

## Séria úloh 6

Rady – kritériá konvergenzie, absolútna a relatívna konvergenca.

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**Príklad 1.** *Vyšetrite relatívnu resp. absolútnu konvergenciu radov.*

- |                                                           |                                                                          |                                                                                                    |
|-----------------------------------------------------------|--------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------|
| a) $\sum_{n=1}^{\infty} \frac{1}{100n+1}$                 | b) $\sum_{n=1}^{\infty} \frac{1+n}{1+n^2}$                               | c) $\sum_{n=1}^{\infty} \frac{1}{(n+2)(n+3)}$                                                      |
| d) $\sum_{n=1}^{\infty} \left(\frac{4n+2}{4n+1}\right)^n$ | e) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$                              | f) $\sum_{n=1}^{\infty} \frac{\sqrt{n+1}-\sqrt{n}}{n}$                                             |
| g) $\sum_{n=1}^{\infty} \sin \frac{\pi}{3^n}$             | h) $\sum_{n=1}^{\infty} \left(\frac{n}{2}\right)^2 \sin \frac{\pi}{2^n}$ | i) $\sum_{n=1}^{\infty} \frac{2n-1}{(\sqrt{2})^n}$                                                 |
| j) $\sum_{n=1}^{\infty} \frac{n}{2^n}$                    | k) $\sum_{n=1}^{\infty} \left(\frac{n+2}{2n-1}\right)^n$                 | l) $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$                                                      |
| m) $\sum_{n=1}^{\infty} \frac{n^5}{2^n+3^n}$              | n) $\sum_{n=1}^{\infty} \frac{3n\sqrt{n+1}}{\sqrt[3]{n^n}}$              | o) $\sum_{n=1}^{\infty} \frac{\sqrt{n!}}{(2+\sqrt{1})(2+\sqrt{2}) \cdot \dots \cdot (2+\sqrt{n})}$ |
| p) $\sum_{n=1}^{\infty} \frac{a^n n!}{n^n}, a > 0$        |                                                                          |                                                                                                    |

**Príklad 2.** *Vypočítajte nasledujúce limity.*

- |                                                    |                                                                                        |
|----------------------------------------------------|----------------------------------------------------------------------------------------|
| a) $\lim_{n \rightarrow \infty} \frac{n^n}{(4n)!}$ | b) $\lim_{n \rightarrow \infty} \left(-n^{-\frac{1}{3}}\right)^n \cdot \sqrt[3]{n!+1}$ |
|----------------------------------------------------|----------------------------------------------------------------------------------------|

**Príklad 3.** *Vyšetrite relatívnu resp. absolútnu konvergenciu radov.*

- |                                                                                  |                                                         |
|----------------------------------------------------------------------------------|---------------------------------------------------------|
| a) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\left(\frac{n+1}{n}\right)^{n^2}}{3^n}$ | b) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{\ln(2n+1)}$ |
| c) $\sum_{n=1}^{\infty} \frac{(-1)^n}{(2\sqrt{n}+(-1)^n)^3}$                     |                                                         |

**Príklad 4.** *Pre aké  $x \in \mathbb{R}$  daný rad absolútne konverguje, relatívne konverguje resp. diverguje?*

- |                                             |                                                          |
|---------------------------------------------|----------------------------------------------------------|
| a) $\sum_{n=1}^{\infty} \frac{nx}{e^{n^x}}$ | b) $\sum_{n=1}^{\infty} \frac{(-1)^n x^{2n+1}}{2^{n+1}}$ |
|---------------------------------------------|----------------------------------------------------------|

**Príklad 5.** Nech  $k \in \mathbb{N}, k > 1, r > 1$ . Dokážte, že platia nasledujúce **vlastné** inklúzie.

$$\mathcal{O}(\log n) \subset \mathcal{O}(n) \subset \mathcal{O}(n \log n) \subset \mathcal{O}(n^k) \subset \mathcal{O}(n^{k+1}) \subset \mathcal{O}(r^n) \subset \mathcal{O}(n!) \subset \mathcal{O}(n^n)$$

$$\Omega(\log n) \supset \Omega(n) \supset \Omega(n \log n) \supset \Omega(n^k) \supset \Omega(n^{k+1}) \supset \Omega(r^n) \supset \Omega(n!) \supset \Omega(n^n)$$

**Domáca úloha:**

úlohy 1 - 4 z témy Kritériá konvergenie radov v mini-zbierke príkladov k cvičeniam 1 a úlohy v tejto sérii