Séria úloh č. 9

1. Vyšetrite konvergenciu (absolútnu/relatívnu) nasledujúcich radov:

a) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{2^{n^2}}$;

b) $\displaystyle\sum_{n=1}^{\infty} \prod_{k=1}^{n} \left(\sqrt{2} - \sqrt[2k+1]{2}\right)$;

c) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n^2}$;

d) $\displaystyle\sum_{n=1}^{\infty} \frac{n^{n+\frac{1}{n}}}{(n+\frac{1}{n})^n}$;

e) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1+\cos n}{2+\cos n}\right)^{2n-\ln n}$;

f) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n + (-1)^n 2^n n}{4^n + (-1)^n n}$;

g) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin \frac{\pi}{3} n}{\sqrt{n+1}}$;

h) $\displaystyle\sum_{n=1}^{\infty} (\sqrt{n^6+n} - n^3) \cos n^2$;

ch) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+3n+4}{(2n^2+5)^2}$;

i) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin n + \sqrt{n} \cos n}{n} \cdot \cos \frac{1}{n}$;

j) $\displaystyle\sum_{n=1}^{\infty} \frac{7^{3n}}{(2n-5)!}$;

k) $\displaystyle\sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^2 a^n}$, $a \in \mathbb{R}$, $a > 0$;

l) $\displaystyle\sum_{n=1}^{\infty} \frac{\arcsin \frac{3+(-1)^n}{4}}{2^n + n}$;

ľ) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{(2n+3)^3}$;

m) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n+1}{2n-\sqrt{13}}\right)^n$;

n) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{[\ln(n+1)]^{n+1}}$;

o) $\displaystyle\sum_{n=1}^{\infty} \frac{\operatorname{arctg} \sqrt{n}}{n} \cdot \sin(2n+1)$;

ô) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin n \cdot \sin n^2}{n}$;

p) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^3 + \ln n}$;

q) $\displaystyle\sum_{n=1}^{\infty} \frac{n^{10}}{2^n+1} \sin(n\sqrt{\pi})$;

r) $\displaystyle\sum_{n=3}^{\infty} \frac{2+(-1)^n n}{n+1}$;

s) $\displaystyle\sum_{n=1}^{\infty} \frac{\left(\frac{n}{n+1}\right)^3}{2n + \frac{100}{n}} \cos \frac{2n\pi}{3}$;

t) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{\operatorname{arctg}^n(1+\frac{1}{n})}$;

u) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \ln\left(1+\frac{1}{n}\right)$;

v) $\displaystyle\sum_{n=1}^{\infty} \sin \frac{1}{n}$;

w) $\displaystyle\sum_{n=1}^{\infty} \ln\left(1-\frac{1}{n^2}\right)$;

x) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n+1}{n(n+2)}$;

y) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{2\sqrt{n}}$;

2. Vyšetrite absolútnu/relatívnu konvergenciu nasledujúcich radov:

a) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{5}{3^n} - \frac{3}{5^n}\right)$;

b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2^n - 1}$;

c) $\displaystyle\sum_{n=1}^{\infty} \frac{n^3}{\mathrm{e}^n}$;

d) $\displaystyle\sum_{n=1}^{\infty} \frac{n+2}{n^2+1}$;

e) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n+3}{n+7}\right)^{3n+1}$;

f) $\displaystyle\sum_{n=1}^{\infty} \binom{2n}{n} \frac{1}{5^n}$;

g) $\displaystyle\sum_{n=1}^{\infty} \left(\sqrt{n^3+1} - \sqrt{n^3-1}\right)$;

h) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \sqrt{n}}{n+1}$;

i) $\displaystyle\sum_{n=3}^{\infty} \frac{3}{2^n - 2n}$;

j) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n + 4^n}{4^n + 5^n}$;

k) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n+1}-\sqrt{n-1}}{\sqrt{n+2}}$

l) $\displaystyle\sum_{n=1}^{\infty} \frac{n^3}{3^n} \cdot a^n$, $a \in \mathbb{R}$, $a > 0$;

m) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{n^2} - n \sin \frac{1}{n^3}\right)$;

n) $\displaystyle\sum_{n=1}^{\infty} \frac{(3n)!}{(n!)^2 4^{3n}}$;

o) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n^2+3n+4}{2n^2+5}$;

p) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \cdot \frac{30^n(n^4+2)}{n^6 \cdot 31^n}$;

r) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n-5}{n+3}\right)^{\frac{n^2}{2}}$;

s) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt[3]{n^2+4}-\sqrt[3]{n^2+1}}{\sqrt[3]{n}}$;

t) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2 \ln n}$;

u) $\displaystyle\sum_{n=1}^{\infty} n^{\ln a}$, $a \in \mathbb{R}$, $a > 0$;

v) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^n}$;

x) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{\left(2+\frac{1}{n}\right)^n}$;

y) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \left(\sqrt[n]{3}-1\right)$;

z) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n+2}-\sqrt{n-2}}{n^a}$, $a \in \mathbb{R}$, $a > 0$.