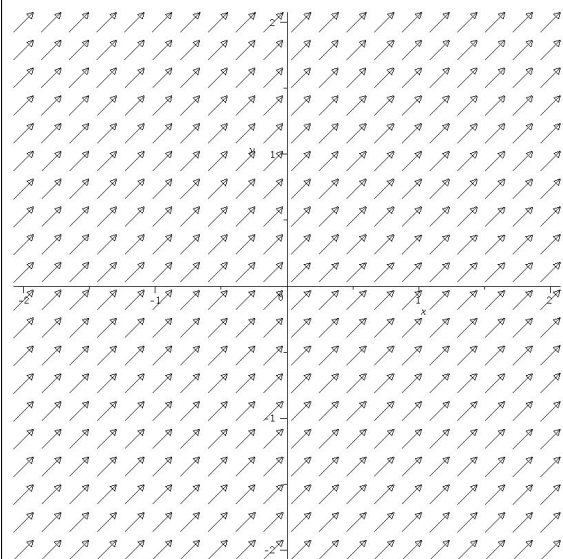
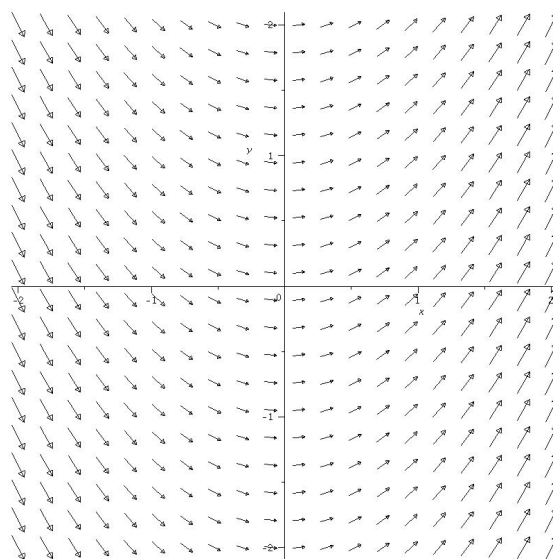


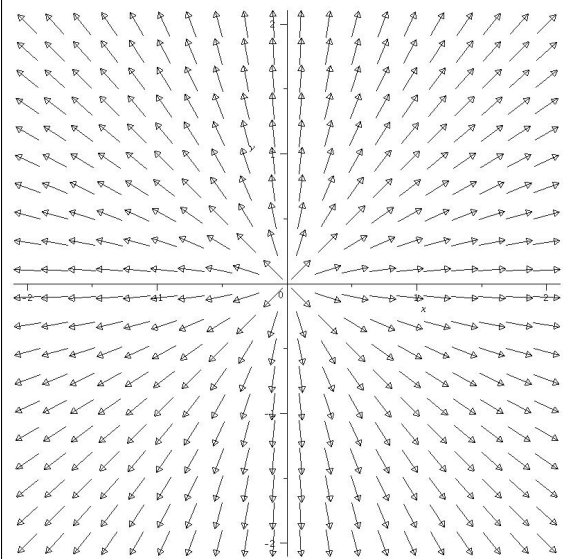
**A. Znázornite vektorové polia**



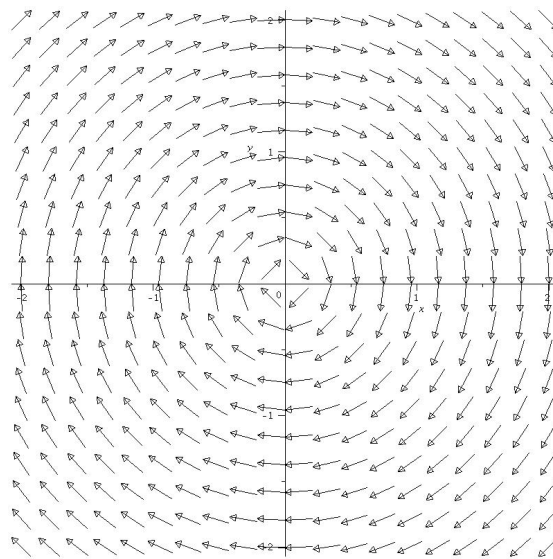
$$\mathbb{F}(x, y) = \left( \frac{1}{2}, \frac{1}{2} \right)$$



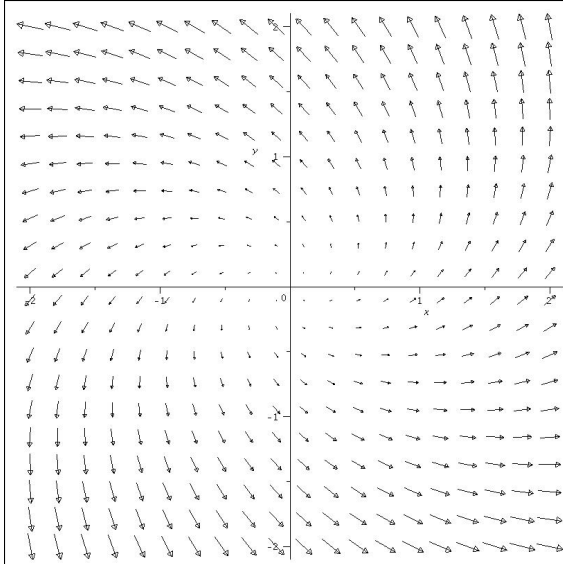
$$\mathbb{F}(x, y) = (1, x)$$



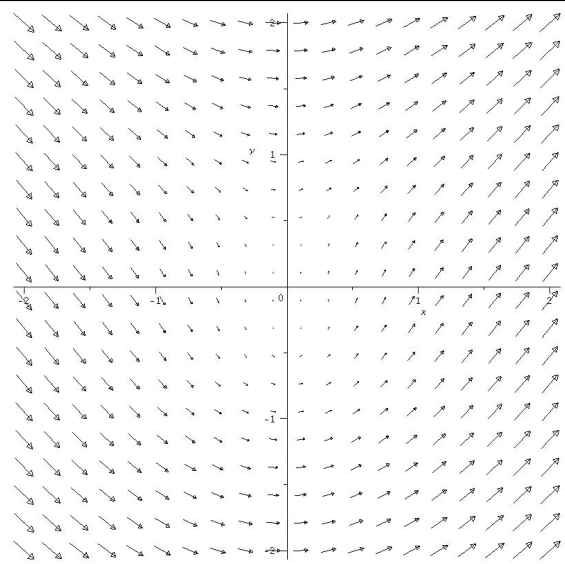
$$\mathbb{F}(x, y) = \left( \frac{x}{\sqrt{x^2+y^2}}, \frac{y}{\sqrt{x^2+y^2}} \right)$$



$$\mathbb{F}(x, y) = \left( \frac{y}{\sqrt{x^2+y^2}}, \frac{-x}{\sqrt{x^2+y^2}} \right)$$

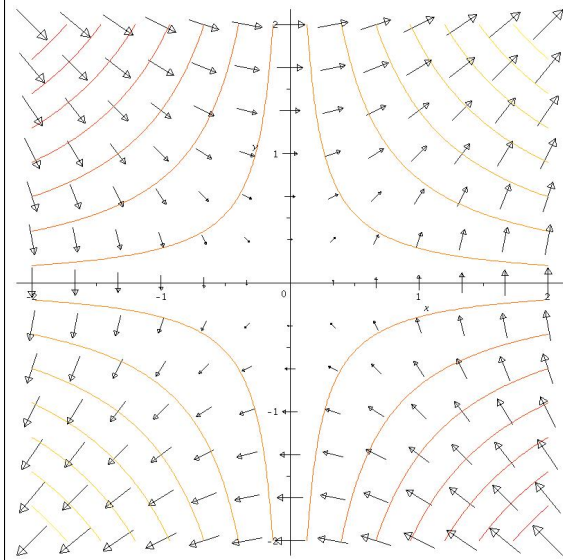


$$\mathbb{F}(x, y) = (2x - 3y, 2x + 3y)$$

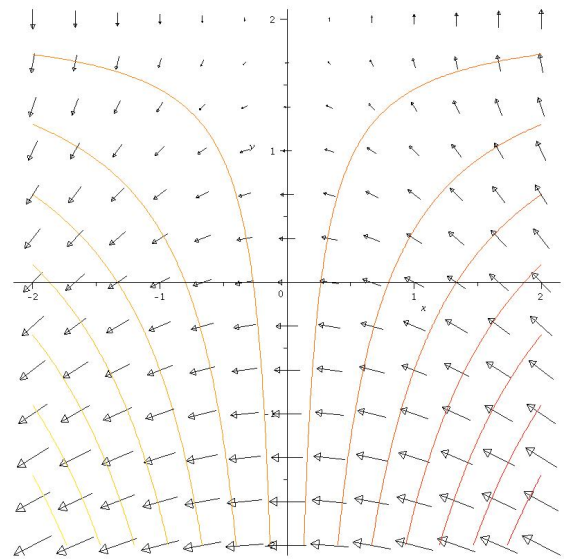


$$\mathbb{F}(x, y) = (\ln(1 + x^2 + y^2), x)$$

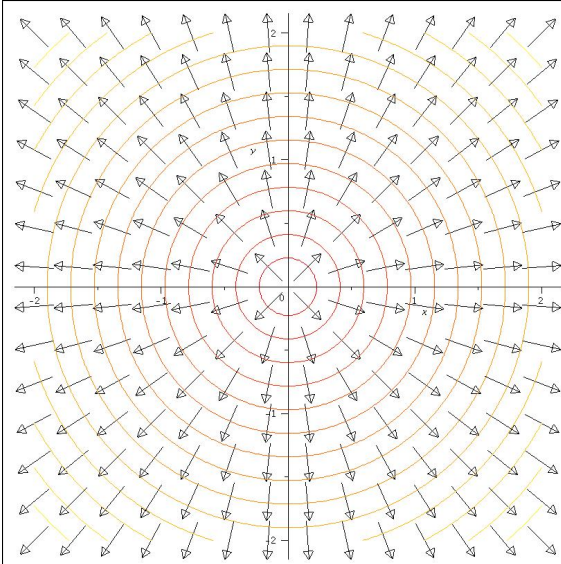
**B. Nakreslite gradientné vektoré polia skalárnych funkcií spolu s ich vrstevnicami.**



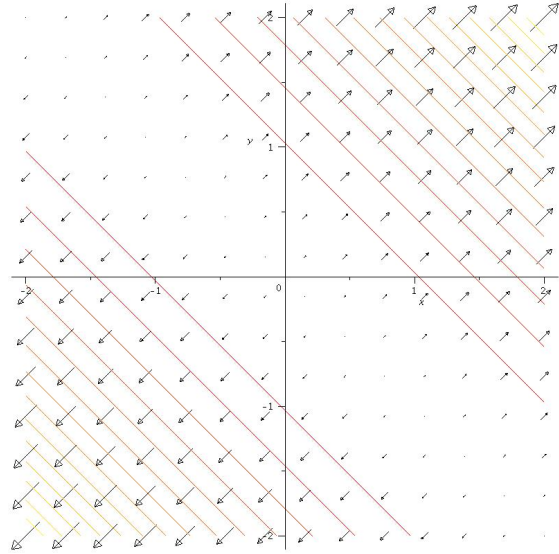
$$f(x, y) = xy$$



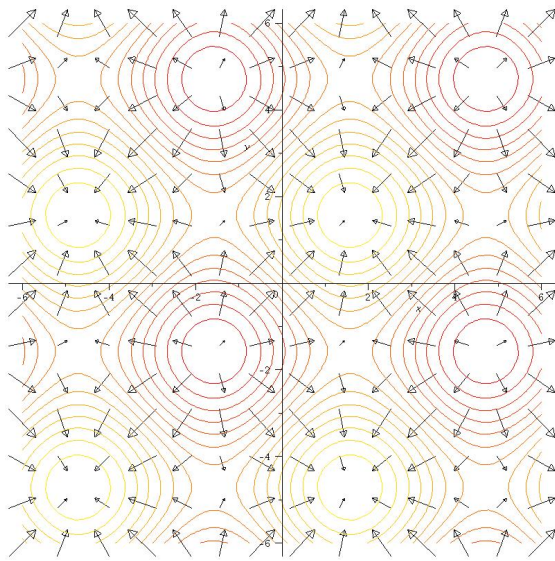
$$f(x, y) = xy - 2x$$



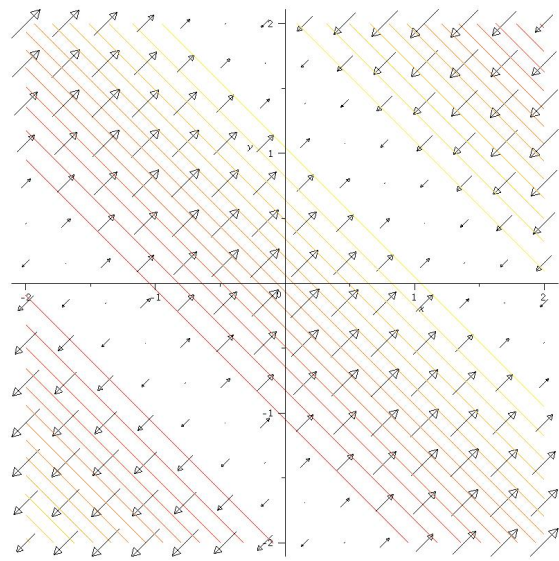
$$f(x, y) = \sqrt{x^2 + y^2}$$



$$f(x, y) = \frac{1}{4}(x + y)^2$$



$$f(x, y) = \sin x + \sin y$$



$$f(x, y) = \sin(x + y)$$

**C. Nájdiť parametrické vyjadrenie nasledujúcich kriviek.**

Jedné z možných parametrizácií:

- |   |   |
|---|---|
| 1. $x = 4t(t-1), y = 16t^3(t-1)^2, t \in \mathbb{R}$                | 4. $x = a \cos t, y = a(1 + \sin t), t \in [0, 2\pi)$   |
| 2. $x = \frac{at}{1+t^4}, y = \frac{at^2}{1+t^4}, t \in \mathbb{R}$ | 5. $x = \frac{3at}{1+t^3}, y = \frac{3at^2}{1+t^3}, t \in \mathbb{R} \setminus \{0\}$                       |
| 3. $x = t, y = \frac{t^2}{2} - \frac{t^3}{2} - 1, t \in \mathbb{R}$ | 6. $x = \frac{at}{(1+t^2) \arctan t}, y = \frac{at^2}{(1+t^2) \arctan t}, t \in \mathbb{R} \setminus \{0\}$ |

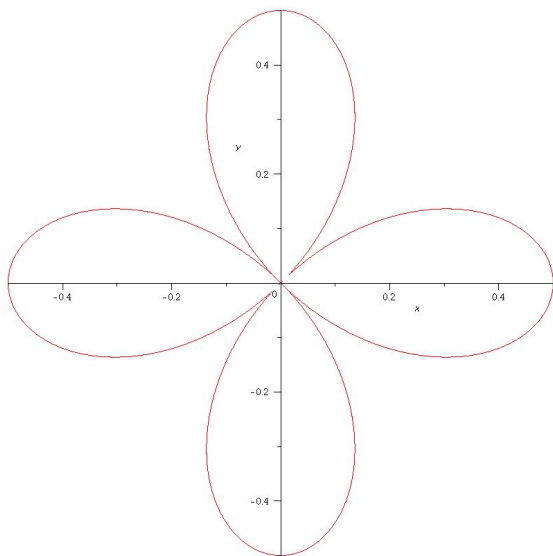
**D. Rovnice kriviek prepíšte na tvar  $F(x, y) = 0$  a zistite tak o aké krivky ide.**

- |   |  |
|---|--|
| 1. elipsa (pre $b \neq d$ ), resp. kružnica (pre $b = d$ ): $\frac{(x-a)^2}{b^2} + \frac{(y-c)^2}{d^2} = 1$ | 4. parabola: $y^2 = -4a(x-a)$                    |
| 2. úsečka: $y = \frac{\pi}{2} - x, x \in (-\pi/2, \pi/2)$   | 5. parabola: $y^2 = 4a(x+a)$                     |
| 3. priamka: $y \cos \phi_0 - x \sin \phi_0 = d$   | 6. lemniskáta: $(x^2 + y^2)^2 = 2a^2(x^2 - y^2)$ |

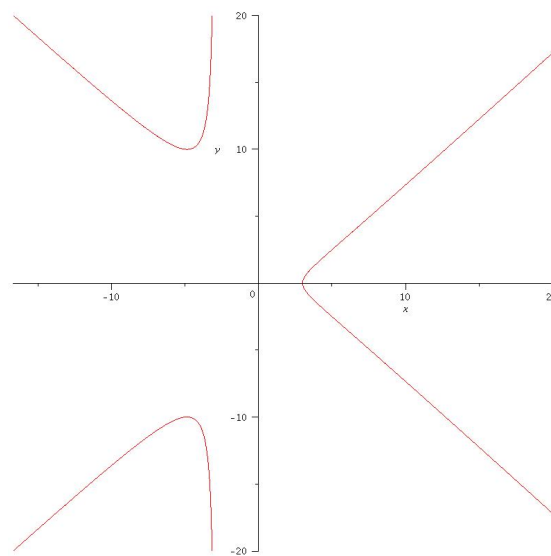
**E. Evolventa kružnice.**

$$x = R(\cos t + t \sin t), y = R(\sin t - t \cos t), t \in \mathbb{R}$$

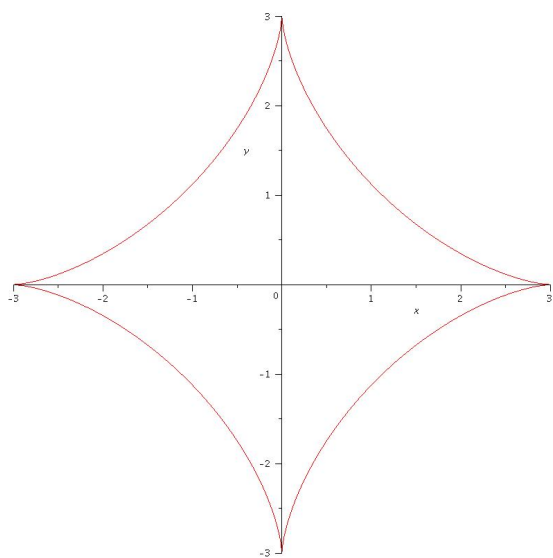
F. Zostrojte nasledujúce krivky.



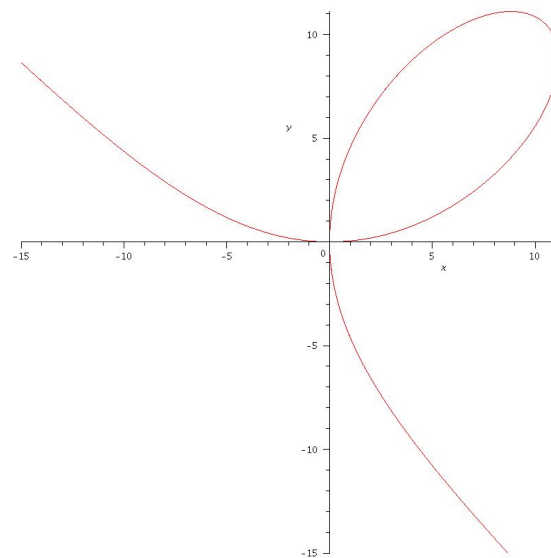
$$(x^2 + y^2)^3 = a^2(x^2 - y^2)^2, a = \frac{1}{2}$$



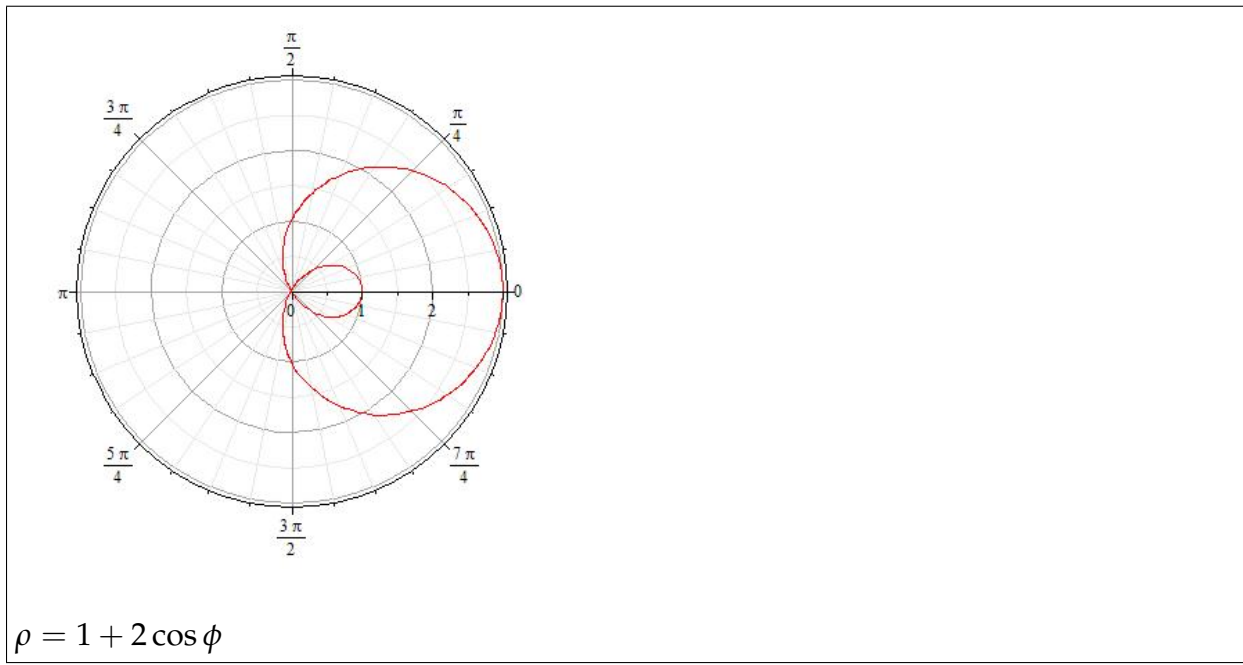
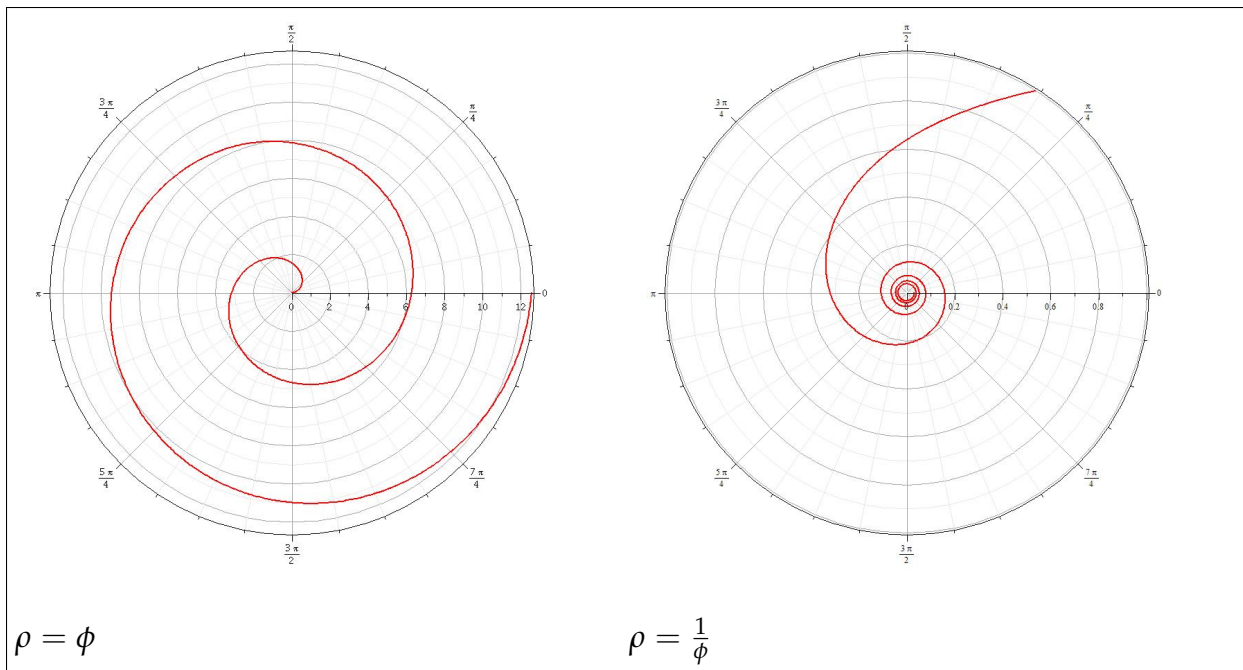
$$x(x^2 - y^2) = a(x^2 + y^2), a = 3$$



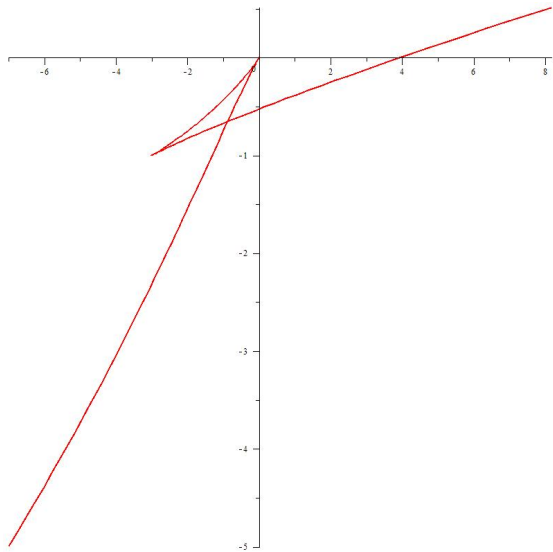
$$x^{2/3} + y^{2/3} = a^{2/3}, a = 3$$



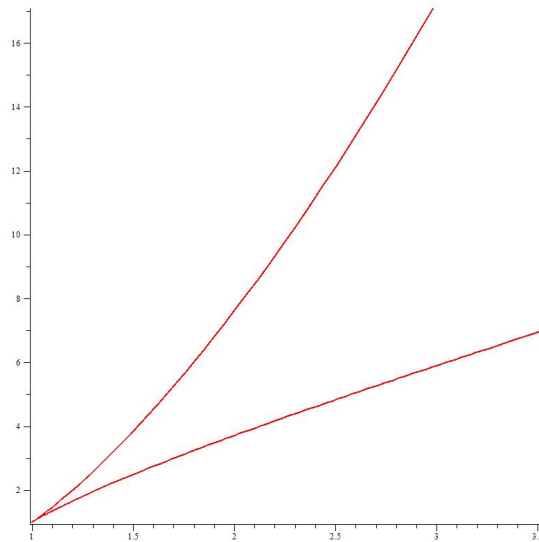
$$x^3 + y^3 = 3axy, a = 7$$



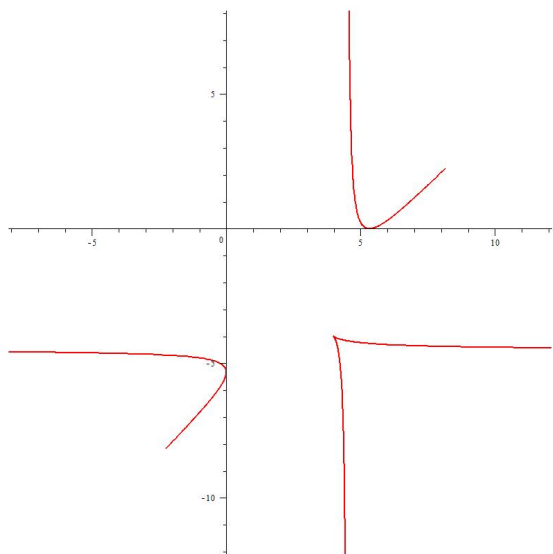
**G. Zostrojte nasledujúce krivky.**



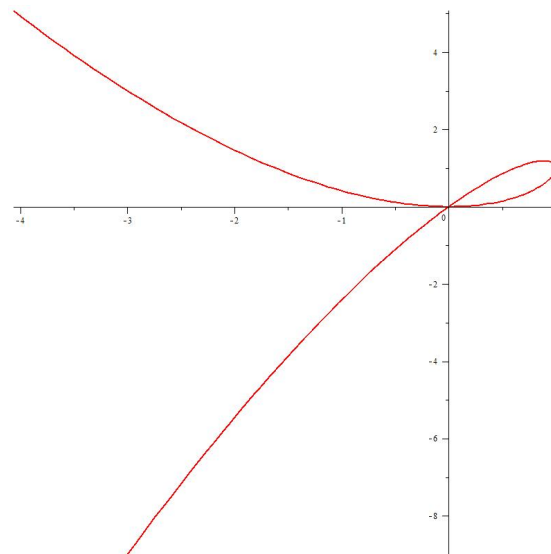
$$x = -5t^2 + 2t^5, y = -3t^2 + 2t^3$$



$$x = t + e^{-t}, y = 2t + e^{-2t}$$



$$x = \frac{(t+2)^2}{t+1}, y = \frac{(t-2)^2}{t-1}$$

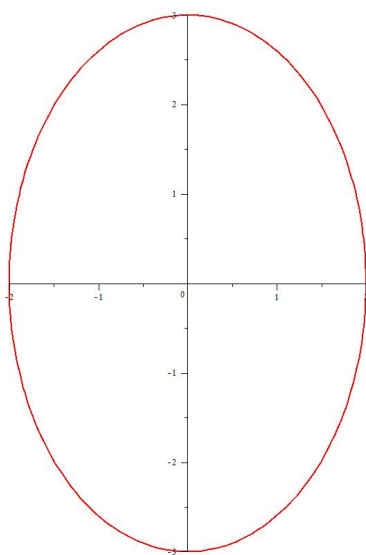


$$x = 2t - t^2, y = 2t^2 - t^3$$

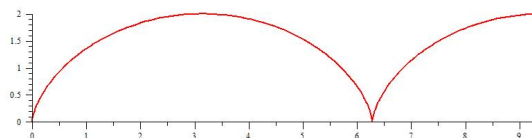
**H. Zistite, či sú zobrazenia Lipschitzovsky spojité, prosté, regulárne.**

1. áno, nie, áno
2. áno, áno, nie
3. nie, áno, áno
4. áno pre  $a \geq 2$ , áno, áno
5. nie, áno, áno
6. áno, nie, nie

**H. Načrtnite ich obory hodnôt.**

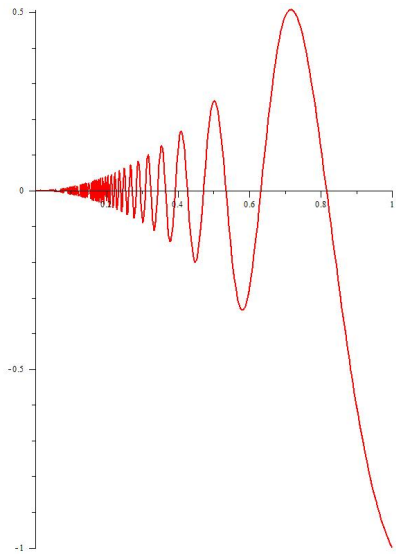


1.  $a=2, b=3$

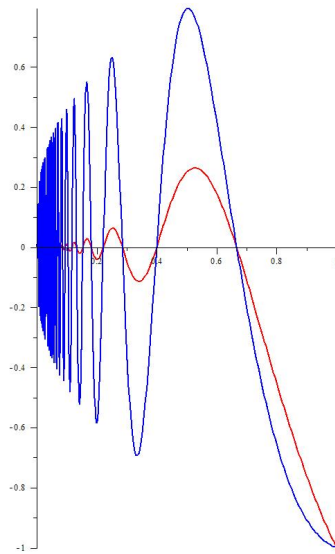


2.

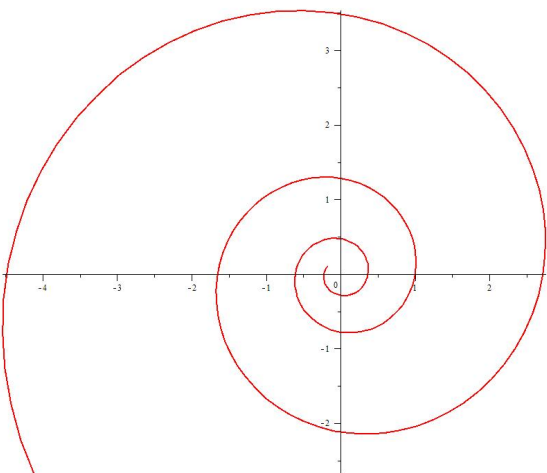




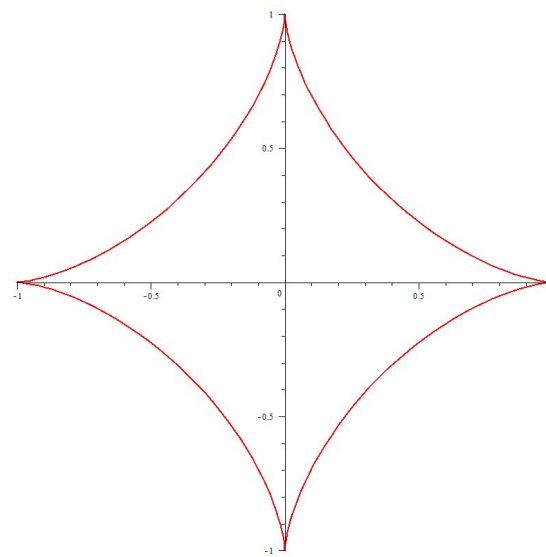
3.



4. red ( $a=2$ ), blue ( $a=\frac{1}{3}$ )

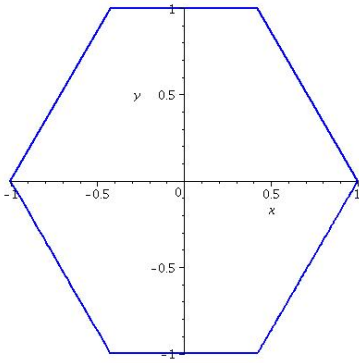


5.



6.

I.



J. Nájďte normu prvku  $u$  v LNP priestore  $(X, \|\cdot\|)$  (s prirodzenou normou, ak nie je povedané inak).

1.  $\frac{2\sqrt{35}}{15}$

2.  $\frac{2\sqrt{3}}{3} + \frac{3\sqrt{2}}{2}$

3.  $\frac{\pi}{\sqrt{6}}$

4.  $\sqrt{2^n n! \sqrt{\pi}}$

5. a) 19, b) 15

6.  $\frac{\sqrt{777}}{21}$

7. 2

8.  $5 + 3^{1-p} 2^p$

9.  $b$

K. Overte, či nasledujúce priestory sú normované.

1. nie, nie

2. nie

3. áno

4. nie

5. áno

6. áno

7. áno

8. nie

9. áno

10. nie

11. nie

12. nie

13. nie

14. áno

15. áno

16. áno

**L. Nájdiť vzdialenosť prvku  $x$  od množiny  $A$  v LNP priestore  $X$ .**

1. 0

2. 1

3.  $\frac{1}{2}$

4.  $\frac{1}{8}$

5.  $\frac{\sqrt{5}}{30} \sqrt{(-720 + 1440\pi - 120\pi^3 + 90\pi^2 + \pi^6)/\pi}$

6.  $\sum_{i=1}^n \frac{1}{i^2} / \sqrt{n}$

7.  $\frac{1}{2}$

8.  $\frac{1}{3}$

**M.  $h \in M : 0 \leq h(x) \leq 2, x \in [0, 1]$**

**O.  $\{a_n\}_{n \in \mathcal{N}} \in \ell^\infty$**