Ťahák pre študentov MAN3c

$$\sin^2\theta + \cos^2\theta = 1 \qquad \cosh^2 x - \sinh^2 x = 1$$

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta \qquad \cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\sin^2\theta = \frac{1-\cos(2\theta)}{2} \qquad \cos^2\theta = \frac{1+\cos(2\theta)}{2}$$

$$2\cos\theta\cos\varphi = \cos(\theta-\varphi) + \cos(\theta+\varphi) \qquad 2\sin\theta\sin\varphi = \cos(\theta-\varphi) - \cos(\theta+\varphi)$$

$$2\sin\theta\cos\varphi = \sin(\theta+\varphi) + \sin(\theta-\varphi) \qquad 2\cos\theta\sin\varphi = \sin(\theta+\varphi) - \sin(\theta-\varphi)$$

$$\sin(2\theta) = 2\cos\theta\sin\theta = \frac{2\tan\theta}{1+\tan^2\theta} \qquad \cos(2\theta) = \cos^2\theta - \sin^2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$$

$$\sin(n\theta) = \sum_{k \text{ nep.}} (-1)^{(k-1)/2}\binom{n}{k}\cos^{n-k}\theta\sin^k\theta \qquad \cos(n\theta) = \sum_{k \text{ pár.}} (-1)^{k/2}\binom{n}{k}\cos^{n-k}\theta\sin^k\theta$$

$$\frac{\pi}{2} = \arcsin x + \arccos x \qquad \frac{\pi}{2} = \arctan x + \operatorname{arccot} x$$

Pre parametricky danú C^2 krivku $\boldsymbol{r} : [a,b] \to \mathbb{R}^3$ $(\boldsymbol{r}' \neq 0)$ je **krivosť** $\kappa(t) = \frac{||\boldsymbol{r}'\times\boldsymbol{r}''||}{||\boldsymbol{r}'||^3}$ a ak je C^3, potom jej **torzia** v neinflexnom bode je $\tau = \frac{(\mathbf{r}'\times\mathbf{r}'')\cdot\mathbf{r}'''}{||\mathbf{r}'\times\mathbf{r}''||^2}$.

Frenetove-Serretove vzorce:

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix}\mathbf{u}\\ \mathbf{a}_n\\ \mathbf{b}\end{bmatrix} = \|\mathbf{r}'\|\begin{bmatrix}0 & \kappa & 0\\ -\kappa & 0 & \tau\\ 0 & -\tau & 0\end{bmatrix}\begin{bmatrix}\mathbf{u}\\ \mathbf{a}_n\\ \mathbf{b}\end{bmatrix}$$

Sférické súradnice v $\mathbb{R}^3$:

$$x = \rho\sin\phi\cos\theta, \qquad \rho = \sqrt{x^2+y^2+z^2},$$
$$y = \rho\sin\phi\sin\theta, \qquad \phi = \arccos\frac{z}{r} = \arctan\frac{\sqrt{x^2+y^2}}{z}$$
$$z = \rho\cos\phi \quad , \qquad \theta = \arctan\left(\frac{y}{x}\right),\ x>0 \quad [-\pi/2 < \theta \leq \pi/2]$$

kde $-\pi < \theta \leq \pi$, $0 < \phi < \pi$, $\rho > 0$ a $h_\rho = 1$, $h_\phi = \rho$, $h_\theta = \rho\sin\phi$.

Sférické súradnice v $\mathbb{R}^n$:

$$x_1 = \rho\cos\phi_1,$$
$$\vdots$$
$$x_j = \rho\cos\phi_j\prod_{i=1}^{j-1}\sin\phi_i,$$
$$\vdots$$
$$x_n = \rho\prod_{i=1}^{n-1}\sin\phi_i$$

$\rho > 0$, $\phi_{n-1} \in (-\pi,\pi]$, $\phi_i \in (0,\pi)$, $i = 1,\dots,n-2$ a $h_\rho = \rho$, $h_{\phi_i} = r\prod_{i=1}^{j-1}\sin\phi_i$

Nech $\phi, \boldsymbol{F} \in C^1$, potom pre (krivočiaré) ortogonálne súradnice platí

- $$\nabla\phi = \left(\frac{1}{h_1}\frac{\partial\phi}{\partial q_1}, \frac{1}{h_2}\frac{\partial\phi}{\partial q_2}, \frac{1}{h_3}\frac{\partial\phi}{\partial q_3}\right)$$
- $$\operatorname{div}\boldsymbol{F} = \frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial q_1}(F_1h_2h_3) + \frac{\partial}{\partial q_2}(F_2h_1h_3) + \frac{\partial}{\partial q_3}(F_3h_2h_1)\right]$$
- $$\operatorname{rot}\boldsymbol{F} = \frac{1}{h_1h_2h_3}\begin{vmatrix} \boldsymbol{h_1} & \boldsymbol{h_2} & \boldsymbol{h_3} \\ \frac{\partial}{\partial q_1} & \frac{\partial}{\partial q_2} & \frac{\partial}{\partial q_3} \\ F_1h_1 & F_2h_2 & F_3h_3 \end{vmatrix}$$
- $$\Delta\phi = \frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial q_1}\left(\frac{h_2h_3}{h_1}\frac{\partial\phi}{\partial q_1}\right) + \frac{\partial}{\partial q_2}\left(\frac{h_1h_3}{h_2}\frac{\partial\phi}{\partial q_2}\right) + \frac{\partial}{\partial q_3}\left(\frac{h_2h_1}{h_3}\frac{\partial\phi}{\partial q_3}\right)\right]$$

Vlastnosti skalárneho a vektorového súčinu:

- $\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$
- $\mathbf{A} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{A}$
- $\mathbf{A} \times \mathbf{B} = -\mathbf{B} \times \mathbf{A}$
- $(\mathbf{A} + \mathbf{B}) \cdot \mathbf{C} = \mathbf{A} \cdot \mathbf{C} + \mathbf{B} \cdot \mathbf{C}$
- $(\mathbf{A} + \mathbf{B}) \times \mathbf{C} = \mathbf{A} \times \mathbf{C} + \mathbf{B} \times \mathbf{C}$
- $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = \mathbf{B} \cdot (\mathbf{C} \times \mathbf{A}) = \mathbf{C} \cdot (\mathbf{A} \times \mathbf{B})$
- $\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \cdot \mathbf{C})\,\mathbf{B} - (\mathbf{A} \cdot \mathbf{B})\,\mathbf{C}$
- $(\mathbf{A} \times \mathbf{B}) \cdot (\mathbf{C} \times \mathbf{D}) = (\mathbf{A} \cdot \mathbf{C})\,(\mathbf{B} \cdot \mathbf{D}) - (\mathbf{B} \cdot \mathbf{C})\,(\mathbf{A} \cdot \mathbf{D})$
- $(\mathbf{A} \times \mathbf{B}) \times (\mathbf{C} \times \mathbf{D}) = (\mathbf{A} \cdot \mathbf{B} \times \mathbf{D})\,\mathbf{C} - (\mathbf{A} \cdot \mathbf{B} \times \mathbf{C})\,\mathbf{D}$

Vzťahy diferenciálnych operátorov

- $\operatorname{div}\boldsymbol{\phi u} = \phi \operatorname{div}\boldsymbol{u} + \nabla\phi \cdot \boldsymbol{u}$
- $\operatorname{div}\boldsymbol{(u \times v)} = \operatorname{rot}\boldsymbol{u} \cdot \boldsymbol{v} - \operatorname{rot}\boldsymbol{v} \cdot \boldsymbol{u}$
- $\operatorname{rot}\boldsymbol{(u \times v)} = (\boldsymbol{v} \cdot \nabla)\boldsymbol{u} - (\boldsymbol{u} \cdot \nabla)\boldsymbol{v} + (\operatorname{div}\boldsymbol{v})\boldsymbol{u} - (\operatorname{div}\boldsymbol{u})\boldsymbol{v}$
- $\operatorname{rot}\boldsymbol{(\phi u)} = \phi \operatorname{rot}\boldsymbol{u} + \nabla\phi \times \boldsymbol{u}$
- $\operatorname{rot}\boldsymbol{\nabla\phi} = \boldsymbol{0}$
- $\operatorname{rot}\boldsymbol{(\mathrm{rot}\, u)} = \nabla(\operatorname{div}\boldsymbol{u}) - \Delta\boldsymbol{u}$
- $\operatorname{div}\boldsymbol{(\mathrm{rot}\, u)} = 0$
- $\nabla(\boldsymbol{u} \cdot \boldsymbol{v}) = \boldsymbol{u} \times \operatorname{rot}\boldsymbol{v} + \boldsymbol{u} \times \operatorname{rot}\boldsymbol{u} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{v} + (\boldsymbol{v} \cdot \nabla)\boldsymbol{u}$
- $(\boldsymbol{u} \cdot \nabla)\boldsymbol{u} = \operatorname{rot}\boldsymbol{u} \times \boldsymbol{u} + \frac{1}{2}\nabla|\boldsymbol{u}|^2$

Eulerov integrál 2. druhu

Gamma funkcia

$$\Gamma(z) = \int_0^\infty t^{z-1} e^{-t}\,\mathrm{d}t, \qquad \Re(z) > 0$$

$$\Gamma(n) = (n-1)!, \quad n \in \mathcal{N}$$

$$\Gamma(z+1) = z\Gamma(z)$$

$$\Gamma(z)\Gamma(1-z) = \frac{\pi}{\sin \pi z}, \quad 0 < z < 1$$

$$\Gamma\left(-\tfrac{3}{2}\right) = \tfrac{4\sqrt{\pi}}{3}$$

$$\Gamma\left(-\tfrac{1}{2}\right) = -2\sqrt{\pi}$$

$$\Gamma\left(\tfrac{1}{2}\right) = \sqrt{\pi}$$

$$\Gamma\left(\tfrac{3}{2}\right) = \tfrac{\sqrt{\pi}}{2}$$

$$\Gamma\left(\tfrac{5}{2}\right) = \tfrac{3\sqrt{\pi}}{4}$$

$$\Gamma\left(\tfrac{7}{2}\right) = \tfrac{15\sqrt{\pi}}{8}$$

Eulerov integrál 1. druhu

Beta funkcia

$$\mathrm{B}(z_1, z_2) = \int_0^1 t^{z_1-1}(1-t)^{z_2-1}\,\mathrm{d}t, \qquad \Re(z_1), \Re(z_2) > 0$$

$$\mathrm{B}(z_1, z_2) = \frac{\Gamma(z_1)\,\Gamma(z_2)}{\Gamma(z_1+z_2)}$$

$$\mathrm{B}(z_1, z_2) = \mathrm{B}(z_2, z_1)$$

$$\mathrm{B}(z_1, z_2) = \mathrm{B}(z_1, z_2+1) + \mathrm{B}(z_1+1, z_2)$$

$$\mathrm{B}(z_1, z_2) \cdot \mathrm{B}(z_1+z_2, 1-z_2) = \frac{\pi}{z_1 \sin(\pi z_2)}$$

$$\mathrm{B}(z_1+1, z_2) = \mathrm{B}(z_1, z_2) \cdot \frac{z_1}{z_1+z_2}$$

$$\mathrm{B}(z_1, z_2+1) = \mathrm{B}(z_1, z_2) \cdot \frac{z_2}{z_1+z_2}$$