

Diferenciálne rovnice 1.rádu

1. $2xyy' + x = -y^2 \quad \dots \quad xy^2 + \frac{x^2}{2} = C$
2. $1 - 2x - y^2 y' = 0 \quad \dots \quad y = \sqrt[3]{3(x - x^2) + C}$
3. $y' = 2 \left(\frac{y+2}{x+y-1} \right)^2 \quad \dots \quad (y+2)e^{2\operatorname{arctg} \frac{y+2}{x-3}} = C$
4. $x + y y' = 0 \quad \dots \quad x^2 + y^2 = C$
5. $xy - x^2 y' = 2a^2$, kde $a \in R \quad \dots \quad y = \frac{a^2}{x} + Cx$
6. Cauchyho úloha: $\frac{x}{1+y} - \frac{y}{1+x} y' = 0$, $y(0) = 1 \quad \dots \quad 2(x^3 - y^3) + 3(x^2 - y^2) + 5 = 0$
7. $(x-2)y' = \frac{3y}{x+1} \quad \dots \quad y = C \cdot \frac{x-2}{x+1}$
8. Cauchyho úloha: $y' + y \cos x - \sin x \cdot \cos x = 0$, $y(0) = 1 \quad \dots \quad y = \sin x - 1 + 2e^{-\sin x}$
9. Cauchyho úloha: $y' = \frac{xy}{x^2 + y^2}$, $y(\sqrt{e}) = \sqrt{e} \quad \dots \quad \frac{x^2}{2y^2} = \ln |y|$
10. $\frac{x}{x^2 + y^2} dy + \left(1 - \frac{y}{x^2 + y^2} \right) dx = 0 \quad \dots \quad \operatorname{arctg} \left(\frac{y}{x} \right) + x = C$
11. Cauchyho úloha: $xy' + y - y^2 = 0$, $y(1) = \frac{1}{2} \quad \dots \quad y = \frac{1}{1+x}$
12. $y' = \frac{1}{x-y} + 1 \quad \dots \quad (x-y)^2 = -2x + C$
13. Cauchyho úloha: $y(2xy - 1)dx + xdy = 0$, $y(1) = 1 \quad \dots \quad y = \frac{1}{x}$
14. $y - 2x + 1 + (y - 2x)y' = 0 \quad \dots \quad \frac{1}{3}(2x - y) + \frac{1}{9} \ln |6x - 3y - 1| - x = C$ a $6x - 3y - 1 = 0$
15. $y' = \sqrt{4x + 2y + 1} \quad \dots \quad \sqrt{4x + 2y + 1} - 2 \ln \left(\sqrt{4x + 2y + 1} + 2 \right) = x + C$
16. $y' - y \operatorname{tg} x = y^4 \cos x \quad \dots \quad y^{-3} = -3 \sin x \cos^2 x + C \cos^3 x$ a $y = 0$
17. $(x-y)dy = (x+y)dx \quad \dots \quad 2 \operatorname{arctg} \frac{y}{x} = \ln(x^2 + y^2) + C$
18. $(y^2 - 3x^2)y' + 2xy = 0 \quad \dots \quad \frac{x}{y} - \frac{x^3}{y^3} = x \cdot C$ a $y = 0$
19. $y' = \frac{2x + y - 1}{4x + 2y + 5} \quad \dots \quad 10y - 5x + 7 \ln |10x + 5y + 9| = C$ a $10x + 5y + 9 = 0$
20. $10^x dx - 10^{-y} dy = 0 \quad \dots \quad y = -\log_{10}(C - 10^x)$
21. $xy' - \frac{x^2}{y} - y = 0 \quad \dots \quad y^2 = x^2(2 \ln |x| + C)$
22. $y' = x - 2xy \quad \dots \quad y = \frac{1 - Ce^{-x^2}}{2}$
23. $(xy' - 1) \ln x = 2y \quad \dots \quad y = -\ln x + C \ln^2 x$

24. $(2x - 3)y' = -3x^2 - 2y \quad \dots \quad y = \frac{C - x^3}{2x - 3}$
25. $y' + y - x\sqrt{y} = 0 \quad \dots \quad y = (Ce^{-\frac{x}{2}} + x - C)^2 \text{ a } y = 0$
26. $(6x^4 + y)dx - xdy = 0 \quad \dots \quad y = x(2x^3 - C)$
27. $y' = \frac{-2x + 4y - 6}{x + y - 3} \quad \dots \quad (y - x - 1)^2 = C(2x - y)^3 \text{ pre } C \neq 0, y = x + 1 \text{ a } y = 2x$
28. $x^2(y - x)y' + 1 - x^2y = 0 \quad \dots \quad \frac{y^2}{2} - xy - \frac{1}{x} = C$
29. $y^2 - xy - x^2y' = 0 \quad \dots \quad y = \frac{2x}{1 - Cx^2} \text{ a } y = 0$
30. $2y - x^3y' = 0 \quad \dots \quad y = C \cdot e^{-\frac{1}{x^2}}$
31. $x - y + xy' = 0 \quad \dots \quad y = x \left(\ln \frac{1}{|x|} + C \right)$
32. $(y^3 + y \cos x)dx + 2xy^2dy = 0 \quad \dots \quad xy^2 + \sin x = C \text{ a } y = 0$
33. $xy^2(xy' + y) = 1 \quad \dots \quad y = \sqrt[3]{\frac{3}{2} \cdot \frac{1}{x} + \frac{C}{x^3}}$
34. $xy + e^x - xy' = 0 \quad \dots \quad y = e^x (\ln|x| + C)$
35. $e^x \sin^3 y + (1 + e^{2x}) \cos y \cdot y' = 0 \quad \dots \quad \operatorname{arctg} e^x + \frac{1}{2 \sin^2 y} = C$
36. !!! $y' + \frac{x}{1 - x^2} y = x\sqrt{y} \quad \dots \quad y = \left[-\frac{1}{3}(1 - x^2) + C\sqrt[4]{1 - x^2} \right]^2 \text{ pre } x \in (-1, 1),$
 $y = \left[\frac{1}{3}(x^2 - 1) + C\sqrt[4]{x^2 - 1} \right]^2 \text{ pre } x \in (-\infty, -1) \text{ alebo } x \in (1, \infty) \text{ a } y = 0$
37. $xy' + y = x \cdot \sin x \quad \dots \quad y = \frac{1}{x} (\sin x - x \cdot \cos x + C)$
38. $(1 + e^x)y \cdot y' = e^x \quad \dots \quad y^2 = 2 \ln(1 + e^x) + C$
39. $xy' = y(1 - \ln x + \ln y) \quad \dots \quad y = x \cdot e^{x \cdot C}$
40. $x^2(y - x)y' + 1 - x^2y = 0 \quad \dots \quad \frac{y^2}{2} - xy - \frac{1}{x} = C$