complete Boolean algebras that carry a Maharam submeasure.

Boolean algebras that carry a finitely additive measure.

Boolean algebras that carry an exhaustive submeasure.

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SPACES OF SMALL CELLULARITY HAVE NOWHERE CONSTANT CONTINUOUS IMAGES OF SMALL WEIGHT

István Juhász

We call a continuous map $f : X \to Y$ *nowhere constant* if it is not constant on any non-empty open subset of its domain X. Clearly, this is equivalent with the assumption that every fiber $f^{-1}(y)$ of f is nowhere dense in X. We call the continuous map $f : X \to Y$ *pseudo-open* if for each nowhere dense $Z \subset Y$ its inverse image $f^{-1}(Z)$ is nowhere dense in X. Clearly, if Y is crowded, i.e. has no isolated points, then f is nowhere constant.

How "small" nowhere constant, resp. pseudo-open continuous images can "large" spaces have? We give two answers to these questions, both of them involve the cardinal function $\widehat{c}(X)$, the "hat version" of cellularity, defined as the smallest cardinal κ such that there is no κ-sized disjoint family of open sets in X. (Thus, for instance, $\widehat{c}(X) = \omega_1$ means that X is CCC.)

Theorem A. *Any crowded Tychonov space X has a crowded Tychonov nowhere constant continuous image Y of weight $w(Y) \le \widehat{c}(X)$. Moreover, in this statement $\le$ may be replaced with $<$ iff there are no $\widehat{c}(X)$-Suslin lines (or trees).*

Theorem B. *Any crowded Tychonov space X has a crowded Tychonov pseudo-open continuous image Y of weight $w(Y) \le 2^{<\widehat{c}(X)}$.*

The latter result is sharp, at least consistently, because if Martin's axiom holds then there is a CCC crowded Tychonov space X such that for any crowded Hausdorff pseudo-open continuous image Y of X we have $w(Y) \ge \mathfrak{c}$ $(= 2^{<\omega_1})$.

This is joint work with L. Soukup and Z. Szentmiklóssy.

Key words and phrases. Nowhere constant maps, weight, cellularity.
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Set-theoretic methods in topology and real functions theory, dedicated to 80th birthday of Lev Bukovský
Košice, Slovakia, September 9–13, 2019

INNER MODELS OF SET THEORY CONSTRUCTED BY USING GENERALIZED LOGICS

Menachem Magidor

The constructible universe L is built by a series of stages where each successor stage is the set of (first order) definable subsets of the previous stage. The problem with L is that it misses many canonical objects like $0^\sharp$. One possible attempt to define a rich class of inner models is by imitating the construction of L but using definability by stronger logic. A classical theorem of Myhill and Scott claims that if we use second order logic we get HOD – the class of sets hereditarily ordinal definable. HOD is not very canonical, it depends very much on the universe of Set Theory from which we start. This work (which is joint work with J. Kennedy and J. Vaananen) studies the inner models we can get by using logics which are between first order logic and second order logic, e.g., the logic of the quantifier $Qxy\Phi(x, y)$ which means "The formula $\Phi(x, y)$ defines a linear order which has cofinality ω". The model we get is rather canonical (in the presence of large cardinals) and contains many canonically definable objects. We shall discuss similar results for other extended logics.

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POLARISED PARTITION RELATIONS FOR ORDER TYPES

Thilo Weinert

Many partition relations have been proved assuming the Generalised Continuum Hypothesis. More precisely, many negative partition relations involving ordinals smaller than ω_2 have been proved assuming the Continuum Hypothesis. Some recent results in this vein for polarised partition relations came from Garti and Shelah. The talk will focus on ordinary partition relations. The negative relations $\omega_1\omega \not\to (\omega_1\omega, 3)^2$ and $\omega_1^2 \not\to (\omega_1\omega, 4)^2$ were both shown to follow from the Continuum Hypothesis, the former in 1971 by Erdős and Hajnal and the latter in 1987 by Baumgartner and Hajnal. The former relation was shown to follow from both the dominating number and the stick number being $\aleph_1$ in 1987 by Takahashi. In 1998 Jean Larson showed that simply the dominating number being $\aleph_1$ suffices for this. It turns out that the unbounding number and the stick number both being $\aleph_1$ yields the same result. Moreover, also the second relation follows both from the dominating number being $\aleph_1$ and from both the unbounding number and the stick number being $\aleph_1$ thus answering a question of Jean Larson.

This is joint work with Chris Lambie-Hanson and both Shimon Garti and William Chen, the paper and the preprint are available at

`https://projecteuclid.org/euclid.jmsj/1542704621`,

`http://www.logic.univie.ac.at/~weinertt92/stick.pdf`,

respectively.

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SUBSPACES OF COUNTABLY COMPACT TOPOLOGICAL SPACES

Taras Banakh, Serhii Bardyla, and Alex Ravsky

Presented by Serhii Bardyla

We investigate subspaces of countably compact and ω-bounded topological spaces. A space X is called $\overline{\omega}$-*normal* if for any two disjoint closed subsets A, B of a closed separable subspace $Y \subset X$ there exist disjoint open sets U, V in X such that $A \subset U$ and $B \subset V$. We show that each Hausdorff ω-bounded space is $\overline{\omega}$-normal and each $\overline{\omega}$-normal space can be embedded into an ω-bounded Hausdorff space. We construct a consistent example of a regular separable scattered sequentially compact space which is not Tychonoff and hence can not be embedded into ω-bounded topological spaces. Separation axioms of subspaces of Hausdorff countably compact topological spaces were investigated. Also, we construct an example of a regular separable scattered topological space which can not be embedded into Urysohn countably compact topological spaces. Some open problems will be posed.

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Key words and phrases. Countably compact spaces, ω-bounded spaces.

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COMPACTIFIABLE CLASSES AND THEIR COMPLEXITY

Adam Bartoš

I will present several results from the joint work with J. Bobok, J. van Mill, P. Pyrih, and B. Vejnar [1], [2].

We say that a class of continua $\mathcal{C}$ is *compactifiable* if there is a metrizable compactum whose set of components is equivalent to $\mathcal{C}$. (We call two classes of spaces equivalent if every member of one class has a homeomorphic copy in the other class and vice versa.) It turns out that a class of continua $\mathcal{C}$ is compactifiable if and only if there is a continuous map $q\colon A \to B$ between some metrizable compacta A and B such that the family of fibers $\{q^{-1}(b) : b \in B\}$ is equivalent to $\mathcal{C}$. This condition may be easily generalized, so we define compactifiable classes of compacta in the obvious way. We also define *Polishable classes by a weaker condition* – it is enough if the witnessing spaces A and B are Polish. Moreover, we define *strongly compactifiable* and *strongly Polishable* classes by the extra requirement that the map q is closed and open. The motivation for these modified notions is their close connection to hyperspaces.

Since the Hilbert cube $[0,1]^\omega$ is universal for metrizable compacta, every class of compacta may be realized by an equivalent family $\mathcal{F} \subseteq \mathcal{K}([0,1]^\omega)$. Having such a realization, we may talk about its topological properties and about its complexity with respect to the Borel hierarchy. We have proved that a class of metrizable compacta $\mathcal{C}$ is strongly compactifiable if and only if it can be realized by a closed family or equivalently by an F_σ family, and that $\mathcal{C}$ is strongly Polishable if and only if it can be realized by a G_δ family or equivalently by an analytic family.

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Key words and phrases. Compactifiable class, Polishable class, homeomorphism equivalence, metrizable compactum, Polish space, hyperspace, complexity.

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ON THE QUASICELLULARITY OF A SPACE

Fortunata Aurora Basile and Nathan Carlson

Presented by Fortunata Aurora Basile

We define the quasicellularity $qc(X)$ of a space X with the property $wL(X) \leq qc(X) \leq c(X)$ for any space X. It is shown that $c(X) = qc(X)dot(X)$, decomposing $c(X)$ into two components, where $dot(X)$ is defined in [1]. Relationships between $qc(X)$ and other cardinal invariants are investigated. In particular we prove that $qc(X) = wL(X)$ for any extremally disconnected space. Cardinality bounds involving $qc(X)$ are given, including $|X| \leq \pi_\chi(X)^{qc(X)dot(X)\psi_c(X)}$ for a Hausdorff space X.

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Key words and phrases. Sets, ...
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ON $\varkappa$-METRISABLE AND STRATIFIABLE SPACES

Wojciech Bielas, Andrzej Kucharski, and Szymon Plewik

Presented by Wojciech Bielas

Among generalisations of metric there is one which resembles “the distance” between a point a regular closed subset. Such a distance (a function satisfying certain conditions) is called a $\varkappa$*-metric*. It is known that the existence of a $\varkappa$-metric can be characterised in terms of the existence of a system of approximations of regular open subsets. The existence of such a system for various classes of open subsets in spaces like the Sorgenfrey line, double arrow space, and the Niemytzki plane, will be discussed.

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NEW CONSISTENCY RESULTS ABOUT CARDINAL INVARIANTS ASSOCIATED WITH THE STRONG MEASURE ZERO IDEAL

Miguel A. Cardona

Yorioka [3] constructed a *matrix* of subsets of the reals, which gives a Tukey isomorphism between the σ-ideal of strong measure zero sets $\mathcal{SN}$ and $\langle \kappa^\kappa, \leq^* \rangle$, to prove that $\mathrm{cof}(\mathcal{SN}) = \mathfrak{d}_\kappa$ (the *dominating number on* κ^κ) whenever $\mathrm{add}(\mathcal{I}_f) = \mathrm{cof}(\mathcal{I}_f) = \kappa$ for all increasing f (the $\mathcal{I}_f$ are the *Yorioka ideals*).

In this talk we introduce a *suitable matrix* (see [1]) that generalizes Yorioka's matrix in some sense, and we construct a *suitable matrix* via a forcing matrix iterations of ccc posets to force

$$\mathrm{add}(\mathcal{SN}) = \mathrm{cov}(\mathcal{SN}) < \mathrm{non}(\mathcal{SN}) < \mathrm{cof}(\mathcal{SN}).$$

On the other hand, the speaker with Mejía and Rivera-Madrid [2] showed that, in Sacks model, $\mathrm{non}(\mathcal{SN}) < \mathrm{cov}(\mathcal{SN}) < \mathrm{cof}(\mathcal{SN})$. These are first results where 3 cardinal invariants associated with $\mathcal{SN}$ are pairwise different.

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Key words and phrases. Strong measure zero sets, cardinal invariants.
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THE UNIFORM BOUNDEDNESS PRINCIPLE FOR (ULTRA)FILTERS

Ben De Bondt and Hans Vernaeve

Presented by Ben De Bondt

Let X be an infinite dimensional Banach space. We formulate for each filter $\mathcal{F}$ on the set of natural numbers a corresponding uniform boundedness principle (UBP) in X, which can either hold true or fail, depending on the properties of the filter $\mathcal{F}$. For the Fréchet filter of cofinite sets, this UBP follows from the classic Banach-Steinhaus theorem. More surprisingly, it follows from a theorem by Benedikt [1] that this UBP holds as well for selective ultrafilters. We will discuss recent work together with Hans Vernaeve which gives a combinatorial characterization of those filters $\mathcal{F}$ for which this filter version of the uniform boundedness principle holds.

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Key words and phrases. Filters on ω, Banach spaces, uniform boundedness principle, . . .
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IF I WERE A RICH DENSITY

Rafał Filipów

Abstract upper densities are monotone and subadditive functions from the power set of positive integers into the unit real interval that generalize the upper densities used in number theory, including the upper asymptotic density, the upper Banach density, and the upper logarithmic density.

At the open problem session of the Workshop "Densities and their application", held at St. Étienne in July 2013, G. Grekos asked a question whether there is a "nice" abstract upper density, whose the family of null sets is precisely a given ideal of subsets of $\mathbb{N}$, where "nice" would mean the properties of the familiar densities consider in number theory.

In 2018, M. Di Nasso and R. Jin (Acta Arith. 185 (2018), no. 4) showed that the answer is positive for the summable ideals (for instance, the family of finite sets and the family of sequences whose series of reciprocals converge) when "nice" density means translation invariant and rich density (i.e. density which is *onto* the unit interval).

In my talk I show how to extend their result to all ideals with the Baire property. This extension was obtained jointly with Jacek Tryba.

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σ-LACUNARY ACTIONS OF POLISH GROUPS

Jan Grebik

We show that every essentially countable orbit equivalence relation induced by a continuous action of a Polish group on a Polish space is σ-lacunary. In combination with [2] we obtain a straightforward proof of the result from [1] that every essentially countable equivalence relation that is induced by the action of abelian non-archimedean Polish group is Borel reducible to $\mathbb{E}_0$, i.e., it is essentially hyperfinite.

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ON A SEMITOPOLOGICAL EXTENDED BICYCLIC SEMIGROUP WITH ADJOINED ZERO

Oleg Gutik and Kateryna Maksymyk

Presented by Oleg Gutik

The *bicyclic monoid* $\mathcal{C}(p,q)$ is the semigroup with the identity 1 generated by two elements p and q subject only to the condition $pq = 1$. The *extended bicyclic semigroup* $\mathcal{C}_{\mathbb{Z}}$ was introduced in [7] and it is a generalizations of the bicyclic monoid.

The following dichotomy for the bicyclic monoid with adjoined zero $\mathcal{C}^{\mathbf{0}} = \mathcal{C}(p,q) \sqcup \{\mathbf{0}\}$ was proved in [3]: every Hausdorff locally compact semitopological bicyclic monoid with adjoined zero $\mathcal{C}^{\mathbf{0}}$ is either compact or discrete. The above dichotomy was extended in [1] to locally compact λ-polycyclic semitopological monoids, in [2] to locally compact semitopological graph inverse semigroups in [5] to locally compact semitopological interassociates of the bicyclic monoid with an adjoined zero, to other generalizations of the bicyclic monoid with adjoined zero in [6], and they are extended in [4] to locally compact semitopological 0-bisimple inverse ω-semigroups with compact maximal subgroups.

We show that every Hausdorff locally compact semigroup topology on the extended bicyclic semigroup with zero $\mathcal{C}_{\mathbb{Z}}^{\mathbf{0}}$ is discrete, but on $\mathcal{C}_{\mathbb{Z}}^{\mathbf{0}}$ there exist $\mathfrak{c}$ many Hausdorff locally compact non-compact shift-continuous topologies. Also, we construct minimal shift-continuous, minimal semigroup and minimal inverse semigroup topologies on $\mathcal{C}_{\mathbb{Z}}^{\mathbf{0}}$ and establish their property.

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Key words and phrases. Semigroup, extended bicyclic semigroup, locally compact.
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SIZE-BASED LEVEL MEASURE AS A STANDARD LEVEL MEASURE

Lenka Halčinová and Jaroslav Šupina

Presented by Jaroslav Šupina

We show that size-based super level measure [3, 1] may be represented as a standard super level measure, i.e., the function $h_{\mu,f}(\alpha) = \mu(\{x \in X : f(x) > \alpha\})$. However, one has to consider different underlying space and minitive measure, see [2].

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Key words and phrases. size, level measure, non-additive measure.

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SMALL $\mathfrak{u}(\kappa)$ FOR A SINGULAR κ WITH COMPACTNESS AT κ^{++}

Radek Honzik

We study the question whether the compactness principles at λ^+ have a non-trivial effect on the generalized cardinal invariants in the neighbourhood of λ (for instance at λ or the cardinal predecessor of λ unless λ is a limit cardinal).

As a first result in this direction, we show that it is consistent that there is a singular strong limit cardinal κ (with countable or uncountable cofinality) such that $\mathfrak{u}(\kappa) = \kappa^+$, $2^\kappa > \kappa^+$, and the tree property, stationary reflection and the failure of approachability hold at κ^{++}. The proof is based on the methods of [1] and [2] for the small ultrafilter number $\mathfrak{u}(\kappa)$ and [3] for the tree property argument. The result will soon appear as a preprint, see [4].

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ON THE SETS WHICH CAN BE MOVED AWAY FROM THE SETS OF A CERTAIN FAMILY

Grażyna Horbaczewska and Sebastian Lindner

Presented by Sebastian Lindner

The strong measure zero sets (*smz*) have been characterized by Galvin, Mycielski and Solovay in the following way:

The set $A \subset \mathbb{R}$ is smz iff A can be translated away from any meager set on $\mathbb{R}$.

We prove that under Continuum Hypotesis there is also a symmetrical result:

The set $A \subset \mathbb{R}$ is meager iff A can be translated away from any smz set on $\mathbb{R}$.

Then we characterize the family of meager-additive sets in a similar way.

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NON "COMPLETE" CASE OF LOUVEAU–SIMPSON THEOREM

Joanna Jureczko

In 1982 A. Louveau and S. G. Simpson proved the essential theorem on functions having the Baire property from the Ellentuck space:

Theorem 1 ([4]). *Let X be a metric space and let $f\colon [\omega]^\omega \to X$ be such a mapping that the inverse image of any open set is completely Ramsey. Then there is an infinite subset T of ω such that $f([\omega]^\omega)$ is separable.*

Unfortuntely, their result was restricted to complete spaces. Our results show that this restriction can be erased, more precisely we showed that

Theorem 2 ([2]). *Let Y be a metric space and M be a non-completely Ramsey subset in Ellentuck space $[\omega]^\omega$. Let $f\colon M \to Y$ be such a mapping that $f^{-1}(U)$ is completely Ramsey for any open set $U \subset Y$. Then there is an infinite set $T \subset \omega$ such that $f([T]^\omega \cap M)$ is separable.*

This result is strictly connected with the following theorem

Theorem 3 ([2]). *Let M be a non-completely Ramsey subset in Ellentuck space. and let $\mathcal{F}$ be a partition of M into nowhere Ramsey sets. Then there is $\mathcal{F}' \subset \mathcal{F}$ such that $\bigcup \mathcal{F}'$ is not completely Ramsey.*

The research around partitions of structures onto "small" sets was started by K. Kuratowski in 1935 and continnued by L. Bukovský with using advanced set-theoretical methods (see [1]).

The methods used for proving Theorem 2 and Theorem 3 allows to get a dip generalization of Kuratowski–Ryll-Nardzewski Theorem on selectors (see e.g. [3, CH. XIV]) for Ellentuck structure and arbitrary metric space (not only Polish space as was originally).

This is the joint work with Ryszard Frankiewicz.

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Key words and phrases. Sets, Ellentuck topology, Baire property Kuratowski partition, separability, selectors.
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Set-theoretic methods in topology and real functions theory, dedicated to 80th birthday of Lev Bukovský
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ON COUNTABLY SATURATED LINEAR ORDERS AND GRAPHS

Ziemowit Kostana

A linear order L is countably saturated if for any countable subsets A, B of L, such that any element of A is less than any element of B, we can find an element of L between them. This obvious generalization of density corresponds to “countable saturation” from model theory. We'll say, that a countably saturated linear order L is prime, if every countably saturated linear order contains an isomorphic copy of L.

I'd like to present a characterization of the prime countably saturated linear order, and outline how it can be used to prove its uniqueness. Also, I will say something about related results concerning certain classes of uncountable graphs.

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Key words and phrases. linear orders, homogeneous structures, countable saturation.
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GAMES ON FINITELY GENERATED STRUCTURES

Adam Krawczyk and Wiesław Kubiś

Presented by Adam Krawczyk

We study the abstract Banach-Mazur game played with finitely generated structures instead of open sets. We characterize the existence of winning strategies aiming at a single countably generated structure. We also introduce the concept of *weak Fraïssé classes*, extending the classical Fraïssé theory, revealing its relations to our Banach-Mazur game.

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TOPOLOGICAL SPACES WITH THE FREESE-NATION PROPERTY

Andrzej Kucharski and Judyta Bąk

Presented by Andrzej Kucharski

The Freese-Nation property was introduced by R. Freese and J.B. Nation [1]. L. Heindorf and L.B. Shapiro [2] showed that a family of all clopen sets of 0-dimensional compact space X has the FNS property if and only if X is openly generated. We will present a proposal of properties related to Freese-Nation property and some relationship between these concepts.

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YET ANOTHER IDEAL VERSION OF THE BOUNDING NUMBER

Adam Kwela

This is a joint work with Rafał Filipów.

Let $\mathcal{I}$ be an ideal on ω. Denote by $\mathcal{D}_\mathcal{I}$ the family of all functions $f \in \omega^\omega$ satisfying $f^{-1}[\{n\}] \in \mathcal{I}$ for all $n \in \omega$. Given $f, g \in \omega^\omega$, write $g \leq_\mathcal{I} f$ whenever $\{n \in \omega : g(n) > f(n)\} \in \mathcal{I}$.

We study two closely connected ideal versions of the bounding number:

$$\mathfrak{b}(\mathcal{D}_{\mathrm{Fin}}, \geq_\mathcal{I}) = \min\left\{|\mathcal{F}| : \mathcal{F} \subseteq \mathcal{D}_{\mathrm{Fin}} \wedge \neg(\exists g \in \mathcal{D}_{\mathrm{Fin}}\, \forall f \in \mathcal{F}\, (g \leq_\mathcal{I} f))\right\};$$

$$\mathfrak{b}(\mathcal{D}_\mathcal{I}, \geq_\mathcal{I}) = \min\left\{|\mathcal{F}| : \mathcal{F} \subseteq \mathcal{D}_\mathcal{I} \wedge \neg(\exists g \in \mathcal{D}_\mathcal{I}\, \forall f \in \mathcal{F}\, (g \leq_\mathcal{I} f))\right\}.$$

It is known that $\mathfrak{b}(\mathcal{D}_{\mathrm{Fin}}, \geq_{\mathrm{Fin}}) = \mathfrak{b}$. We study those two invariants in the case of nice ideals (ideals with the Baire property, coanalytic ideals, P-ideals, etc.) as well as show some consistency results distinguishing $\mathfrak{b}$, $\mathfrak{b}(\mathcal{D}_{\mathrm{Fin}}, \geq_\mathcal{I})$ and $\mathfrak{b}(\mathcal{D}_\mathcal{I}, \geq_\mathcal{I})$.

Although the topic is interesting itself, we are also motivated by the studies of ideal versions of QN-spaces, as $\mathfrak{b}(\mathcal{D}_{\mathrm{Fin}}, \geq_\mathcal{I})$ and $\mathfrak{b}(\mathcal{D}_\mathcal{I}, \geq_\mathcal{I})$ describe uniformity numbers of such spaces. This topic is intensively studied by Lev Bukovský and his group.

Key words and phrases. Bounding number, ideals, QN-spaces.

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PATTERNS OF STATIONARY REFLECTION

Maxwell Levine

We will present an Easton-style result for stationary reflection. If S is a stationary subset of a cardinal κ, the reflection principle $\mathsf{SR}(S)$ asserts that every stationary subset of S reflects. It is known that $\mathsf{SR}(\kappa \cap \mathrm{cof}(\aleph_n))$ has the following trivial ZFC constraints: (1) $\mathsf{SR}(\kappa \cap \mathrm{cof}(\aleph_n))$ holds if and only if $\mathsf{SR}(\mathrm{cf}(\kappa) \cap \mathrm{cof}(\aleph_n))$ holds; (2) $\mathsf{SR}(\aleph_{n+1} \cap \mathrm{cof}(\aleph_n))$ fails; and of course (3) $\mathsf{SR}(\kappa \cap \mathrm{cof}(\aleph_n))$ holds vacuously if $\kappa \leq \aleph_n$. Assuming supercompact cardinals (which are necessary to make stationary reflection fail at successors of singulars), we prove that given a fixed $n < \omega$, these are the only ZFC constraints on $\mathsf{SR}(\kappa \cap \mathrm{cof}(\aleph_n))$.

This is joint work with Sy-David Friedman.

Key words and phrases. Large cardinals, forcing.

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ROSENTHAL FAMILIES AND THE CARDINAL INVARIANTS OF CICHOŃ'S DIAGRAM

Arturo Martínez-Celis

Joint work with Piotr Koszmider. A family $\mathcal{R} \subseteq [\omega]^\omega$ is Rosenthal if for every matrix $M = (m_{i,j})_{i,j\in\omega}$ of non-negative numbers such that for every $i \in \omega$, $\sum_{j\neq i} m_{i,j} \leq 1$, there is a $A \in \mathcal{R}$ such that for every $i \in A$, $\sum_{j\in A\setminus\{i\}} m_{i,j} \leq \frac{1}{2}$. H. Rosenthal proved that $[\omega]^\omega$ is a Rosenthal family. In this talk we will study the cardinal invariant $\mathfrak{ros}$, the smallest size of a Rosenthal family. We will mention some basic properties of these families which will lead us to compare $\mathfrak{ros}$ with the cardinal invariants related to the ideals of the Meager sets and the Null sets in the reals. Finally we will use a forcing argument to show that $\mathfrak{ros}$ is a cardinal invariant which is consistently different from the cardinal invariants in Cichoń's diagram.

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MYCIELSKI AMONG TREES – CATEGORY CASE

Marcin Michalski, Robert Rałowski, and Szymon Żeberski

Presented by Marcin Michalski

The 2-dimensional version of classical Mycielski Theorem says that every comeager or conull subset of $[0,1]^2$ contains a square of a form $P \times P$, where P is a perfect subset of $[0,1]$. We consider generalizations of this theorem by replacing a perfect square with a rectangle of the form $[T_1] \times [T_2]$, where $T_1 \subseteq T_2$ are trees of some type (perfect, uniformly perfect, Silver, Miller or Laver) and $[T]$ denotes a body of a tree T.

In this talk we will focus on the category case. In particular we will show that for every comeager G_δ set $G \subseteq \omega^\omega \times \omega^\omega$ there exists a Miller tree $M \subseteq \omega^{<\omega}$ and a uniformly perfect tree $P \subseteq M$ such that $[P] \times [T] \subseteq G$. We will also show that this result is somewhat optimal – we cannot replace P with a Silver tree or a Miller tree and no comeager G_δ set contains a square of bodies of Silver trees or Laver trees.

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MYCIELSKI AMONG TREES – NONSTANDARD PROOFS

Marcin Michalski, Robert Rałowski, and Szymon Żeberski

Presented by Robert Rałowski

The 2-dimensional version of classical Mycielski Theorem says that every comeager or conull subset of $[0,1]^2$ contains a square of a form $P \times P$, where P is a perfect subset of $[0,1]$. In this talk we present nonstandard proof by using Shoenfield absolutness theorem of the Mycielski Theorem for category case but replacing perfect set by the body of slalom perfect tree.

The Eggleston like Theorem says that if $\mathcal{N}$ is σ-ideal of null sets on the interval $[0,1]$ and $G \subseteq [0,1]^2$ has measure equal to 1, then there are perfect sets $P, Q \subseteq [0,1]$ such that $P \times Q \subseteq G$ and Q has measure 1. Analogously we can formulate Eggleston like theorem for category case. We present nostandard proof of Eggleston like theorem for the measure and category case.

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PROPERTIES OF LOCAL CLOSURE FUNCTIONS IN IDEAL TOPOLOGICAL SPACES

Anika Njamcul and Aleksandar Pavlović

Presented by Aleksandar Pavlović

A triple $\langle X, \tau, \mathcal{I} \rangle$, where $\langle X, \tau \rangle$ is a topological space and $\mathcal{I}$ an ideal on X is known as *ideal topological space*. In it, a local function for a set $A \subset X$, defined by $A^* = \{x \in X : A \cap U \notin \mathcal{I} \text{ for each } U \in \tau(x)\}$, is a generalization of topological closure (for more details see [2]). A. Al-Omari and T. Noiri [1] defined a generalization of θ-closure $\Gamma(A) = \{x \in X : A \cap \mathrm{Cl}(U) \notin \mathcal{I} \text{ for each } U \in \tau(x)\}$, called *local closure function*. We examine differences and similarities between those two functions depending on properties of the topological space and the ideal. We extend results published in [3] by results considering closure compatibility, idempotency of Γ and cases when $\Gamma(X) = X$.

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Key words and phrases. ideal, ideal topological spaces, local function, local closure function, θ-open set.

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TOPOLOGY EXPANSIONS VIA SPECIFIC IDEALS

Anika Njamcul and Aleksandar Pavlović

Presented by Anika Njamcul

For a topological space (X, τ), the local function $A^* = \{x \in X : A \cap U \notin \mathcal{I}$ for each $U \in \tau(x)\}$, where $\mathcal{I}$ is an ideal on X and A a subset of X, can be used to define an expansion τ^* of τ [1]. We describe the specific ideals which generate new topologies τ^* making certain sets open, while additionally preserving significant topological properties, i.e. regular open sets (as in [2]) or connectedness. Furthermore, we study other properties of the newly formed topology depending on the characteristics of the added sets.

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Key words and phrases. connectedness, ideal, ideal topological spaces, local function, preopen sets, submaximal space.

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TOPOLOGICAL GAMES AND SELECTION PROPERTIES OF HYPERSPACES

Alexander V. Osipov

Given a Hausdorff space X we denote by 2^X the family of all closed subsets of the space X.

In this report we continue to research relationships between closure-type properties of hyperspaces over a space X and covering properties of X. We investigate selectors for sequence of subsets of the space 2^X with the upper Fell topology ($\mathbf{F}^+$-topology) and the $\mathbf{Z}^+$-topology. Also we consider the topological games and the selection properties of the bitopological space $(2^X, \mathbf{F}^+, \mathbf{Z}^+)$.

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Key words and phrases. Hyperspaces, upper Fell topology, selection principles, $\mathbf{Z}^+$-topology, bitopological space, topological games.

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CLOSED HEREDITARY COREFLECTIVE SUBCATEGORIES IN CERTAIN CATEGORIES OF TOPOLOGICAL SPACES

Veronika Pitrová

Denote by **Tych** (**ZD**) the category of all Tychonoff (zero-dimensional) spaces. Let **A** be an epireflective subcategory of the category **Top** of all topological spaces such that $\mathbf{ZD} \subseteq \mathbf{A} \subseteq \mathbf{Tych}$. Our goal is to describe closed hereditary coreflective subcategories of **A**.

Let α be a regular cardinal. By $\mathbf{Top}(\alpha)$ we denote the subcategory of **Top** consisting of such spaces X that if $\mathcal{U}$ is a non-empty family of open subsets of X with $|\mathcal{U}| < \alpha$, then the intersection $\bigcap_{U\in\mathcal{U}} U$ is open in X. The subcategories $\mathbf{Top}(\alpha) \cap \mathbf{A}$ are closed hereditary and coreflective in **A**.

Let $C(\alpha)$ be the space on the set $\alpha \cup \{\alpha\}$ such that a subset U is open in $C(\alpha)$ if and only if $\alpha \notin U$ or $|\alpha \setminus U| < \alpha$. In [1] we showed that if MA (Martin's Axiom) holds and measurable cardinals do not exist, then the closed hereditary coreflective hull of the space $C(\omega_0)$ in **A** is the whole category **A**.

In our talk we show that if MA holds and measurable cardinals do not exist, then the closed hereditary coreflective hull of the space $C(\alpha)$ in **A** is $\mathbf{Top}(\alpha) \cap \mathbf{A}$ for any regular cardinal α. We obtain that if **B** is a closed hereditary coreflective subcategory of **A** such that $\mathbf{B} \neq \mathbf{Top}(\alpha) \cap \mathbf{A}$ and $\mathbf{B} \neq \mathbf{Dis}$ (the category of all discrete spaces), then **B** consists only of sums of connected spaces. Hence, the only closed hereditary coreflective subcategories of **ZD** are **Dis** and $\mathbf{Top}(\alpha)\cap\mathbf{ZD}$, where α is a regular cardinal.

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Key words and phrases. Closed hereditary subcategory, coreflective subcategory, epireflective subcategory.
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ON l-EQUIVALENCE AND THE MENGER PROPERTY

Masami Sakai

For a Tychonoff space X, we denote by $C_p(X)$ the space of all real valued continuous functions with the topology of pointwise convergence. We remark that if $C_p(X)$ and $C_p(Y)$ are linearly homeomorphic and X is first countable and Menger (resp., Hurewicz), then Y is also Menger (resp., Hurewicz).

Key words and phrases. Menger; projectively Menger; Hurewicz; projectively Hurewicz; l-equivalence; l-invariant; function space.

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DIVISIBILITY IN βN AND ${}^*N$

Boris Šobot

We investigate a quasiorder $\widetilde{\mid}$ on the set βN of ultrafilters on the set N of natural numbers that is a natural extension of the divisibility relation on N. The "lower" ultrafilters of the $\widetilde{\mid}$-hierarchy are nicely organized in ω-many levels, resembling divisibility on N. Above these levels the situation is more complex. The connection of βN with nonstandard extensions ${}^*N$ of N via monads of ultrafilters proves to be useful in finding out more about the relation $\widetilde{\mid}$, as it turns out that two ultrafilters are divisible if and only if there are divisible hypernatural numbers in their respective monads. Some recent results on this connection by Di Nasso and Luperi Baglini provide another useful tool.

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CONVERGENCE OF MEASURES ON MINIMALLY GENERATED BOOLEAN ALGEBRAS

Damian Sobota

Minimally generated Boolean algebras were introduced and investigated by Koppelberg in 1989. Later on they were intensively used and studied in the context of the Efimov problem (Geschke, Dow and Pichardo-Mendoza, Dow and Shelah), measure theory (Borodulin-Nadzieja) or both (Džamonja and Plebanek). During my talk we will follow the latter way.

It is already a folklore fact that each minimally generated Boolean algebra $\mathcal{A}$ admits a sequence of signed finite measures (μ_n) for which the integrals $\int_{St(\mathcal{A})} f\,d\mu_n$ converge to 0 for every continuous real-valued function f on the Stone space $St(\mathcal{A})$ but not for every bounded Borel real-valued function (i.e. $\mathcal{A}$ does not have the Grothendieck property). We will show that this fact can always be witnessed even by a sequence of measures being a finite linear combination of point measures. Assuming CH, we will also provide an example of a minimally generated Boolean algebra $\mathcal{A}$ having the property that for every sequence of measures (μ_n) on $\mathcal{A}$ such that the integrals $\int_{St(\mathcal{A})} 1_A\,d\mu_n$ $(= \mu_n(A))$ converge to 0 for every $A \in \mathcal{A}$, the integrals $\int_{St(\mathcal{A})} f\,d\mu_n$ converge to 0 for every $f \in C(St(\mathcal{A}))$, too (i.e. $\mathcal{A}$ has the Nikodym property). The Stone space of such a minimally generated Boolean algebra must necessarily be an Efimov space (but we will show that under CH the converse does not hold). The existence of $\mathcal{A}$ has important measure-theoretic consequences.

This is a joint work with Lyubomyr Zdomskyy.

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Set-theoretic methods in topology and real functions theory, dedicated to 80th birthday of Lev Bukovský
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CARDINAL SEQUENCES AND UNIVERSAL SPACES

Lajos Soukup

If X is a locally compact, scattered Hausdorff (in short: LCS) space, we let $CS(X)$ denote the cardinal sequence of X, i.e. the sequence of the cardinalities of the Cantor Bendixson levels of X.

If α is an ordinal, let $\mathcal{C}(\alpha)$ denote the class of all cardinal sequences of LCS spaces of height α and put

$$\mathcal{C}_\lambda(\alpha) = \{s \in \mathcal{C}(\alpha) : s(0) = \lambda \wedge \forall \beta < \alpha \ s(\beta) \geq \lambda\}.$$

Given a family $\mathcal{C}$ of sequences of cardinals we say that an LCS space X is *universal for* $\mathcal{C}$ if $CS(X) \in \mathcal{C}$, and for each $s \in \mathcal{C}$ there is an open subspace $Y \subset X$ with $CS(Y) = s$.

Constructing universal spaces we will prove theorems claiming that certain $\mathcal{C}_\lambda(\alpha)$-classes are quite rich in elements. For example, we can prove the following generalization of a classical result of Baumgartner and Shelah:

Theorem (Martinez and S. [3])**.** *It is consistent, that 2^ω is as large as you wish and for each $\delta < \omega_3$ we have*

$$\{f \in {}^{\delta}([\omega, 2^\omega] \cap Card) : f(\alpha) = \omega \text{ whenever } \alpha = 0 \text{ or } \mathrm{cf}(\alpha) = \omega_2\} \subset \mathcal{C}_\omega(\delta).$$

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Key words and phrases. scattered spaces, locally compact, cardinal sequence, univesal space, consistency result.

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CLUB STATIONARY REFLECTION AND THE TREE PROPERTY

Šárka Stejskalová

It is known that the consistency of the tree property and the stationary reflection at $\aleph_2$ can be obtained from a weakly compact cardinal. More precisely, if λ is weakly compact, then forcing with the Mitchell poset $\mathbb{M}(\aleph_0, \lambda)$ produces a model in which $\lambda = \aleph_2$, there are no $\aleph_2$-Aronszajn trees and every stationary subset of $\aleph_2 \cap \mathrm{cof}(\aleph_0)$ reflects at a point of $\aleph_1$.

We improve this result by showing that it is consistent with the tree property that every stationary subset S of $\aleph_2 \cap \mathrm{cof}(\aleph_0)$ reflects in a stronger sense, i.e. there is a club $C \subseteq \aleph_2$ such that every point of C of cofinality $\aleph_1$ is a reflection point for S (this stronger form of reflection was introduced by [2]). We use the optimal consistency assumption of a weakly compact cardinal.

Our method is based on forcing with an iteration of club shooting posets after forcing with the Mitchell forcing, and hence it represents a progress with regard to the more general question of which kinds of forcing preserve the tree property. The result will appear soon [1].

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Key words and phrases. Tree property; Stationary reflection.
The work was supported by FWF-GAČR grant Compactness principles and combinatorics, 19-29633L.
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GENERALIZED TOWERS AND PRODUCTS

Piotr Szewczak and Magdalena Włudecka

Presented by Piotr Szewczak

Let X be a set of reals and $\mathrm{C}_p(X)$ be the set of all continuous real-valued functions on X with the pointwise convergence topology. By the result of Gerlits and Nagy [1] the space $\mathrm{C}_p(X)$ has the Fréchet–Urysohn property (a generalization of first-countability) if and only if the set X is a γ-set, i.e., has a combinatorial covering property. The existence of uncountable γ-sets of reals is independent of ZFC. Tsaban proved [3] that sets with some special combinatorial structure are γ-sets. We generalize this class of sets and prove that their products have the property γ. We also show that for every set X from our class and every gamma set Y, the product space $X \times Y$ have a strong property weaker than the property γ. These investigations are motivated by the result of Miller, Tsaban and Zdomskyy [2] that under CH, there are two γ-sets whose product space is not even Menger (in particular it is not γ).

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LINEARLY SENSITIVE PROPERTIES

Małgorzata Terepeta

A function f is called *linearly sensitive with respect to the property (or condition)* (P) if f has property (P) and for any $\alpha \neq 0$ the function $f + \alpha \cdot \mathrm{id}$ has not the property (P). We remind Mazurkiewicz construction of a function linearly sensitive with respect to (N)-Lusin condition and examine linearly sensitivity with respect to strong Świątkowski property and Świątkowski property.

The results are obtained with Artur Bartoszewicz and Małgorzata Filipczak.

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2010 *Mathematics Subject Classification.* Primary: 26A30; Secondary: 26A15, 26A21.

Key words and phrases. (N)-Lusin property, Darboux property, Świątkowski property.

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IDEAL CONVERGENCE AND MATRIX SUMMABILITY

Jacek Tryba

These results were obtained jointly with Rafał Filipów.

We examine relationship between ideal convergence and matrix summability in the realm of bounded and unbounded sequences. We present the Problem 5 from The Scottish Book [3], stated by Mazur, that can be described as "is the notion of statistical convergence of bounded sequences equivalent to some matrix summability method?"

We investigate the claims written by Mazur in the book that would lead to the negative answer to that problem and present the results of Khan and Orhan [2], which gave us a positive answer to the Problem 5 from The Scottish Book.

We also examine when ideal convergence is equal to either intersection or union of some matrix summability methods. In particular, we solve a problem posed by Gogola, Mačaj and Visnyai [1].

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Key words and phrases. Sets, ideals, ideal convergence, matrix summability.

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STRUCTURE OF THE RK-ORDER OF P-POINTS

Jonathan Verner

Our work starts with the following question, posed by A. Blass:

Question 1 ([1]). What ordinals can be embedded into the RK-ordering of P-points?

Of course, for this question to make any sense, one needs to assume some axiom which guarantees that there are (sufficiently many) P-points. Typically, axioms such as *MA* or *CH* are assumed. Since the RK-order is witnessed by functions from ω to ω it is immediately obvious that every ultrafilter can have at most $\mathfrak{c}$-many RK-predecessors and that the largest ordinal one can hope to embed is $\mathfrak{c}^+$. The following result of A. Blass shows that $\mathfrak{c}$ is possible:

Theorem 1 ([2]). *Assume MA. The ordinal $\mathfrak{c}$ can be embedded into the RK-ordering of P-points.*

The next step is given by the following theorem of D. Raghavan and S. Shelah whose immediate corollary is that, under MA every $\alpha < \mathfrak{c}^+$ embeds into the RK-ordering of P-points.

Theorem 2 ([4]). *Assume MA. The ordering $\mathcal{P}(\omega)/fin$ can be embedded into the RK-ordering of P-points.*

B. Kuzeljević and D. Raghavan ([3]) were then able use CH together with a complicated construction to embed $\mathfrak{c}^+$ into the RK-order of (rapid) P-points. Finally, the D. Raghavan together with the author have shown that the ordering of rapid P-points is, in fact, strongly closed:

Theorem 3 ([5]). *Assume CH. The RK-order of rapid P-points is upwards $\mathfrak{c}^+$-closed!*

We aim to present a complementary theorem due to B. Kuzeljević, D. Raghavan and the author, showing that the ordering is also strongly closed in the other direction:

Theorem 4. *Assume MA. The RK-order below a $P_{\mathfrak{c}}$-point is $\mathfrak{c}$-closed.*

Key words and phrases. Ultrafilters; Rudin-Keisler order; P-points.

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ON $\mathcal{S}$-APPROXIMATELY CONTINUOUS FUNCTIONS

Renata Wiertelak

It is well known that a point $x_0 \in \mathbb{R}$ is a density point of a Lebesgue measurable set A if

$$\lim_{h\to 0^+} \frac{\lambda(A \cap [x_0 - h, x_0 + h])}{2h} = 1,$$

where $\lambda(A)$ denote the Lebesgue measure of a measurable set A.

In the paper [1] is presented generalization of notion of density point. By $\mathcal{S}$ we will denote a **sequence** of sets with positive Lebesgue measure $\{S_n\}_{n\in\mathbb{N}}$ **tending to zero**, that means $\operatorname{diam}\{S_n \cup \{0\}\} \underset{n\to\infty}{\longrightarrow} 0$.

We shall say that a point $x_0 \in \mathbb{R}$ is a **$\mathcal{S}$-density point** of a set $A \in \mathcal{L}$, if

$$\lim_{n\to\infty} \frac{\lambda(A \cap (S_n + x_0))}{\lambda(S_n)} = 1.$$

Let

$$\Phi_{\mathcal{S}}(A) = \{x \in \mathbb{R} : x \text{ is a } \mathcal{S}\text{-density point of } A\},$$
$$\mathcal{T}_S = \{A \in \mathcal{L} : A \subset \Phi_{\mathcal{S}}(A)\}.$$

Then family $\mathcal{T}_S$ contains topology natural topology.

For sequnce $\mathcal{S}$ of measurable sets tending to zero we consider four families of continuous functions defined as follows:

$$\mathcal{C}_{nat,nat} = \{f : (\mathbb{R}, \mathcal{T}_{nat}) \to (\mathbb{R}, \mathcal{T}_{nat})\},$$
$$\mathcal{C}_{nat,\mathcal{S}} = \{f : (\mathbb{R}, \mathcal{T}_{nat}) \to (\mathbb{R}, \mathcal{T}_S)\},$$
$$\mathcal{C}_{\mathcal{S},nat} = \{f : (\mathbb{R}, \mathcal{T}_S) \to (\mathbb{R}, \mathcal{T}_{nat})\},$$
$$\mathcal{C}_{\mathcal{S},\mathcal{S}} = \{f : (\mathbb{R}, \mathcal{T}_S) \to (\mathbb{R}, \mathcal{T}_S)\}.$$

The aim of the presentation are the properties of continuous functions equipped with the $\mathcal{S}$-density topology or natural topology in the domain or the range.

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2010 *Mathematics Subject Classification.* 54A10, 26A15, 54A20.

Key words and phrases. density topologies, $\mathcal{S}$-density topologies, approximately continuous functions, $\mathcal{S}$-approximately continuous functions.

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QN-SPACES AND COVERING PROPERTIES OF HUREWICZ

Lyubomyr Zdomskyy

The talk will be devoted to the following result motivated by the previous work of Bukovský, Fremlin, Haleš, Scheepers, and others:

Theorem (Tsaban-Z. 2007)**.** *A set of reals X is a QN space if, and only if, each Borel image of X in ω^ω is bounded.*

Having all Borel images in ω^ω bounded is equivalent to the Hurewicz covering property for countable Borel covers. We shall also discuss the relation of QN-spaces with other variants of the Hurewicz property.

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Key words and phrases. QN-space, spaces of functions, Hurewicz property.
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NONMEASURABLE UNIONS WITH RESPECT TO TREE IDEALS

Szymon Żeberski and Robert Rałowski

Presented by Szymon Żeberski

Let T be a tree $T \subseteq \omega^{<\omega}$. Body of T is $[T] = \{x \in \omega^\omega : \forall n\ x|n \in T\}$.

A tree T is called

- *perfect* or *Sacks* if $(\forall \sigma \in T)(\exists \tau \in T)(\sigma \subseteq \tau \wedge (\exists n \neq m)(\tau^\frown n, \tau^\frown m \in T)$;
- *superperfect* or *Miller* if $(\forall \sigma \in T)(\exists \tau \in T)(\sigma \subseteq \tau \wedge (\exists^\infty n)(\tau^\frown n \in T)$;
- *Laver* if there is a node $s \in T$ such that, for every node $t \in T$ if $s \subseteq t$ then t is infinitely spliting i.e. $\{n \in \omega : s^\frown n \in T\}$ is infinite.

Let $\mathcal{T}$ be a family of trees. We say that $A \in P(\omega^\omega)$ is in t_0 iff

$$(\forall P \in \mathcal{T})(\exists Q \in \mathcal{T})\ Q \subseteq P \wedge [Q] \cap A = \emptyset.$$

We say that $A \in P(\omega^\omega)$ is t-measurable iff

$$(\forall P \in \mathcal{T})(\exists Q \in \mathcal{T})\ Q \subseteq P \wedge ([Q] \subseteq A \vee [Q] \cap A = \emptyset).$$

The first result is connected to s_0 ideal i.e. a classic Marczewski ideal. Let $\mathcal{A} \subseteq s_0$ be point-finite family of subsets of ω^ω such that $\bigcup \mathcal{A} \notin s_0$. Then there is a subfamily $\mathcal{A}' \subseteq \mathcal{A}$ such that $\bigcup \mathcal{A}'$ is not s-measurable.

Analogous result is true in the case of Miller ideal m_0 and m-measurability.

We also show that it is relatively consistent with ZFC that there is ω_1-point family $\mathcal{A} \subseteq s_0 \cap l_0 \cap m_0$ such that $\bigcup \mathcal{A} = \omega^\omega$ and union of any subfamily of $\mathcal{A}$ is I-measurable where $I \in \{s_0, l_0, m_0\}$.

Key words and phrases. Marczewski ideal, Sacks tree, Miller tree, Laver tree, measurable set.

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