Séria úloh 6A: Výpočet derivácie

Úloha 1. Zderivujte zadané funkcie (výsledok upravte!!!) na príslušných množinách:

(a) $y = \frac{1}{8} \ln \frac{x^8 - 1}{x^8 + 1}$;

(b) $y = \frac{x\,\mathrm{e}^x \operatorname{arctg} x}{\ln^5 x}$;

(c) $y = \ln \frac{\sqrt{1 + \mathrm{e}^x + \mathrm{e}^{2x}} - \mathrm{e}^x - 1}{\sqrt{1 + \mathrm{e}^x + \mathrm{e}^{2x}} - \mathrm{e}^x + 1}$;

(d) $y = \sqrt{1 + 2x - x^2} - \arcsin \frac{x - 1}{\sqrt{2}}$;

(e) $y = (2x + 3)^4 \arcsin \frac{1}{2x + 3} + \frac{2}{3}(4x^2 + 12x + 11)\sqrt{x^2 + 3x + 2}$, kde $2x + 3 > 0$;

(f) $y = \sqrt{9x^2 - 12x + 5}\, \operatorname{arctg}(3x - 2) - \ln(3x - 2 + \sqrt{9x^2 - 12x + 5})$;

(g) $y = \frac{\ln(\operatorname{cotg} x + \operatorname{cotg} \alpha)}{\sin \alpha}$;

(h) $y = x\sqrt{a^2 - x^2} + a^2 \arcsin \frac{x}{a}$, $a \neq 0$;

(i) $y = \frac{1}{2} \ln \frac{1 + \cos x}{1 - \cos x} - \frac{1}{\cos x} - \frac{1}{3 \cos^3 x}$;

(j) $y = \frac{1}{3(1 + x^3)} + \frac{1}{3} \ln \frac{x^3}{1 + x^3}$;

(k) $y = -\frac{x + 1}{2} + \frac{x^2 + 1}{2} \operatorname{arctg} x$;

(l) $y = \frac{(x + 5)^2 (x - 4)^3}{(x + 2)^5 (x + 4)^2}$;

(m) $y = \sqrt{\frac{\operatorname{tg} x + \sqrt{2 \operatorname{tg} x} + 1}{\operatorname{tg} x - \sqrt{2 \operatorname{tg} x} + 1}}$;

(n) $y = (x + 5)^2 (2x + 7)^3 (x - 2)(x - 3)$;

(o) $y = \frac{5^x (\sin 3x \ln 5 - 3 \cos 3x)}{9 + \ln^2 5}$;

(p) $y = \operatorname{argsinh} x$;

(q) $y = 3\mathrm{e}^{\sqrt[3]{x}} \left(\sqrt[3]{x^5} - 5\sqrt[3]{x^4} + 20x - 60\sqrt[3]{x^2} + 120\sqrt[3]{x} - 120 \right)$;

(r) $y = x^{29^x} \cdot 29^x + (x \sin x)^{8 \ln(x \sin x)}$;

(s) $y = \operatorname{arctg} \frac{x}{2} + \ln \sqrt{\frac{x - 2}{x + 2}}$;

(t) $y = \frac{\cos x}{3(2 + \sin x)} + \frac{4}{3\sqrt{3}} \operatorname{arctg} \frac{2 \operatorname{tg} \frac{x}{2} + 1}{\sqrt{3}}$;

(u) $y = \left(\frac{a}{b}\right)^x \left(\frac{b}{x}\right)^a \left(\frac{x}{a}\right)^b$, $a, b > 0$;

(v) $y = \frac{1}{4(1 + x^4)} + \frac{1}{4} \ln \frac{x^4}{1 + x^4}$;

(w) $y = \frac{\arcsin x}{\sqrt{1 - x^2}} + \frac{1}{2} \ln \frac{1 - x}{1 + x}$;

Úloha 2. Určte $A, B, C \in \mathbb{R}$ tak, aby na zadanej množine M platil daný vzťah:

(a) $M = \mathbb{R}$

$$\left[\frac{2}{\sqrt{3}} \operatorname{arctg} \left(\frac{\cos x}{\sqrt{3}} \right) + \frac{1}{4} \ln \frac{2 + \sin x}{2 - \sin x} \right]' = \frac{A \cos x + B \sin x}{C + \cos^2 x};$$

(b) $M = (0, +\infty)$

$$\left[\log \left(\cos^2 x + \sqrt{1 + \cos^4 x} \right) + \operatorname{arctg} x + \arcsin \frac{1}{\sqrt{1 + x^2}} \right]' = C \frac{\sin 2x}{\sqrt{1 + \cos^4 x}};$$

(c) $M = \mathbb{R}$

$$\left[A + x - \operatorname{arctg} x + \left(\frac{1}{2}(1 + x^2) \operatorname{arctg} x - \frac{1}{2}x \right) \left(\ln(1 + x^2) - 1 \right) \right]' = (Ax + B) \operatorname{arctg} x \cdot \ln(1 + x^2).$$