Sufficient conditions for the existence of some nonoscillatory solutions of third-order nonlinear differential equations

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Abstract. The aim of this paper is to study the asymptotic behavior of solutions of nonlinear differential equations of the third-order with quasiderivatives. In particular, we state the sufficient conditions ensuring the existence of some nonoscillatory solutions with a specified asymptotic property as $t$ tends to infinity. The basic tool used in proving our results is the classical Banach contraction mapping principle.

1. Introduction

This paper is concerned with the asymptotic behavior of solutions of the third-order nonlinear differential equations with quasiderivatives of the form

$$\left( \frac{1}{p(t)} \left( \frac{1}{r(t)} x'(t) \right) \right)' + q(t)f(x(t)) = 0, \quad t \geq a \tag{N}$$

where

$$r, p, q \in C([a, \infty), \mathbb{R}), \quad r(t) > 0, \quad p(t) > 0, \quad q(t) > 0 \text{ on } [a, \infty),$$

$$f \in C(\mathbb{R}, \mathbb{R}), \quad f(u)u > 0 \text{ for } u \neq 0.$$

For the sake of convenience, we introduce the following notation

$$x[0] = x, \quad x[1] = \frac{1}{r} x', \quad x[2] = \frac{1}{p} \left( \frac{1}{r} x' \right)' = \frac{1}{p} \left( x[1] \right)' = \frac{1}{p} \left( x[0]' \right)' = x[3]' = \left( x[2] \right)'.$$

These functions $x[i], i = 0, 1, 2, 3$, we call the quasiderivatives of $x$.

By a solution of an equation of the form (N), we mean a function $w : [a, \infty) \to \mathbb{R}$ such that quasiderivatives $w[i](t), 0 \leq i \leq 3$ exist and are continuous on the interval $[a, \infty)$ and it satisfies the equation (N) for all $t \geq a$. A solution $w$ of equation (N) is said to be proper if it satisfies the following condition

$$\sup \{ |w(s)| : t \leq s < \infty \} > 0 \text{ for any } t \geq a.$$

A proper solution is said to be oscillatory if it has a sequence of zeros converging to $\infty$; otherwise it is said to be nonoscillatory.

Fixed point theorems are important tools in the oscillation and nonoscillation theory of ordinary differential equations. Especially, when one proves the existence of nonoscillatory solutions with a specified asymptotic behavior as $t$ tends
to infinity. We refer to the books [1, 13] for various interesting results on this subject and fairly comprehensive bibliography of the earlier work. Now, we state the Banach contraction mapping principle in a shortened form that we will need later. For a complete statement of this fixed point result, we refer for example to the recent monograph [4].

**Theorem 1.1** (Banach fixed point theorem). Any contraction mapping of a complete non-empty metric space $\mathcal{M}$ into $\mathcal{M}$ has a unique fixed point in $\mathcal{M}$.

Let $\mathcal{N}(N)$ denote the set of all proper nonoscillatory solutions of equation $(N)$. The set $\mathcal{N}(N)$ can be divided into the following four classes in the same way as in [5, 6, 17]:

- $\mathcal{N}_0 = \{ x \in \mathcal{N}(N), \exists t_x : x(t)x^{(1)}(t) < 0, x(t)x^{(2)}(t) > 0 \text{ for } t \geq t_x \}$
- $\mathcal{N}_1 = \{ x \in \mathcal{N}(N), \exists t_x : x(t)x^{(1)}(t) > 0, x(t)x^{(2)}(t) < 0 \text{ for } t \geq t_x \}$
- $\mathcal{N}_2 = \{ x \in \mathcal{N}(N), \exists t_x : x(t)x^{(1)}(t) > 0, x(t)x^{(2)}(t) > 0 \text{ for } t \geq t_x \}$
- $\mathcal{N}_3 = \{ x \in \mathcal{N}(N), \exists t_x : x(t)x^{(1)}(t) < 0, x(t)x^{(2)}(t) < 0 \text{ for } t \geq t_x \}$

The object of our interest are nonoscillatory solutions of equation $(N)$ in the classes $\mathcal{N}_0$ and $\mathcal{N}_3$. Therefore, with respect to asymptotic behavior of the solutions in these classes, we can divide the class $\mathcal{N}_0$ into the following two disjoint subclasses

$$\mathcal{N}_0^B = \left\{ x \in \mathcal{N}_0 : \lim_{t \to \infty} x(t) = l_x \neq 0 \right\}, \quad \mathcal{N}_0^0 = \left\{ x \in \mathcal{N}_0 : \lim_{t \to \infty} x(t) = 0 \right\}$$

$$\left[ \mathcal{N}_3^B = \left\{ x \in \mathcal{N}_3 : \lim_{t \to \infty} x(t) = l_x \neq 0 \right\}, \quad \mathcal{N}_3^0 = \left\{ x \in \mathcal{N}_3 : \lim_{t \to \infty} x(t) = 0 \right\} \right].$$

Various types of differential equations (without or with deviating argument) of the third-order have been subject of intensive studying in the literature. There are many results devoted to the classification, existence, oscillatory and asymptotic properties of solutions of these differential equations. Among the extensive literature on this topic, we mention here [2, 3, 5, 6, 17] for the differential equations without deviating argument and [9, 10, 11, 15, 16, 20, 21, 22] for those with deviating argument. For other papers devoted to the study of differential, integral and integro-differential equations by means of Banach contraction mapping principle, we also refer to [8, 14, 19].

The aim of this paper is to continue the study of existence and asymptotic behavior of nonoscillatory solutions of equation $(N)$. We state the sufficient conditions guaranteeing the existence of nonoscillatory solutions in the classes $\mathcal{N}_0$, $\mathcal{N}_3$ with given asymptotic behavior as $t \to \infty$ (see the subclasses introduced above).

Our research is based on a study of the asymptotic properties of nonoscillatory solutions as well as on a topological approach via the Banach fixed point theorem. We mention here that the existence of nonoscillatory solutions of $(N)$ in the classes $\mathcal{N}_1$, $\mathcal{N}_2$ has been largely investigated in the paper [18]. Hence, we complement the results given there. Moreover, our results complement and extend some other results that have been stated in [7] and [12], respectively. Several examples illustrating presented theorems are also provided.
Finally, we introduce the following notation:

\[ I(u_i) = \int_a^\infty u_i(t) \, dt, \quad I(u_i, u_j) = \int_a^\infty u_i(t) \int_a^t u_j(s) \, ds \, dt, \quad i, j = 1, 2 \]

\[ I(u_i, u_j, u_k) = \int_a^\infty u_i(t) \int_a^t u_j(s) \int_a^s u_k(z) \, dz \, ds \, dt, \quad i, j, k = 1, 2, 3, \]

where \( u_i, i = 1, 2, 3 \) are continuous positive functions on the interval \([a, \infty)\).

2. MAIN RESULTS

We start our investigation with the results regarding the asymptotic behavior of nonoscillatory solutions of equation (N) in the class \( N^3 \). The following result provides the sufficient condition for the existence of solutions in the class \( N^3 \).

**Theorem 2.2.** Let \( I(r, p, q) < \infty \) and assume that function \( f \) satisfies Lipschitz condition on some interval \([c, d]\) where \( c, d \) are constants such that \( 0 < c < d \). Then equation (N) has a solution \( x \) in the class \( N^3 \) such that \( \lim_{t \to \infty} x(t) \neq 0 \), i.e. \( N^3 \neq \emptyset \).

**Proof.** In the following, we prove the existence of a positive solution of equation (N) in the class \( N^3 \) which approaches to positive constant as \( t \to \infty \).

Let \( L \) denote Lipschitz constant of function \( f \) on the interval \([c, d]\) and \( K = \max \{ f(u) : u \in [c, d] \} \). Further, let \( t_0 \geq a \) be such that

\[ \int_{t_0}^{\infty} r(\tau) \int_{t_0}^{\tau} p(s) \int_{t_0}^{s} \min \{ d - c, \frac{1}{K}, \frac{1}{L+1} \} \, dz \, ds \, d\tau \leq \min \{ d - c, \frac{1}{K}, \frac{1}{L+1} \}. \tag{2.1} \]

Let us define the set

\[ \Delta = \{ u \in C([t_0, \infty), \mathbb{R}) : c \leq u(t) \leq d \}, \]

where \( C([t_0, \infty), \mathbb{R}) \) will denote the Banach space of all continuous and bounded functions defined on the interval \([t_0, \infty)\) with the sup norm \( \| u \| = \sup \{ |u(t)|, t \geq t_0 \} \). It is obvious that \( \Delta \) is a non-empty closed subset of \( C([t_0, \infty), \mathbb{R}) \) and so \( \Delta \) is a non-empty complete metric space. For every \( u \in \Delta \) we consider a mapping \( T_1 : \Delta \to C([t_0, \infty), \mathbb{R}) \) given by

\[ x_u(t) = (T_1u)(t) = c + \int_{t_0}^{\infty} r(\tau) \int_{t_0}^{\tau} p(s) \int_{t_0}^{s} q(z) f(u(z)) \, dz \, ds \, d\tau, \quad t \geq t_0. \]

We prove that \( T_1 \) maps \( \Delta \) into itself and \( T_1 \) is a contraction mapping in \( \Delta \) in order to apply to the mapping \( T_1 \) the Banach fixed point theorem (Theorem 1.1).

\( T_1 \) maps \( \Delta \) into \( \Delta \). Really, \( x_u(t) \geq c \) and in view of (2.1), we obtain

\[ x_u(t) = c + \int_{t_0}^{\infty} r(\tau) \int_{t_0}^{\tau} p(s) \int_{t_0}^{s} q(z) f(u(z)) \, dz \, ds \, d\tau \]

\[ \leq c + \int_{t_0}^{\infty} r(\tau) \int_{t_0}^{\tau} p(s) \int_{t_0}^{s} q(z) f(u(z)) \, dz \, ds \, d\tau \]

\[ \leq c + K \int_{t_0}^{\infty} r(\tau) \int_{t_0}^{\tau} p(s) \int_{t_0}^{s} q(z) \, dz \, ds \, d\tau \leq d. \]
Now, let $u_1, u_2 \in \Delta$ and $t \geq t_0$. Taking into account the inequality (2.1) and the fact that function $f$ satisfies Lipschitz condition on the interval $[c, d]$, we have the following

$$|T_1 u_1(t) - T_1 u_2(t)| \leq \int_t^\infty r(\tau) \int_{t_0}^\tau p(s) \int_{t_0}^s q(z) |f(u_1(z)) - f(u_2(z))| \, dz \, ds \, d\tau$$

$$\leq \int_t^\infty r(\tau) \int_{t_0}^\tau p(s) \int_{t_0}^s q(z) |u_1(z) - u_2(z)| \, dz \, ds \, d\tau$$

$$\leq L \int_t^\infty r(\tau) \int_{t_0}^\tau p(s) \int_{t_0}^s q(z) \, dz \, ds \, d\tau$$

$$\leq \frac{L}{L + 1} \|u_1 - u_2\| = Q_1 \|u_1 - u_2\|$$

These inequalities immediately implies that for every $u_1, u_2 \in \Delta$

$$\|T_1 u_1 - T_1 u_2\| \leq Q_1 \|u_1 - u_2\| \quad \text{where } 0 < Q_1 < 1.$$

Hence, we proved that $T_1$ is a contraction mapping in $\Delta$. Consequently, the Banach fixed point theorem yields the existence of the unique fixed point $x \in \Delta$ such that

$$x(t) = c + \int_t^\infty r(\tau) \int_{t_0}^\tau p(s) \int_{t_0}^s q(z) f(x(z)) \, dz \, ds \, d\tau, \quad t \geq t_0.$$

As

$$x^{[1]}(t) = - \int_{t_0}^t p(s) \int_{t_0}^s q(z) f(x(z)) \, dz \, ds < 0$$

and

$$x^{[2]}(t) = - \int_{t_0}^t q(z) f(x(z)) \, dz < 0,$$

it is clear that $x$ is a positive solution of the equation (N) in the class $N_3$ which approaches to positive constant as $t \to \infty$, i.e. $x \in N_3^3$. \qed

Now, we prove the sufficient condition for the existence of solutions of equation (N) in the class $N_3^3$.

**Theorem 2.3.** Let $I(r, p) < \infty$, $I(q) < \infty$ and assume that function $f$ satisfies Lipschitz condition on the interval $[0; 2I(r, p)]$. Then equation (N) has a solution $x$ in the class $N_3$ such that $\lim_{t \to \infty} x(t) = 0$, i.e. $N_3^3 \neq \emptyset$.

**Proof.** We prove the existence of a positive solution of equation (N) in the class $N_3$ which approaches to zero as $t \to \infty$.

Let $L$ denote Lipschitz constant of function $f$ on the interval $[0; 2I(r, p)]$. Further, let $t_0 \geq a$ be such that

$$\int_{t_0}^\infty r(\tau) \int_{t_0}^\tau p(s) \, ds \, d\tau \leq \frac{1}{L + 1}$$

(2.2)
and

\[(2.3) \quad \int_{t_0}^{\infty} q(t) \, dt \leq \min \left\{ \frac{1}{K}, 1 \right\}, \]

where

\[ K = \max \left\{ f(u) : u \in \left[ 0; 2 \int_{t_0}^{\infty} r(\tau) \int_{t_0}^{\tau} p(s) \, ds \, d\tau \right] \right\}. \]

For the sake of convenience, we introduce the following notation

\[ H(t) = \int_{t}^{\infty} r(\tau) \int_{t_0}^{\tau} p(s) \, ds \, d\tau, \quad t \geq t_0. \]

Let us define the set

\[ \Delta = \{ u \in C([t_0, \infty), \mathbb{R}) : H(t) \leq u(t) \leq 2H(t) \}, \]

where \( C([t_0, \infty), \mathbb{R}) \) denotes the Banach space of all continuous and bounded functions defined on the interval \([t_0, \infty)\) with the sup norm \( \| u \| = \sup\{|u(t)|, \, t \geq t_0\} \). Clearly, \( \Delta \) is a non-empty closed subset of \( C([t_0, \infty), \mathbb{R}) \) and so \( \Delta \) is a non-empty complete metric space. For every \( u \in \Delta \) we consider a mapping \( T_2 : \Delta \to C([t_0, \infty), \mathbb{R}) \) given by

\[ x_u(t) = (T_2u)(t) = H(t) + \int_{t}^{\infty} r(\tau) \int_{t_0}^{\tau} p(s) \int_{t_0}^{s} q(z) f(u(z)) \, dz \, ds \, d\tau, \quad t \geq t_0. \]

In order to apply to the mapping \( T_2 \) the Banach fixed point theorem (Theorem 1.1), it is sufficient to prove that \( T_2 \) maps \( \Delta \) into itself and \( T_2 \) is a contraction mapping in \( \Delta \).

\( T_2 \) maps \( \Delta \) into \( \Delta \). In fact, \( x_u(t) \geq H(t) \) and in view of (2.3), we have

\[ x_u(t) = H(t) + \int_{t}^{\infty} r(\tau) \int_{t_0}^{\tau} p(s) \int_{t_0}^{s} q(z) f(u(z)) \, dz \, ds \, d\tau \]
\[ \leq H(t) + K \int_{t}^{\infty} r(\tau) \int_{t_0}^{\tau} p(s) \int_{t_0}^{s} q(z) \, dz \, ds \, d\tau \]
\[ \leq H(t) + K \left( \int_{t_0}^{\infty} q(z) \, dz \right) \left( \int_{t}^{\infty} r(\tau) \int_{t_0}^{\tau} p(s) \, ds \, d\tau \right) \]
\[ \leq H(t) + H(t) = 2H(t). \]

Now, let \( u_1, u_2 \in \Delta \) and \( t \geq t_0 \). Then, the fact that function \( f \) satisfies Lipschitz condition on the interval \([0; 2I(r, p)]\) and the inequalities (2.2) and (2.3), give the
Thus, we proved that the Banach theorem there exists the unique fixed point following

\[ x \in \text{the set} \]

\[ \text{satisfies Lipschitz condition on interval } [c, d] \text{ where } c, d \text{ are constants such that } 0 < c < d. \text{ Then equation (N) has a solution } x(t) \text{ in the class } N_0 \text{ which approaches to zero as } t \to \infty, \text{i.e. } x \in N_0^B. \]

In the sequel, we consider the solutions of equation (N) in the class \( N_0 \). We give here the sufficient conditions that guarantee the existence of solutions in the classes \( N_0^B \) and \( N_0^B \). The following theorems hold.

**Theorem 2.4.** Let \( I(q, p, r) < \infty \) and assume that function \( f \) satisfies Lipschitz condition on some interval \([c, d]\) where \( c, d \) are constants such that \( 0 < c < d \). Then equation (N) has a solution \( x(t) \) in the class \( N_0 \) such that \( \lim_{t \to \infty} x(t) \neq 0 \), i.e. \( N_0^B \neq \emptyset \).

**Proof.** We prove the existence of a positive solution of equation (N) in the class \( N_0 \) which approaches to positive constant as \( t \to \infty \).

Let \( L \) denote Lipschitz constant of function \( f \) on the interval \([c, d]\) and \( K = \max\{f(u) : u \in [c, d]\} \). Further, let \( t_0 \geq a \) be such that

\[
\int_{t_0}^{\infty} q(z) \int_{t_0}^{z} p(s) \int_{t_0}^{s} r(\tau) d\tau ds dz \leq \min\left\{ \frac{d - c}{K}, \frac{1}{L+1} \right\}.
\]

Let us define the set \( \Delta \) in the same way as in the proof of Theorem 2.2. For every \( u \in \Delta \) we consider a mapping \( T_3 : \Delta \to C([t_0, \infty), \mathbb{R}) \) given by

\[
x_u(t) = (T_3u)(t) = c + \int_{t}^{\infty} r(\tau) \int_{\tau}^{\infty} p(s) \int_{s}^{\infty} q(z) f(u(z)) dz d\tau, \quad t \geq t_0.
\]

Taking into account the inequality (2.4) and the fact that function \( f \) satisfies Lipschitz condition on interval \([c, d]\) and using similar arguments as in the proof of Theorem 2.2, it is easy to verify that \( T_3 \) maps \( \Delta \) into itself and \( T_3 \) is a contraction mapping in \( \Delta \).
mapping in $\Delta$. Consequently, the Banach theorem ensures the existence of the unique fixed point $x \in \Delta$ such that

$$x(t) = c + \int_t^\infty r(\tau) \int_\tau^\infty p(s) \int_s^\infty q(z) f(x(z)) \, dz \, ds \, d\tau, \quad t \geq t_0.$$  

It is clear that $x$ is a positive solution of the equation $(N)$ in the class $N_0$ which approaches to positive constant as $t \to \infty$, i.e. $x \in N_0^\infty$.  

**Theorem 2.5.** Let $I(r) < \infty$, $I(q, p) < \infty$ and assume that function $f$ satisfies Lipschitz condition on the interval $[0; 2I(r)]$. Then equation $(N)$ has a solution $x$ in the class $N_0$ such that $\lim_{t \to \infty} x(t) = 0$, i.e. $N_0^\infty \neq \emptyset$.

**Proof.** We prove the existence of a positive solution of equation $(N)$ in the class $N_0$ which approaches to zero as $t \to \infty$.  

Let $L$ denote Lipschitz constant of function $f$ on the interval $[0; 2I(r)]$. Further, let $t_0 \geq a$ be such that

\begin{equation}
\int_{t_0}^\infty r(t) \, dt \leq \frac{1}{L + 1}
\end{equation}

and

\begin{equation}
\int_{t_0}^\infty q(s) \int_{t_0}^s p(\tau) \, d\tau \, ds \leq \min \left\{ \frac{1}{R}, 1 \right\},
\end{equation}

where

$$K = \max \left\{ f(u) : u \in [0; 2\int_{t_0}^\infty r(t) \, dt] \right\}.$$  

For the sake of convenience, we introduce the following notation

$$H(t) = \int_t^\infty r(\tau) \, d\tau, \quad t \geq t_0.$$  

Let us define the set

$$\Delta = \{ u \in C([t_0, \infty), \mathbb{R}) : H(t) \leq u(t) \leq 2H(t) \},$$

where $C([t_0, \infty), \mathbb{R})$ denotes the Banach space of all continuous and bounded functions defined on the interval $[t_0, \infty)$ with the sup norm $\|u\| = \sup\{|u(t)|, t \geq t_0\}$. Clearly, $\Delta$ is a non-empty closed subset of $C([t_0, \infty), \mathbb{R})$ and so $\Delta$ is a non-empty complete metric space. For every $u \in \Delta$ we consider a mapping $T_4 : \Delta \to C([t_0, \infty), \mathbb{R})$ given by

$$x_u(t) = (T_4u)(t) = H(t) + \int_t^\infty r(\tau) \int_\tau^\infty p(s) \int_s^\infty q(z) f(u(z)) \, dz \, ds \, d\tau, \quad t \geq t_0.$$  

By easy computation, we obtain the following inequality:

\begin{equation}
\int_t^\infty r(\tau) \int_\tau^\infty p(s) \int_s^\infty q(z) \, dz \, ds \, d\tau \leq \int_{t_0}^\infty q(z) \int_{t_0}^z p(s) \, ds \, dz \cdot \int_t^\infty r(\tau) \, d\tau
\end{equation}

Taking into account the inequalities (2.5), (2.6) and (2.7) and the fact that function $f$ satisfies Lipschitz condition on interval $[0; 2I(r)]$ and using similar arguments as in the proof of Theorem 2.3, we again verify that $T_4$ maps $\Delta$ into itself and $T_4$ is
a contraction mapping in $\Delta$. Now, the Banach fixed point theorem can be applied to the mapping $T_4$. Hence, there exists the unique fixed point $x \in \Delta$ such that

$$x(t) = H(t) + \int_0^t r(\tau) \int_\tau^\infty p(s) \int_s^\infty q(z) f(x(z)) \, dz \, ds \, d\tau, \quad t \geq t_0.$$ 

It is clear that $x$ is a positive solution of the equation (N) in the class $N_0$ which approaches to zero as $t \to \infty$, i.e. $x \in N_0^0$. □

The following examples illustrate the meaning of our results.

Example 2.1. We consider the differential equation

$$\left( \frac{1}{t^2} \left( t^7 x'(t) \right)' \right)' + \frac{24t^6}{(2t^2 + 3)^3} x^3(t) = 0, \quad t \geq 3.$$ 

This is the equation of the form (N), where $r(t) = \frac{1}{t^7}$, $p(t) = t^2$, $q(t) = \frac{24t^6}{(2t^2 + 3)^3}$, and $f(u) = u^3$. It is easy to verify that the integral $I(r, p, q)$ is convergent and the function $f$ satisfies Lipschitz condition on the interval $[2, 3]$ with the Lipschitz constant $L = 27$. Hence, Theorem 2.2 gives that the equation (2.8) has a solution in the class $N_0^B$. One such solution is the function $x(t) = \frac{2t^2 + 3}{t}$.

Example 2.2. The following differential equation of the form (N)

$$\left( \frac{1}{t} \left( \frac{1}{t^2} x'(t) \right)' \right)' + \frac{24}{t^2(t + 1)^2} x^2(t) \text{sgn} x(t) = 0, \quad t \geq 2$$

satisfies all the assumptions of Theorem 2.4. Therefore, the equation (2.9) has a solution in the class $N_0^B$. In fact, one such solution is the function $x(t) = \frac{t + 1}{t}$.

Remark 2.1. Theorem 2.2 (Theorem 2.3) is still valid if instead of the assumption that function $f$ satisfies Lipschitz condition on an interval $[c, d]$ where $c$, $d$ are constants such that $0 < c < d$ (function $f$ satisfies Lipschitz condition on the interval $[0; 2I(r, p)]$, we will require that function $f$ satisfies Lipschitz condition on an interval $[d_1, c_1]$ where $c_1$, $d_1$ are constants such that $d_1 < c_1 < 0$ (function $f$ satisfies Lipschitz condition on the interval $[-2I(r, p); 0]$). In Theorem 2.2 (Theorem 2.3), taking into account this assumption and using similar arguments, we can prove the existence of a negative solution of equation (N) in the class $N_0^B$ which approaches to negative constant (to zero) as $t \to \infty$. Moreover, we observe that this fact about a negative solution also holds for Theorems 2.4, 2.5.

Remark 2.2. Similar investigation of the asymptotic behavior of solutions of nonlinear differential equations of the second order with quasiderivatives has already been completed by the authors in [7]. Our results also complement and extend some other ones that have been published in [18] and [12], respectively. We refer the reader to the books [1, 9, 13] and to the references contained therein for other interesting results on this topic.

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