Séria úloh č. 10

1. Vyšetrite absolútnu/relatívnu konvergenciu nasledujúcich radov:

a) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(n\pi/2)}{n^2+1}$;

b) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \ln\left(1+\frac{1}{n}\right)$;

c) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n-1}{n+1}\right)^n$;

d) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n\ln^p n}$, $p \in \mathbb{R}$;

e) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \sqrt{\frac{n}{n^2+1}}$;

f) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin\frac{1}{n}}{n}$;

g) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}} \cos\frac{1}{n}$;

h) $\displaystyle\sum_{n=1}^{\infty} \frac{2^{2n}n^2}{\mathrm{e}^n n!} \frac{1}{\ln^2 n}$.

2. Pre ktoré $x \in \mathbb{R}$ sú rady absolútne konvergentné, relatívne konvergentné a divergentné?

a) $\displaystyle\sum_{n=1}^{\infty} \ln^n x$;

b) $\displaystyle\sum_{n=1}^{\infty} \mathrm{e}^{-n^2 x}$;

c) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+1}{n^3\cdot 2^n}\cdot x^n$;

d) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{x}{\operatorname{arctg} n}\right)^n$;

e) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1}\,\mathrm{e}^{-n\sin x}$;

f) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^x}$.

3. Nájdite funkciu f, ktorá je bodovou limitou postupnosti $(f_n)_1^\infty$, ak

a) $f_n = \dfrac{nx^2}{x+3n+2}$, $x > 0$;

b) $f_n = \sin^n x + \cos^n x$;

c) $f_n = n\left(x^{1/n} - x^{1/2n}\right)$, $x > 0$;

d) $f_n = n^2 x^4 \sin\dfrac{x}{n^2+n}$;

e) $f_n = n^3 x^2\,\mathrm{e}^{-nx}$, $x \geq 0$;

f) $f_n = n \operatorname{arccotg} nx^2$, $x > 0$;

g) $f_n = \dfrac{x}{n}\ln nx$;

h) $f_n = \begin{cases} 1, & x \in \langle \frac{1}{n}, +\infty) \\ -1, & x \in (-\infty, -\frac{1}{n}\rangle \\ nx, & x \in (-\frac{1}{n}, \frac{1}{n}) \end{cases}$;

i) $f_n = \begin{cases} 1-nx, & x \in \langle 0, \frac{1}{n}\rangle \\ 0, & x \in \langle \frac{1}{n}, 1\rangle \end{cases}$;

j) $f_n = \begin{cases} n^2 x, & x \in \langle 0, \frac{1}{n}\rangle \\ \frac{1}{x}, & x \in \langle \frac{1}{n}, 1\rangle \end{cases}$.