Semester – 60% ← 35b ZÁPOČET

10b MALÉ PÍSOMKY

8+ $\boxed{7b}$ PÍSOMKY NA PREDNÁŠKE B

alebo semestr. projekt $\boxed{7b}$ A

$\Downarrow$

aspoň 30b

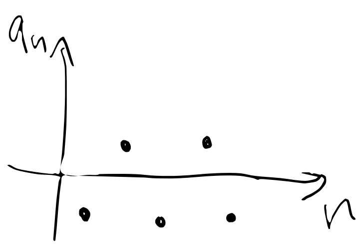
$\min(A+B, 7)$

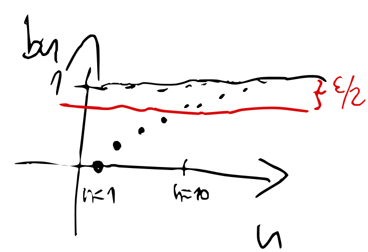
Skúška – 40%

umv.science.upjs.sk/analyza/texty/predmety/FRPb.html

[2.5.–3.6.]

$a_n = (-1)^n \to \{a_n\}_{n\in\mathbb{N}_0} = \{-1, 1\}$





$b_n = \frac{n-1}{n} \to \left\{0, \frac{1}{2}, \frac{2}{3}, \dots, \frac{100}{101}, \dots, \underbrace{\frac{1000}{1001}}_{\doteq 1}, \dots\right\}$

$|b_n - 1| < 0.001$

pre ľubovoľné $n \in \mathbb{N}$ platí

$|b_n - 1| < \varepsilon \quad \varepsilon > 0$ ľub. , $\varepsilon << 1$

$$\left|\frac{n-1}{n} - 1\right| = \left|\frac{n-1-n}{n}\right| = \frac{1}{n} \leq 0.001$$

$$n^* = \frac{1}{0.001} = 1000$$

pre $b_n \quad n^* = \frac{1}{\varepsilon}$

Jednozn. limity : nech $\{a_n\}_{n \in \mathbb{N}_0}$ má 2 limity

$a \neq b$

zoľme $\varepsilon = \frac{|a-b|}{2} > 0$

z def $\Rightarrow \exists\, n_1 \in \mathbb{N} \;\forall n > n_1 \; |a_n - a| < \varepsilon \quad (*)$

$\exists\, n_2 \in \mathbb{N} \;\forall n > n_2 \; |a_n - b| < \varepsilon \quad (\Delta)$

položme $n_0 = \max(n_1, n_2)$

potom pre $n > n_0$ platia $(*), (\Delta)$

~~take~~ $|a-b| = |a - a_n + a_n - b| \leq |a - a_n| + |a_n - b| < \varepsilon + \varepsilon = \frac{|b-a|}{2}$

SPOR

Veta 4.3 D:

nech $\lim_{n\to\infty} a_n = a$

vieme, že pre $\varepsilon = 1$ $\exists n_0 \in \mathbb{N}_0 : \forall n > n_0, n \in \mathbb{N}$

$$|a_n - a| < 1$$

ale $\forall n > n_0$ $|a_n| = |a_n - a + a| \leq |a_n - a| + |a| < 1 + |a|$

zoberme $c = \max(1 + |a|, |a_1|, |a_2|, \dots, |a_{n_0}|)$

$\Rightarrow |a_n| \leq c \quad \forall n \in \mathbb{N}$

$$\lim_{n\to\infty} \sqrt[k]{a_n} = \sqrt[k]{\lim_{n\to\infty} a_n}$$

Pr. $\frac{(-1)^n}{n} = c_n \rightarrow c_n = a_n \cdot b_n = \frac{1}{n} \cdot (-1)^n$

$a_n = \frac{1}{n} \to 0 \quad |b_n| = |(-1)^n| = 1$

Veta 4.6 $\Rightarrow c_n \to 0$

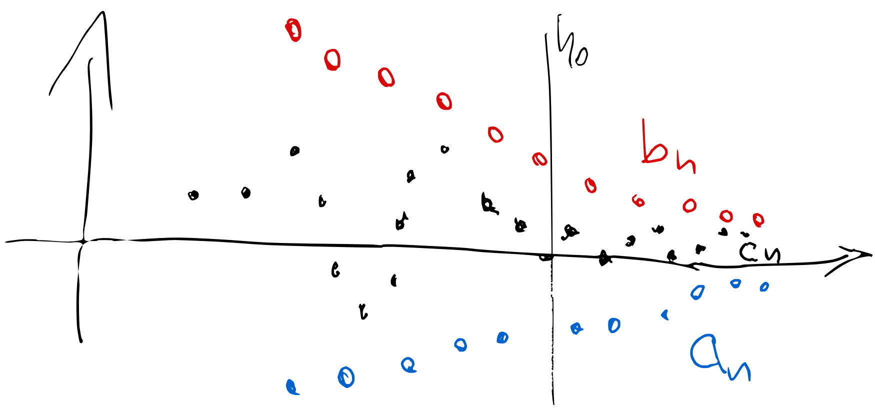
D. Veta 4.6

z def. $\exists\, c>0 : \forall n\in\mathbb{N}_0\ |b_n|\leq c$

$\frac{\varepsilon}{c}>0\ \exists\, n_0\in\mathbb{N}_0 : \forall n\in\mathbb{N}_0,\ n>n_0\ |a_n|<\frac{\varepsilon}{c}$

$\forall n>n_0$

$|a_n b_n - 0| < \frac{\varepsilon}{c}\cdot c = \varepsilon$ ✓



c_n nemá inú možnosť

$\exists\, n_0\in\mathbb{N}\quad |c_n|\leq\tilde{a}_n\quad \forall n>n_0$

x fixnuté $\quad x>1 \quad \lim_{n\to\infty} x^{\frac{1}{n}} = 1$

$\sqrt[n]{x} > 1 \qquad x_n := \sqrt[n]{x} - 1 > 0$

potom $\quad \sqrt[n]{x} = x_n + 1 \quad$ (binomická veta)

$$x = (x_n+1)^n = \sum_{k=0}^{n} \binom{n}{k} x_n^k > n\, x_n$$

$\Rightarrow 0 < x_n < \frac{x-1}{n}$ t.j. $0 < \sqrt[n]{x} - 1 < \frac{x-1}{n}$

resp. $\quad 1 < \sqrt[n]{x} < 1 + \frac{x-1}{n} \to 0$

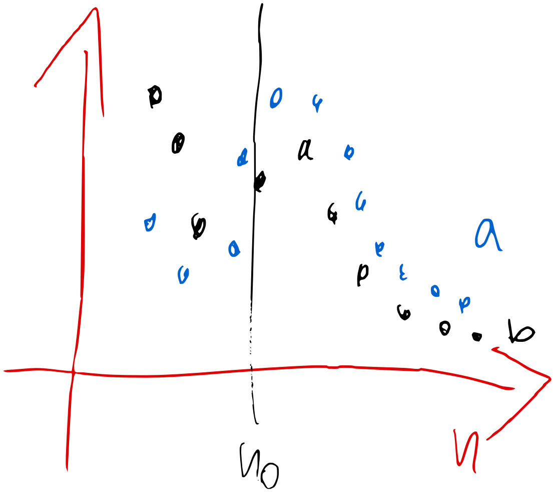
„policajti"

$$\binom{n}{0}1 + \binom{n}{1}x_n + \binom{n}{2}x_n^2 + \dots + \binom{n}{n}x_n^n >$$ DÚ

Bernoulli ner. $\quad (1+u)^n \geq 1 + nu, \quad n \in \mathbb{N}_0, \quad u \geq -1$

$$x \geq 1 + n x_n$$

$$0 \leq x_n \leq \frac{x-1}{n}$$



$\forall n > n_0$

$a > b \quad a_n > b_n$