Pr. 4 Obor konv. radu $\sum_{n=1}^{\infty} \frac{(x-3)^n}{n5^n}$

$\Downarrow$ moch. rad so str. $a=3$

$x-3=y$

$\Downarrow$

$\sum_{n=1}^{\infty} \frac{y^n}{n5^n}$, $a_n = \frac{1}{n5^n}$

počítajme $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{\frac{1}{(n+1)5^{n+1}}}{\frac{1}{n5^n}} = \lim_{n\to\infty} \frac{n}{n+1} \frac{1}{5} =$

$= \frac{1}{5}$

$\Rightarrow$ polomer konv. je $r = \frac{1}{1/5} = 5$

$\Rightarrow$ interval konv. je $(3-5, 3+5) = (-2, 8)$

Ďalej pre $x=8$ máme $\sum_{n=1}^{\infty} \frac{1}{n}$, teda harm. rad, ktorý diverguje

Pre $x=-2$ máme Leibn. rad $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$, ktorý konv. podľa Leib. krit.

$\Rightarrow$ obor konv. je $\langle -2, 8)$

Pr. 1 Glob. extr. fcie h na D_h, ak

$$h(x,y) = xy\sqrt{1-x^2-y^2} \Rightarrow 1-x^2-y^2 \geq 0$$

$$D_h = \{(x,y) \in \mathbb{R}^2 : x^2+y^2 \leq 1\} = B_1 \quad \text{(kruh)}$$

Keďže $f \in C(B_1) \Rightarrow \exists$ glob. extr.

Na ∂B_1 platí $h = 0$

Vo vnútri, teda na B_1^0 platí $\frac{\partial h}{\partial x} = \frac{y(1-2x^2-y^2)}{\sqrt{1-x^2-y^2}}$

a $\frac{\partial h}{\partial y} = \frac{x(1-x^2-2y^2)}{\sqrt{1-x^2-y^2}}$. Navyše $\nabla h = \vec{0} \Leftrightarrow$

$y(1-2x^2-y^2)=0 \ \wedge \ x(1-x^2-2y^2)=0 \ \Leftrightarrow$

$(y=0 \wedge x=0) \vee (y=0 \wedge 1-x^2-2y^2=0) \vee$

$(x=0 \wedge y=0) \vee (x=0 \wedge 1-2x^2-y^2=0) \vee (1-x^2-2y^2=0 \wedge 1-2x^2-y^2=0)$

$\Downarrow$ SB $(0,0)$

$\Downarrow$ SB ležiace na ∂B_1

$\Downarrow$ 4 SB: $|x| = |y| = \frac{1}{\sqrt{3}}$

$\Rightarrow h(0,0) = 0$

Taže $h(a,a) = h(-a,-a) = a^2\sqrt{1-2a^2} = \frac{1}{3\sqrt{3}}$

a $h(a,-a) = h(-a,a) = -h(a,a) = -\frac{1}{3\sqrt{3}}$ pre

$a = \frac{1}{\sqrt{3}}$.

Nakoniec glob. min. je a glob. max. je .

Pr. 5 Dodefinujte $f = \frac{x^3 y}{x^2+y^2}$ tak, aby mala tot. dif. všade na $\mathbb{R}^2$.

① mimo bodu $(0,0)$ platí, že parc. der. sú spojité
$\Rightarrow$ že je tam dif.

② v $(0,0)$ musí byť sp. – defin. $f(0,0)$:

$$\Rightarrow \lim_{(x,y)\to(0,0)} \frac{x^3 y}{x^2+y^2} = f(0,0)$$

$$\| $$

$$\lim_{(x,y)\to(0,0)} \frac{x^2}{x^2+y^2}\cdot yx = 0 \Rightarrow f(0,0)=0$$

„ohr. · 0"

ďalej

$$\frac{\partial f}{\partial x}(0,0) = \frac{\partial f}{\partial y}(0,0) = 0, \text{ lebo } \begin{array}{l} f(x,0)-f(0,0)=0 \quad \forall x\neq 0 \\ f(0,y)-f(0,0)=0 \quad \forall y\neq 0 \end{array}$$

Overíme dif:

$$\omega(x,y) = \frac{f(x,y)-f(0,0)-0\cdot x-0\cdot y}{\sqrt{x^2+y^2}} = \frac{x^3 y}{(x^2+y^2)^{3/2}}$$

$$\lim_{(x,y)\to(0,0)} \omega(x,y) = \lim_{-\|-} \frac{x^3}{(x^2+y^2)^{3/2}}\cdot y = 0$$

„ohr. · 0" ✓

Pr. 6. Nech $F(u,t) = f(x,y)$, $x = u\cos t$, $y = u\sin t$

$f \in C^2(\mathbb{R}^2)$ ...

$$\frac{\partial F}{\partial u} = \frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial u} + \frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial u} = \frac{\partial f}{\partial x}\cos t + \frac{\partial f}{\partial y}\cdot\sin t$$

$$\frac{\partial F}{\partial t} = \frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial t} = \left(-\frac{\partial f}{\partial x}\sin t + \frac{\partial f}{\partial y}\cos t\right)\cdot u$$

$$\frac{\partial^2 F}{\partial u^2} = \frac{\partial^2 f}{\partial x^2}\cos^2 t + 2\frac{\partial^2 f}{\partial y\partial x}\cos t\sin t + \frac{\partial^2 f}{\partial y^2}\sin^2 t$$

$$\frac{\partial^2 F}{\partial t^2} = \left(\frac{\partial^2 f}{\partial x^2}\sin^2 t - \frac{\partial f}{\partial x}\cos t - 2\frac{\partial^2 f}{\partial y\partial x}\sin t\cos t + \frac{\partial^2 f}{\partial y^2}\cos^2 t - \frac{\partial f}{\partial y}\sin t\right)u^2$$

Ale z toho

$$\frac{\partial^2 F}{\partial u^2} + \frac{\partial^2 F}{\partial t^2}\cdot\frac{1}{u^2} + \frac{\partial F}{\partial u}\cdot\frac{1}{u} = \frac{\partial^2 f}{\partial x^2}(\cos^2 t + \sin^2 t) + \frac{\partial^2 f}{\partial y^2}(-||-)$$

$$= \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = \Delta f$$

$\Downarrow$

Laplacián v pol. súrad.

Pr. 3 $y'' + 2y' + y = e^{x} + e^{-x} = 2\cosh(x)$

hľ. riešenie v tvare $y = e^{\lambda x}$ [rovnice bez PS]

$\Leftrightarrow \lambda^2 + 2\lambda + 1 = 0$

$\Leftrightarrow (\lambda+1)^2 = 0$, teda $\lambda_{1,2} = -1$

$\Rightarrow$ FSR $\{e^{-x}, xe^{-x}\}$

$y_h(x) = c_1 e^{-x} + c_2 e^{-x} \cdot x$

y_p: 1 $\det W = \begin{vmatrix} e^{-x} & xe^{-x} \\ -e^{-x} & e^{-x}(1-x) \end{vmatrix} = e^{-2x}$

$\det W_1 = \begin{vmatrix} 0 & xe^{-x} \\ e^{x}+e^{-x} & (1-x)e^{-x} \end{vmatrix} = -x(1+e^{-2x})$

$\det W_2 = \begin{vmatrix} e^{-x} & 0 \\ -e^{-x} & e^{x}+e^{-x} \end{vmatrix} = 1 + e^{-2x}$

$\Rightarrow y_p = e^{-x} \int \frac{-x(1+e^{-2x})}{e^{-2x}} dx + xe^{-x} \int \frac{1+e^{-2x}}{e^{-2x}} dx$

$$y_p = -e^{-x}\int x(e^{2x}+1)\,dx + xe^{-x}\int e^{2x}+1\,dx =$$

$$= \frac{e^{-x}x^2}{2} + \frac{e^x}{4}$$

alebo $\boxed{\text{Veta 7.17}}$ + $\boxed{\text{Veta 7.19}}$

2°:

$$y_p^1 = Ae^x \quad \text{(dosadíme do rovnice)}$$

$$\Rightarrow \quad Ae^x + 2Ae^x + Ae^x = e^x$$

$$4Ae^x = e^x \Rightarrow A = \frac{1}{4}$$

$$y_p^2 = Bx^2e^{-x}$$

$$\Rightarrow \quad e^{-x}\left(\underbrace{B(x^2-4x+2) - 2B(x-2)x + Bx^2}_{2B}\right) = \underbrace{e^{-x}}_{1}$$

$$2B = 1 \Rightarrow B = \frac{1}{2}$$

Teda $\quad y = C_1e^{-x} + C_2e^{-x}\cdot x + \frac{e^x}{4} + \frac{e^{-x}x^2}{2}$

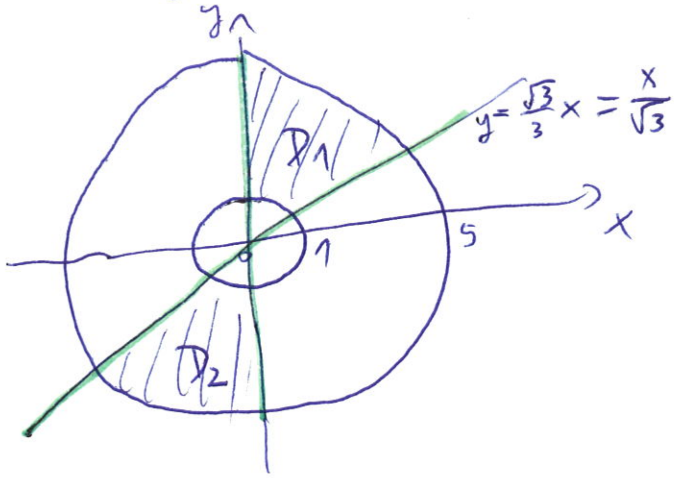
→ spojitá na komp. $D \Rightarrow \exists$ integrál

Pr. 2 $I = \iint_D \frac{x}{x^2+2y^2}\, d(x,y)$, D: $x=0$ $\sqrt{3}x = 3y$

$1 = x^2+y^2$, $25 = x^2+y^2$

$\downarrow$

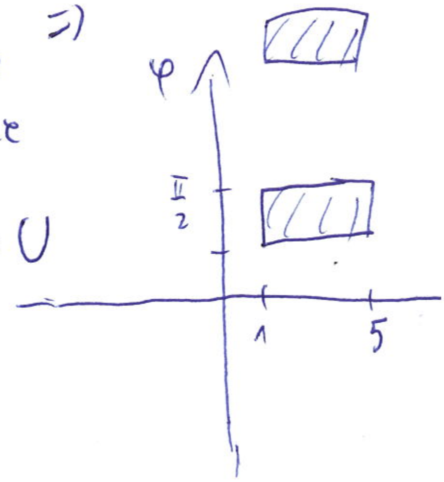
$r \in [1,5]$



$\Rightarrow$

Ψ: $x = r\cos\varphi$, $y = r\sin\varphi$

$\Psi^{-1}(D) = U$



Jakobián

$$I = \iint_U \frac{r\cos\varphi}{r^2(\cos^2\varphi + 2\sin^2\varphi)} \cdot r \; d(r,\varphi)$$

$\sqrt{3}\,\cancel{r}\cos\varphi = 3\,\cancel{r}\sin\varphi$

$\frac{\sqrt{3}}{3} = tg\,\varphi \Rightarrow \varphi = \frac{\pi}{6}$

Teda $\underbrace{\varphi \in \left[\frac{\pi}{6}, \frac{\pi}{2}\right] \cup \left[\frac{7}{6}\pi, \frac{3}{2}\pi\right]}_{J}$

$$I = \int_1^5 r \cdot r^{-1}\, dr \cdot \int_J \frac{\cos\varphi}{2-\cos^2\varphi}\, d\varphi =$$

$$= 4 \cdot \left[\int_{\pi/6}^{\pi/2} \frac{\cos\varphi}{2-\cos^2\varphi}\, d\varphi + \int_{7/6\pi}^{3/2\pi} \frac{\cos\varphi}{2-\cos^2\varphi}\, d\varphi\right] = 4\left[+\int_{1/2}^{1} \frac{du}{1+u^2} + \int_{-1}^{-1/2} -\frac{du}{1+u^2}\right] =$$

$\frac{\cos\varphi}{2-\cos^2\varphi} = \frac{\cos\varphi}{1+\sin^2\varphi} \rightarrow \left|\begin{matrix} \sin\varphi = u \\ \cos\varphi\, d\varphi = du \end{matrix}\right|$

$$= -4\left[arctg(1) + arctg(1/2) + arctg\left(-\tfrac{1}{2}\right) - arctg(-1)\right] =$$

$$= -8 \cdot 0 = 0$$

Alebo: $f(x,y) = \frac{x}{x^2+2y^2}$ pre $\forall (x,y) \in D_1$ platí $f(x,-y) = -f(x,y)$

symetria