

① Zintegrujme rovnici $y'' y = y'$
 pre $y > 0$ ($y < 0$) $y'' = \frac{y'}{y} \Leftrightarrow y'' - \frac{y'}{y} = 0$
 $\Leftrightarrow \frac{d}{dx} (y' - \ln|y|) = 0 \Leftrightarrow y' - \ln|y| = K, K \in \mathbb{R}$

$\Leftrightarrow y' = \ln|cy|, c > 0$

$\Leftrightarrow \int \frac{dy}{\ln|cy|} = x + B, B \in \mathbb{R}, c > 0$
 (nedá sa najst prim. v tr. elem. fcii)
 (iba špec. fcia exp. integrál Ei)

② $(y''')^2 - y' y'''' = \left(\frac{y'}{x}\right)^2, x > 0$ ($x < 0$)

$y' = z \Rightarrow (z')^2 - z \cdot z'' = \left(\frac{z}{x}\right)^2 \quad (z \neq 0) \Leftrightarrow \frac{(z')^2 - z z''}{z^2} = \frac{1}{x^2}$

$\Leftrightarrow -\frac{d}{dx} \left(\frac{z'}{z}\right) = \frac{1}{x^2}$

$\Leftrightarrow -\frac{z'}{z} = c + \left(\frac{-1}{x}\right)$ (separovat.) $\Leftrightarrow z(x) = K x e^{-cx}, K > 0$
 $c \in \mathbb{R}$

$y(x) = K \int x e^{-cx} dx = -\frac{K}{c} e^{-cx} \left(x + \frac{1}{c}\right) + D, D \in \mathbb{R}$

③ ~~$\sin^2(t) \ddot{x} - 2x = 0$~~

$2t \ddot{x} + \dot{x} - 2x = 0, t > 0$

transf: $x(t) = \psi(\tau) \Rightarrow \dot{x} = x' \cdot \dot{\tau}, \ddot{x} = x'' (\dot{\tau})^2 + x' \ddot{\tau} \quad \left[\frac{d}{dt} = \frac{d}{d\tau}\right]$

$\Rightarrow 2t [x'' (\dot{\tau})^2 + x' \ddot{\tau}] + x' \dot{\tau} - 2x = 0$

$x' [\dot{\tau} + 2t \ddot{\tau}] + 2t x'' (\dot{\tau})^2 - 2x = 0$

$\Rightarrow \psi = c_1 + c_2 t^{1/2} \Rightarrow 2t \cdot \left(\frac{1}{2}\right)^2 \cdot t^{-1/2} \cdot x'' = 2x$
 $c_1 = 0, c_2 = 1 \Rightarrow \left(\frac{1}{2}\right)^2 x'' - 2x = 0$

~~$x(t) = \frac{c_1}{2t^2} + c_2 t^2$~~
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$$X(\tau) = c_1 e^{-2\tau} + c_2 e^{2\tau}$$

$$X(t) = c_1 e^{-2\sqrt{t}} + c_2 e^{2\sqrt{t}}$$

①

Einsteigende Funktion

die 200 (200)

$$= (f'(t) - f''(t)) \frac{1}{x^2}$$

$$= f'(t) - f''(t) = 0$$

(für die hier exp. integriert ist)
 (in der nächsten Aufgabe mit exp. integrieren)
 (hier in der nächsten Aufgabe mit exp. integrieren)

②

$$\left(\frac{1}{x} \right)' = -\frac{1}{x^2}$$

$$\frac{1}{x} = \int -\frac{1}{x^2} dx = \frac{1}{x} + C$$

$$\frac{1}{x} = \frac{1}{x} + C$$

$$\Rightarrow -\frac{1}{x^2} = C + \left(\frac{1}{x} \right)' \quad (\text{Ebenenwert}) \Rightarrow 5x/x = K \times 5$$

$$Kx = Kx - Cx = -\frac{K}{5} \Rightarrow x = \frac{1}{5} + D, \quad D \in \mathbb{R}$$

③

~~$$2x^2 + 3x - 5 = 0$$~~

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$$x = x, \quad \dot{x} = \dot{x}, \quad \ddot{x} = \ddot{x}$$

$$0 = x^2 - \dot{x}^2 + (\ddot{x}^2 + \dot{x}^2 - 5x) = 0$$

$$x^2 + \dot{x}^2 + \ddot{x}^2 + 5x = 0$$

$$x^2 = \frac{1}{5} \Rightarrow x = \frac{1}{\sqrt{5}}$$

1/5
1/5