Spočítajme $I_n = \int_0^1 \int_0^{1-x_1} \cdots \int_0^{1-x_1-\dots-x_{n-1}} \sqrt{x_1+\dots+x_n}\, dx$

$$I_n = \left| \begin{array}{l} 1 - \sum_1^n x_i = y_n \\ 1 - \sum_1^{n-1} x_i = y_{n-1} \\ \quad \vdots \\ 1 - x_1 = y_1 \\ |\det J| = 1 \end{array} \right| = \int_0^1 \int_0^{y_1} \cdots \int_0^{y_{n-1}} \sqrt{1-y_n}\, dy_n \cdots dy_1$$

nech $f(b_n)$: $f^{(-n)}$ $\exists$ $\forall x \in D_f$, $a \in D_f$

$$f^{(-n)}(x) := \int_a^x \int_a^{b_1} \cdots \int_a^{b_{n-1}} f(b_n)\, db_n \cdots db_1$$

Ukážte MI, že antiderivácia n-tého rádu

$$f^{(-n)}(x) = \frac{1}{(n-1)!} \int_a^x (x-t)^{n-1} f(t)\, dt$$

$\Rightarrow I_n = g^{(-n)}(1)$ pre $g(z) = \sqrt{1-z}$, $a=0$

teda

$$I_n = \frac{1}{(n-1)!} \int_0^1 (1-t)^{n-1} \sqrt{1-t}\, dt$$

ale

$$\int_0^1 (1-t)^{n-\frac{1}{2}} dt = \left[\frac{-2(1-t)^{n+\frac{1}{2}}}{(2n+1)} \right]_0^1 = \frac{2}{2n+1}$$