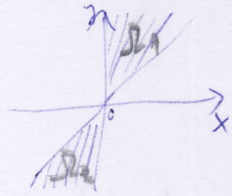
Pr: $$xy' = 3y - 2x - 2\sqrt{xy - x^2}$$

$\Omega := \{(x,y)\in\mathbb{R}^2 \mid x(y-x)\ge 0\} = \Omega_1 \cup \Omega_2$

graf riešenia musí ležať v Ω



$Q(x,y) = x$

$P(x,y) = 3y - 2x - 2\sqrt{xy-x^2}$

pre $t>0$ $P(tx,ty) = t(3y-2x) - |t|\cdot 2\sqrt{xy-x^2}$

$= t\,P(x,y)$

pre $t<0$, $t=-\mu>0$

$P(-\mu x,-\mu y) = -\mu(3y-2x) - |\mu|\cdot 2\sqrt{xy-x^2}$

$\neq -\mu\,P(x,y)$!!!

pozitívne homog. fcia st. 1

Každopádne použ. subst. $y = zx \Rightarrow y' = z'x + z$

Čiže $x(z'x+z) = 3zx - 2x - 2\sqrt{x^2 z - x^2} \Leftrightarrow z'x^2 = 2zx - 2x - 2\sqrt{x^2(z-1)}$

$\Leftrightarrow z'x^2 = 2x(z-1) - 2|x|\sqrt{z-1}$ nech $z\neq 1 \wedge z\neq 2$

pre $x>0 \Rightarrow \dfrac{dz}{2\sqrt{z-1}\left(\sqrt{z-1}-1\right)} = \dfrac{dx}{x} \Leftrightarrow \ln\left|\sqrt{z-1}-1\right| = \ln x + \ln c$, $c>0$

$\Rightarrow \sqrt{z-1} = 1 + kx$, $k\neq 0$, $x>0$

$\Rightarrow z-1 = (1+kx)^2 \Rightarrow y = x\left[1+(1+kx)^2\right]$

$x>0$, $k\neq 0$

pre $x<0$ podobne dostaneme $y = x\left[1+(1+mx)^2\right]$, $m\in\mathbb{R}\setminus\{0\}$, $x<0$

$z=2 \Rightarrow y = 2x$, čo je na hranici Ω $n=0$

$z=1$ je sing. riešenie $\Rightarrow y = x$

SPOLU $\boxed{y(x) = x\left[1+(1+nx)^2\right],\ n\in\mathbb{R},\ x\in\mathbb{R}}$, lebo

ak $x\geq 0 \Rightarrow y-x\geq 0$ $\Rightarrow 1+nx\geq 0$ a naopak

pre $n\in\mathbb{R}, x\in\mathbb{R}$

$x\cdot\left[1+(1+nx)^2 + 2x(1+nx)n\right] = 3x\left[1+(1+nx)^2\right] - 2x - 2\sqrt{x^2(1+nx)^2}$

$2x^2(1+nx)n = 2x\left[1+(1+nx)^2\right] - 2x - 2|x|\cdot|1+nx|$

$2x\left[nx(1+nx) + (1+nx)\right] = 2x\left[1+nx\right]^2$ ✓