① Zintegrujme rovnicu $y'' y = y'$

pre $y>0$ $(y<0)$ $\quad y'' = \frac{y'}{y} \iff y'' - \frac{y'}{y} = 0$

$\iff \frac{d}{dx}\left(y' - \ln|y| \right) = 0 \iff y' - \ln|y| = K \quad | K \in \mathbb{R}$

$\iff y' = \ln|cy| \quad | c>0$

$\iff \int \frac{dy}{\ln|cy|} = x + B \quad | B \in \mathbb{R} \; | c>0$

(nedá sa nájsť prim. v tr. elem. fcií)
(iba špec. fcia exp. integrál E_i)

② $|y''|^2 - y' y''' = \left(\frac{y'}{x}\right)^2 \quad | \quad x>0 \; (x<0)$

$y' = z \Rightarrow |z'|^2 - z \cdot z'' = \left(\frac{z}{x}\right)^2 \overset{(z \neq 0)}{\iff} \frac{|z'|^2 - z z''}{z^2} = \frac{1}{x^2}$

$\iff -\frac{d}{dx}\left(\frac{z'}{z}\right) = \frac{1}{x^2}$

$\iff -\frac{z'}{z} = c + \left(\frac{-1}{x}\right)$ (separovat.) $\iff z(x) = K x e^{-cx} \quad | K>0$

$c \in \mathbb{R}$

$y(x) = K \int x e^{-cx} dx = -\frac{K}{c} e^{-cx}\left(x + \frac{1}{c}\right) + D \quad | D \in \mathbb{R}$

③ ~~$\sin^2(t)\ddot{x} - 2x = 0$~~

$2t\ddot{x} + \dot{x} - 2x = 0 \quad | t>0$

transf: $\psi(t) = \tau(t) \Rightarrow \dot{x} = x' \cdot \dot{\psi} \quad | \ddot{x} = x''|\dot{\psi}|^2 + x'\ddot{\psi} \quad \left[\frac{d}{dt} = \cdot\right]$

$\Rightarrow 2t\left[x''|\dot{\psi}|^2 + x'\ddot{\psi}\right] + x'\dot{\psi} - 2x = 0$

$x'\underbrace{\left[\dot{\psi} + 2t\ddot{\psi}\right]}_{=0} + 2t x''|\dot{\psi}|^2 - 2x = 0$

$\Rightarrow \psi = c_1 + c_2 t^{1/2} \Rightarrow 2t \cdot \left(\frac{1}{2}\right)^2 \left(\frac{-1}{t^{\frac{1}{2}}}\right)^2 \cdot x'' = 2x$

$c_1 = 0, \; c_2 = 1 \iff \left(\frac{1}{2}\right)^2 x'' - 2x = 0$ (Eulerova) $\Rightarrow x(t) = \frac{d_1}{t^{\frac{1}{2}}} + d_2 t^2$

$$X(T) = C_1 e^{-2T} + C_2 e^{2T}$$

$$X(t) = C_1 e^{-2\sqrt{a}t} + C_2 e^{+2\sqrt{a}t}$$