Séria úloh 11A: Neurčitý integrál a iracionálne funkcie

Úloha 1. Vypočítajte nasledujúce integrály.

a) $\displaystyle\int \frac{c^2+\sqrt{1+c}}{\sqrt[3]{1+c}}\,\mathbf{d}c$ b) $\displaystyle\int \frac{1}{\sqrt{-z^2-4z-3}}\,\mathbf{d}z$ c) $\displaystyle\int \frac{z+2}{\sqrt{-z^2-4z-3}}\,\mathbf{d}z$

d) $\displaystyle\int \frac{1}{\sqrt{1-a^2-6a}}\,\mathbf{d}a$ e) $\displaystyle\int \frac{a+3}{\sqrt{1-a^2-6a}}\,\mathbf{d}a$ f) $\displaystyle\int \frac{a}{\sqrt{1-a^2-6a}}\,\mathbf{d}a$

g) $\displaystyle\int \frac{1}{\sqrt{5-4a^2-8a}}\,\mathbf{d}a$ h) $\displaystyle\int \frac{5}{\sqrt{a+4}}\,\mathbf{d}a$ i) $\displaystyle\int \frac{1}{\sqrt{-2a-3}}\,\mathbf{d}a$

j) $\displaystyle\int \frac{\mathbf{d}x}{(x-\sqrt{x^2-1})^2}$ k) $\displaystyle\int \frac{2x-10}{\sqrt{1+x-x^2}}\,\mathbf{d}x$ l) $\displaystyle\int \frac{x^3+5x^2+8x+3}{\sqrt{x^2+4x+3}}\,\mathbf{d}x$

m) $\displaystyle\int \sqrt{\frac{1-x}{1+x}}\cdot\frac{1}{(x+1)^2}\,\mathbf{d}x$ n) $\displaystyle\int \frac{1}{(1+z)\sqrt{z}}\,\mathbf{d}z$ o) $\displaystyle\int \frac{z}{z+\sqrt{z}}\,\mathbf{d}z$

p) $\displaystyle\int \frac{1}{c\sqrt{c+1}}\,\mathbf{d}c$ q) $\displaystyle\int \frac{\sqrt{a}}{\sqrt[3]{a^2}-\sqrt[4]{a}}\,\mathbf{d}a$ r) $\displaystyle\int \frac{\sqrt[6]{y}+1}{\sqrt[6]{y^7}+\sqrt[4]{y^5}}\,\mathbf{d}y$

Úloha 2. Nájdite primitívnu funkciu k funkcii $y=f(x)$, ak

a) $y=\sqrt{\dfrac{x+2}{2x+3}}\dfrac{1}{(x+2)(3x+5)}$ b) $y=\dfrac{1}{x+\sqrt{x^2-x+1}}$ c) $y=\dfrac{1}{\sqrt{-x^2-4x}}$

d) $y=\dfrac{x+\sqrt{x^2-1}}{x-\sqrt{x^2-1}}$ e) $y=\dfrac{1}{\sqrt{2x^2-x+3}}$ f) $y=\dfrac{1}{\sqrt{3-2x-5x^2}}$

g) $y=\dfrac{1}{x\sqrt{3+2x+x^2}}$ h) $y=\dfrac{1}{(x-1)\sqrt{x^2-x-1}}$ i) $y=\dfrac{1}{x^2\sqrt{7-x^2}}$

j) $y=\dfrac{1}{(x-1)\sqrt{1+x-x^2}}$ k) $y=\dfrac{1}{\sqrt{3-2x-5x^2}}$ l) $y=\dfrac{1}{x-\sqrt{x^2-x+1}}$

m) $y=\dfrac{1}{(2-x)\sqrt{1-x}}$ n) $y=\sqrt{\dfrac{1+x}{1-x}}$ o) $y=\dfrac{\sqrt{x+1}+2}{(x+1)^2-\sqrt{x+1}}$

p) $y=\dfrac{1}{\sqrt{x+1}+\sqrt{x}}$

Úloha 3. Dokážte rovnosti

$$\arcsin x = 2\,\mathrm{arctg}\,\frac{x}{1+\sqrt{1-x^2}} = -2\,\mathrm{arctg}\,\sqrt{\frac{1-x}{1+x}}+\frac{\pi}{2} = 2\,\mathrm{arctg}\,\sqrt{\frac{1+x}{1-x}}-\frac{\pi}{2}$$

pre $x\in(-1,1)$ tak, že na výpočet integrálu $\displaystyle\int\frac{\mathrm{d}x}{\sqrt{1-x^2}}$ postupne použijete 3. Eulerovu substitúciu $\sqrt{1-x^2}=1+tx$ a dve možnosti 2. Eulerovej substitúcie $\sqrt{1-x^2}=(1+x)t$ a $\sqrt{1-x^2}=(1-x)t$.