1 Goniometrické a iné funkcie

$x \cdot \alpha$	0	30	45	60	90	180
$x \cdot \beta$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{\pi}{1}$
$\sin x$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0
$\cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1
$\operatorname{tg} x$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	N/A	0
$\operatorname{ctg} x$	N/A	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	N/A

$$\begin{aligned}
\sin(x \pm y) &= \sin x \cdot \cos y \pm \cos x \cdot \sin y \\
\cos(x \pm y) &= \cos x \cdot \cos y \mp \sin x \cdot \sin y \\
\sin x \pm \sin y &= 2 \cdot \sin\left(\frac{x \pm y}{2}\right) \cdot \cos\left(\frac{x \mp y}{2}\right) \\
\cos x + \cos y &= 2 \cdot \cos\left(\frac{x + y}{2}\right) \cdot \cos\left(\frac{x - y}{2}\right) \\
\cos x - \cos y &= -2 \cdot \sin\left(\frac{x + y}{2}\right) \cdot \sin\left(\frac{x - y}{2}\right)
\end{aligned}$$

$$\left|\sin \frac{x}{2}\right| = \sqrt{\frac{1 - \cos x}{2}} \qquad \left|\cos \frac{x}{2}\right| = \sqrt{\frac{1 + \cos x}{2}}$$

$$\sin 2x = 2 \cdot \sin x \cdot \cos x \qquad \cos 2x = \cos^2 x - \sin^2 x$$

$$\cos^2 x + \sin^2 x = 1 \qquad 1 + \operatorname{tg}^2 x = \frac{1}{\cos^2 x}$$

$$\cos^2 x = \frac{1 + \cos 2x}{2} \qquad \sin^2 x = \frac{1 - \cos 2x}{2}$$

$$1 + \cos x = 2\cos^2 \frac{x}{2} \qquad 1 - \cos x = 2 \sin^2 \frac{x}{2}$$

$$\operatorname{tg} 2x = 2 \cdot \frac{\operatorname{tg} x}{1 - \operatorname{tg}^2 x} \qquad \operatorname{tg} \frac{x}{2} = \frac{1 - \cos x}{\sin x} = \frac{\sin x}{1 + \cos x}$$

$$\sin(\arccos x) = \sqrt{1 - x^2} \qquad \sin(\operatorname{arctg} x) = \frac{x}{\sqrt{1+x^2}} \qquad \sin^2(\operatorname{arctg} x) = \frac{x}{1 + x^2}$$

$$\cos(\arcsin x) = \sqrt{1 - x^2} \qquad \cos(\operatorname{arctg} x) = \frac{1}{\sqrt{1+x^2}} \qquad \cos^2(\operatorname{arctg} x) = \frac{1}{1 + x^2}$$

$$\sinh x = \frac{e^x - e^{-x}}{2} \qquad \cosh^2 x = 1 + \sinh^2 x \qquad \cosh x = \frac{e^x + e^{-x}}{2}$$

$$\tanh x = \frac{\sinh x}{\cosh x} \qquad \sinh 2x = 2 \cdot \sinh x \cosh x$$

2 Logaritmy

$$\log_a (x \cdot y) = \log_a x + \log_a y \qquad \log_a x = \frac{\log_z x}{\log_z a}$$

$$\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y \qquad \log_a b = \frac{1}{\log_b a}$$

$$\log_a x^r = r \cdot \log_a x \qquad x = a^{\log_a x} \qquad x^a = e^{a \cdot \ln x}$$

3 Limity $(k \in \mathbb{R}, a > 1)$

$$\lim_{x\to\infty} \frac{1}{x} = 0 \qquad \lim_{x\to 0} \frac{\sin x}{x} = 1 \qquad \lim_{x\to 0} \frac{1 - \cos x}{x^2} = \frac{1}{2}$$

$$e = \lim_{x\to 0} (1 + x)^{\frac{1}{x}} = \lim_{x\to\infty} \left(1 + \tfrac{1}{x}\right)^x \qquad \lim_{x\to 0} \frac{\ln(1 + x)}{x} = 1$$

$$\lim_{x\to 0} \frac{a^x - 1}{x} = \ln a \qquad \lim_{n\to\infty} \frac{n^k}{a^n} = 0 \qquad \lim_{x\to 0} \frac{e^x - 1}{x} = 1$$

$$\lim_{n\to\infty} \frac{1}{\sqrt[n]{n!}} = 0 \qquad \lim_{n\to\infty} \frac{n}{\sqrt[n]{n!}} = e \qquad \lim_{n\to\infty} \sqrt[n]{n} = 1 \qquad \lim_{n\to\infty} \frac{a^n}{n!} = 0$$

3.1 Nedefinované limity

$$[0 \cdot (\pm\infty)] \qquad \left[\frac{\pm\infty}{\pm\infty}\right] \qquad \left[\frac{0}{0}\right] \qquad [\infty - \infty] \qquad \left[1^{\pm\infty}\right] \qquad \left[0^0\right] \qquad \left[+\infty^0\right]$$

3.2 Definované limity

$$[a \pm \infty = \pm\infty \, (a \in \mathbb{R})] \qquad [\pm\infty \pm \infty = \pm\infty] \qquad \left[\frac{1}{\pm\infty} = 0\right]$$

$$[\pm\infty \cdot \infty = \pm\infty] \qquad \left[a \cdot (\pm\infty) = \begin{cases} \pm\infty & a > 0 \\ \mp\infty & a < 0 \end{cases}\right]$$

4 Derivácie

$$\begin{aligned}
c' &= 0 \quad (c \in \mathbb{R}) \\
(c \cdot f)'(a) &= c \cdot f'(a) \quad (c \in \mathbb{R}) \\
(f \pm g)'(a) &= f'(a) \pm g'(a) \\
(f \cdot g)'(a) &= f'(a) \cdot g(a) + f(a) \cdot g'(a) \\
\left(\frac{1}{g}\right)'(a) &= \frac{g'(a)}{g^2(a)} \quad (g(a) \neq 0) \\
\left(\frac{f}{g}\right)'(a) &= \frac{f'(a)g(a) - f(a)g'(a)}{g^2(a)} \quad (g(a) \neq 0) \\
(h = g \circ f) \;\; h'(a) &= f'(a) \cdot g'(f(a))
\end{aligned}$$

$$(x^n)' = n \cdot x^{n-1} \qquad (n \in \mathbb{R} \setminus \{0\})$$

$$(\sin x)' = \cos x \qquad (\cos x)' = -\sin x$$

$$(\operatorname{tg} x)' = \frac{1}{\cos^2 x} \qquad (\operatorname{ctg} x)' = -\frac{1}{\sin^2 x}$$

$$(\arcsin x)' = \frac{1}{\sqrt{1 - x^2}} \qquad (\arccos x)' = -\frac{1}{\sqrt{1 - x^2}}$$

$$(\operatorname{arctg} x)' = \frac{1}{1 + x^2} \qquad (\operatorname{arcctg} x)' = -\frac{1}{1 + x^2}$$

$$(a^x)' = a^x \cdot \ln a \qquad (e^x)' = e^x$$

$$\left(e^{f(x)}\right)' = e^{f(x)} \cdot f'(x) \qquad (\ln x)' = \frac{1}{x} \quad (x > 0)$$

$$(\log_a x)' = \frac{1}{x \cdot \ln a} \quad (x > 0)$$

4.1 Derivácie inverznej funkcie

$f : M \to \mathbb{R}$ je prostá, diferencovateľná v $f^{-1}(a)$, $a \in f(M)$. Ak f^{-1} je v a spojitá, tak:

1. ak $f'(f^{-1}(a)) \in \mathbb{R} \setminus \{0\}$, tak $\left(f^{-1}\right)'(a) = \frac{1}{f'(f^{-1}(a))}$;
2. ak $f'(f^{-1}(a)) = 0$ a f je v bode $f^{-1}(a)$ rastúca, tak $\left(f^{-1}\right)'(a) = \infty$;
3. ak $f'(f^{-1}(a)) = 0$ a f je v bode $f^{-1}(a)$ klesajúca, tak $\left(f^{-1}\right)'(a) = -\infty$.

4.2 Leibnitzov vzorec

Nech $f, g : I \to \mathbb{R}$ sú n-krát dif. na I. Potom:

$$(f \cdot g)^{(n)}(a) = \sum_{k=0}^{n} \binom{n}{k} \cdot f^{(k)}(a) \cdot g^{(n-k)}(a)$$

4.3 Derivácie vyšších rádov

$$(x^m)^{(n)} = \begin{cases} \frac{m!}{(m-n)!} \cdot x^{m-n} & (n \le m;\ m \in \mathbb{N}) \\ 0 & (n > m;\ m \in \mathbb{N}) \end{cases}$$

$$(\sin x)^{(n)} = \sin\left(x + \frac{n\pi}{2}\right) \qquad (\cos x)^{(n)} = \cos\left(x + \frac{n\pi}{2}\right)$$

$$(a^x)^{(n)} = a^x \cdot \ln^n a \qquad (e^x)^{(n)} = e^x$$

$$(\log_a x)^{(n)} = \frac{(-1)^{n-1} \cdot (n-1)!}{x^n \cdot \ln^n a} \qquad (\ln x)^{(n)} = \frac{(-1)^{n-1} \cdot (n-1)!}{x^n}$$

4.4 L'Hospitalovo pravidlo

Nech

1. funkcie f, g sú diferencovateľné v niektorom prstencovom okolí $P(a)$ bodu $a \in \mathbb{R}^*$;
2. $\forall x \in P(a) : g'(x) \neq 0$;
3. $\lim_{x\to a} f(x) = \lim_{x\to a} g(x) = 0$ alebo $|\lim_{x\to a} g(x)| = +\infty$;
4. existuje vlastná alebo nevlastná $\lim_{x\to a} \frac{f'(x)}{g'(x)}$.

Potom existuje aj $\lim_{x\to a} \frac{f(x)}{g(x)}$ a platí $\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$.

5 Taylorove a MacLaurinove polynómy

Nech fcia f je n-krát dif. v $a \in \mathbb{R}$. Potom **Tay. pol. st. n fcie f v a** je pol. (v prem. x)

$$T_n(a) = f(a) + \frac{f^{(1)}(a)}{1!}(x-a)^1 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n$$

Ak $a = 0 \to$ MacLaurinov pol. Ak je fcia f n-krát dif. v $a \in \mathbb{R}$ a T_n je jej Tay. pol. st. n v a, tak

$$\lim_{x\to a} \frac{f(x) - T_n(x)}{(x-a)^n} = 0$$

5.1 Vybrané polynómy ($\alpha \in \mathbb{R}$)

$$\begin{aligned}
e^x &= 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!} + o(x^n) \\
\sin x &= x - \frac{x^3}{3!} + \cdots + (-1)^{n-1}\frac{x^{2n-1}}{(2n-1)!} + o\left(x^{2n}\right) \\
\cos x &= 1 - \frac{x^2}{2!} + \cdots + (-1)^n \frac{x^{2n}}{(2n)!} + o\left(x^{2n+1}\right) \\
\ln(1+x) &= x - \frac{x^2}{2} + \cdots + (-1)^{n-1}\frac{x^n}{n} + o(x^n) \\
\frac{1}{1-x} &= 1 + x + x^2 + \cdots + x^n + o(x^n) \\
(1+x)^\alpha &= 1 + \alpha x + \cdots + \frac{\alpha(\alpha-1)\cdots(\alpha-n+1)}{n!}x^n + o(x^n)
\end{aligned}$$

5.2 Implikácie ($x \to 0,\ m, n \in \mathbb{N}$)

$$o(x^n) + o(x^n) \Rightarrow o(x^n) \qquad o(x^m) \Rightarrow o(x^n) \quad (\text{pre } m > n)$$

$$o^m(x^n) \Rightarrow o(x^{m\cdot n}) \qquad x^m \cdot o(x^n) \Rightarrow o\left(x^{m+n}\right)$$

$$o(x^m) \cdot o(x^n) \Rightarrow o\left(x^{m+n}\right)$$

5.3 Tvary zvyšku

Nech $f^{(n)}$ je spojitá v $O(a)$, nech $\forall x \in O(a) \setminus \{a\}\ \exists$ vlastná $f^{(n+1)}$. Nech T_n je TP funkcie f stupňa n v bode a. Potom

1. Lagrangeov tvar zvyšku: $\forall x \in O(a),\ x > a$ $(x < a)$ $\exists \vartheta(x) \in (a, x)$ $(\vartheta(x) \in (x, a))$ také, že

$$f(x) - T_n(x) = \frac{f^{(n+1)}(\vartheta(x))}{(n+1)!}(x-a)^{n+1}$$

2. Cauchyho tvar zvyšku: $\forall x \in O(a),\ x > a$ $(x < a)$ $\exists \vartheta(x) \in (a, x)$ $(\vartheta(x) \in (x, a))$ také, že

$$f(x) - T_n(x) = \frac{f^{(n+1)}(\vartheta(x))}{n!}(x-a)(x-\vartheta(x))^n$$

6 Neurčitý integrál

$$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + C \qquad \int \frac{dx}{x} = \ln|x| + C$$

$$\int \frac{dx}{1+x^2} = \operatorname{arctg} x + C = -\operatorname{arcctg} x + C$$

$$\int \frac{dx}{\sqrt{1-x^2}} = \arcsin x + C = -\arccos x + C$$

$$\int \frac{dx}{1-x^2} = \frac{1}{2}\ln\left|\frac{1+x}{1-x}\right| + C \qquad \int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right| + C$$

$$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \qquad \int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$$

$$\int \frac{dx}{\sqrt{x^2 \pm 1}} = \ln\left|x + \sqrt{x^2 \pm 1}\right| + C$$

$$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left|x + \sqrt{x^2 \pm a^2}\right| + C$$

$$\int a^x\ dx = \frac{a^x}{\ln a} + C \qquad \int e^x\ dx = e^x + C$$

$$\int \sin x\ dx = -\cos x + C \qquad \int \cos x\ dx = \sin x + C$$

$$\int \frac{dx}{\sin^2 x} = -\operatorname{ctg} x + C \qquad \int \frac{dx}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \operatorname{tg} x\ dx = -\ln|\cos x| + C$$

6.1 Metóda rozkladu

$$\int (c_1 f_1(x) + \cdots + c_n f_n))dx = c_1 \int f_1(x)dx + \cdots + c_n \int f_n(x)dx$$

6.2 Metóda substitúcie

$$\int g(x)\ dx = \int f(\varphi(x))\varphi'(x)\ dx = \left| \begin{array}{c} \varphi(x) = t \\ \varphi'(x)dx = dt \end{array} \right| =$$

$$= \int f(t)\ dt = F(t) + C = F(\varphi(x)) + C$$

6.3 Metóda per partes

$$\int u'(x)v(x)\ dx = u(x)v(x) - \int u(x)v'(x)\ dx$$