DIFERENCIÁLNE ROVNICE | PF UPJŠ ÚMV/DFR/10 | Semester 5/1

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Sada úloh na precvičenie III

Séria úloh 1: Transformujte rovnice do polárnych súradníc.

(a) $\dfrac{\mathrm{d}x}{\mathrm{d}t} = y + kx(x^2 + y^2), \quad \dfrac{\mathrm{d}y}{\mathrm{d}t} = -x + ky(x^2 + y^2)$

(b) $(xy' - y)^2 = 2xy(1 + y'^2)$

(c) $(x^2 + y^2)^2 y'' = (x + yy')^3$

(d) $x\dfrac{\partial u}{\partial y} - y\dfrac{\partial u}{\partial x} = 0$

(e) $\left(\dfrac{\partial u}{\partial y}\right)^2 + \left(\dfrac{\partial u}{\partial x}\right)^2 = 0$

(f) $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 0$

Séria úloh 2: Zavedením nových nezávislých premenných transformujte rovnice.

(a) $x^4y'' + xyy' - 2y^2 = 0$, ak je $x = \mathrm{e}^t,\ y = u\mathrm{e}^{2t}$, kde $u = u(t)$

(b) $y'' + (x+y)(1+y')^3 = 0$, ak je $x = u + t,\ y = u - t$, kde $u = u(t)$

(c) $\left(x\dfrac{\partial z}{\partial x}\right)^2 + \left(y\dfrac{\partial z}{\partial y}\right)^2 = r^2\dfrac{\partial z}{\partial x}\dfrac{\partial z}{\partial y}$, ak je $x = u\mathrm{e}^w,\ y = v\mathrm{e}^w,\ z = w\mathrm{e}^w$

(d) $x\dfrac{\partial z}{\partial x} + y\dfrac{\partial z}{\partial y} = \dfrac{x}{z}$, ak je $u = 2x - z^2,\ v = \dfrac{y}{z}$

(e) $\dfrac{\partial^2 z}{\partial x^2} + \dfrac{\partial^2 z}{\partial y^2} = 0$, ak je $u = \dfrac{x}{x^2+y^2},\ v = -\dfrac{y}{x^2+y^2}$

(f) $\dfrac{\partial^2 z}{\partial x^2} + 2\dfrac{\partial^2 z}{\partial x \partial y} + \dfrac{\partial^2 z}{\partial y^2} = 0$, ak je $u = x + y,\ v = x - y,\ w = xy - z$

Séria úloh 3: Zavedením nových nezávislých premenných riešte rovnice.

(a) $z_x = z_y$, ak je $u = x + y,\ v = x - y$

(b) $y\,z_x = x\,z_y$, ak je $u = x,\ v = x^2 + y^2$

(c) $x\,z_x + y\,z_y = z$, ak je $u = x,\ v = \dfrac{y}{x}$

(d) $(z_y)^2\,z_{xx} - 2z_xz_yz_{xy} + (z_x)^2z_{yy} = 0$, ak je $x = u,\ y = v,\ z = w\ \ (u = u(v,w))$

Séria úloh 4: Nájdite všeobecné riešenie LDR

(a) $(1-t^2)\ddot{x} - 2t\dot{x} + 2x = 0$

(b) $(2t+1)\ddot{x} + (4t-2)\dot{x} - 8x = 0$

(c) $\sin^2(t)\ddot{x} - 2x = 0$

(d) $\ddot{x} - \dfrac{2t}{1+t^2}\dot{x} + \dfrac{2}{1+t^2}x = (1+t^2)\mathrm{e}^t$

(e) $\ddot{x} + \dfrac{t}{1-t}\dot{x} - \dfrac{1}{1-t}x = t - 1$

(f) $\ddot{x} + \dfrac{1}{t^2\ln t}x = \mathrm{e}^t\left(\dfrac{2}{t} + \ln t\right)$

Séria úloh 5: Nájdite všeobecné riešenie LDR

(a) $x'' - 2tx' + t^2x = 0$

(b) $2tx'' + x' - 2x = 0$

(c) $(1+t^2)^2x'' + 2t(1+t^2)x' + x$

(d) $x'' - x' + \mathrm{e}^{2t}x = 0$

(e) $t^4x'' + 2t^3x' - 4x = \dfrac{1}{t}$

(f) $t^2x'' + tx' + \left(t^2 + \dfrac{1}{4}\right)x = 0$

(g) $tx'' + 2x' - tx = \mathrm{e}^t$

(h) $x'' + \dfrac{2}{t}x' - a^2x = 2$

(i) $t^4x'' + k^2x = 0$

Séria úloh 6: Nájdite všeobecné riešenie LDR

(a) $y'' = 2yy'$

(b) $yy'' = y'$

(c) $y'' = (1 + (y')^2)^{\frac{3}{2}}$

(d) $(1 + (y')^2)y''' = 3y'(y'')^2$

(e) $2(y')^2 = (y-1)y''$

(f) $y'' = (y')^2y$

(g) $yy'' = y'y''$

(h) $y'' + y'\cos x - y\sin x = 0$

Séria úloh 7: Nájdite prvý integrál a načrtnite fázový portrét sústavy dvoch diferenciálnych rovníc.

(a) $x' = x + y^2, \quad y' = 2y$

(b) $x' = x(x^2+y^2), \quad y' = y(x^2+y^2)$

(c) $x' = y, \quad y' = x - x^3$

(d) $x' = x^2y, \quad y' = x(1+y)$

(e) $x' = -3x + 5y, \quad y' = -2x + 3y$

(f) $x' = -x - y, \quad y' = x - y$

Séria úloh 8: Nájdite aspoň dva nezávislé prvé integrály sústavy troch diferenciálnych rovníc.

(a) $x' = y - z, \quad y' = z - x, \quad z' = x - y$

(b) $x' = y + z, \quad y' = z + x, \quad z' = x + y$

(c) $x' = y - x, \quad y' = y + z + x, \quad z' = x - y$

(d) $x' = y(1 + z), \quad y' = -x(z + 1), \quad z' = -2xy$

(e) $x' = y(z - 1), \quad y' = -x(1 + z), \quad z' = -2xy$

(f) $x' = y + 2zy + y^2 + 2zy^2, \quad y' = -2x(1 + 2z), \quad z' = -2x(1 + 2y)$

(g) $Ap' = (B - C)qr, \quad Bq' = (C - A)rp, \quad Cr' = (A - B)pq, \quad A \geq B \geq C > 0$

(h) $x' = z^2 - y^2, \quad y' = z, \quad z' = -y$

(i) $x' = xz, \quad y' = yz, \quad z' = xy\sqrt{z^2 + 1}$

(j) $x' = x + z^2 + y^2, \quad y' = y, \quad z' = z$

(k) $x' = x^3 + 3xy^2, \quad y' = 2y^3, \quad z' = 2y^2 z$

(l) $x' = xy, \quad y' = -y\sqrt{1 - y^2}, \quad z' = z\sqrt{1 - y^2} - ax^2$

Séria úloh 9: Nájdite všeobecné riešeniev PDR prvého rádu.

(a) $(1 + x^2)u_x + u_y = 0$

(b) $x\, u_x + y\, u_y = 0$

(c) $y^2\, u_x + ax^2\, u_y = 0, \quad a \neq 0$

(d) $u_x + u_y + u_z = 0$

(e) $a\, u_x + by\, u_y + cz\, u_z = 0$

Séria úloh 10: Riešte PDR prvého rádu s danými podmienkami.

(a) $u_t - 3u_x = 0, \quad u(x,0) = \mathrm{e}^{-x^2}$ $\qquad u(x,t) = \mathrm{e}^{-(x+3t)^2}$

(b) $u_t + 3u_x = 0, \quad u(x,0) = \sin x$ $\qquad u(x,t) = \sin(x - 3t)$

(c) $\sqrt{1-x^2}u_x + u_y = 0, \quad u(0,y) = y$ $\qquad u(x,y) = y - \arcsin x$

(d) $u_t + t^\alpha u_x = 0, \ \alpha > -1, \quad u(x,0) = \phi(x)$ $\qquad u(x,t) = \phi\left(x - \frac{t^{\alpha+1}}{\alpha+1}\right)$

(e) $u_t + 2u_x - 3u = 0, \quad u(x,0) = \dfrac{1}{1+x^2}$ $\qquad u(x,t) = \dfrac{\mathrm{e}^{-3t}}{1+4t^2-4tx+x^2}$

Séria úloh 11: Riešte kvázilineárne PDR prvého rádu s danými podmienkami.

(a) $u_y + u\,u_x = -\dfrac{1}{2u}, \quad u(x,0) = \sin x$ $\qquad x = \arcsin\sqrt{u^2+y} - \dfrac{2(u^2+y)^{\frac{3}{2}}}{3} + \dfrac{2u^3}{3}$

(b) $u_y + u\,u_x = -u, \quad u(x,0) = -\dfrac{x}{2}$ $\qquad u(x,y) = -\dfrac{x}{\mathrm{e}^y + 1}$

(c) $u(x+u)\,u_x - y(y+u)\,u_y = 0, \quad u(1,y) = \sqrt{y}$ $\qquad u(x,y) = \sqrt{xy}$

(d) $u\,u_x + uy = 1, \quad u(\tau,\tau) = \dfrac{\tau}{2}, \quad \tau \in (0,1)$ $\qquad u(x,y) = \dfrac{x-y}{y-2} + \dfrac{y}{2}$

(e) $x\,u_x + xy\,u_y = \mathrm{e}^{-u}, \quad u = 1$ na krivke $y = x^2$ $\qquad u = x\mathrm{e}^{\mathrm{e}-\mathrm{e}^u} - 2(\mathrm{e} - \mathrm{e}^u - \ln x) - x + \ln y$

(f) $(x^2 + 3y^2 + 3u^2)u_x - 2xy\,u_y + 2xu = 0$ $\qquad F\left(\frac{u}{y}, x^2y + y^3 + u^2y\right) = 0$