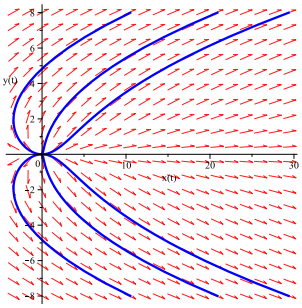
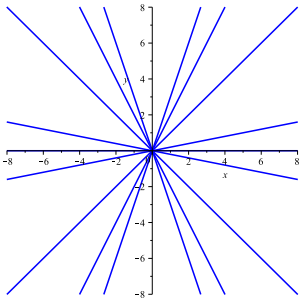
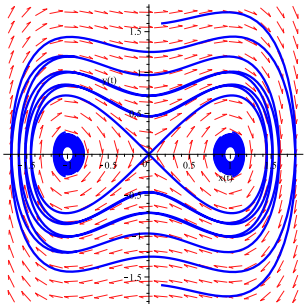
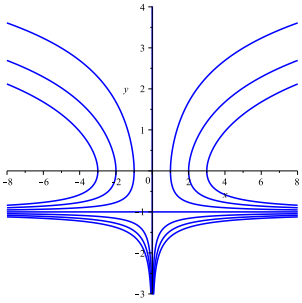
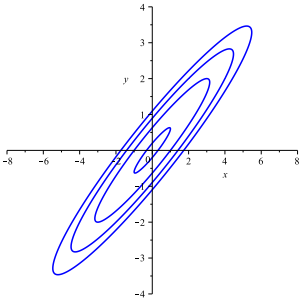
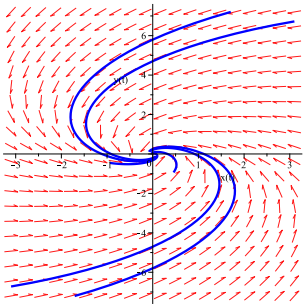
Jozef Kiseľák

Výsledky úloh na precvičenie III.
March 19, 2015

A. Nájdite prvý integrál a načrtnite fázový portrét sústavy dvoch diferenciálnych rovníc.

1. $\left(x - \frac{y^2}{3}\right)^2 = c|y|$
2. $y = cx$
3. $\frac{y^2 - x^2}{2} + \frac{x^4}{4} = c$
4. $(1+y)x\mathrm{e}^{-y} = c$
5. $\frac{5}{2}y^2 + x^2 - 3xy = c$
6. $-\frac{1}{2}\ln\sqrt{\left(\frac{x}{y}\right)^2 + 1} + \arctan\frac{x}{y} - \ln|y| = c$

B. Nájdite aspoň dva nezávislé prvé integrály sústavy troch diferenciálnych rovníc.

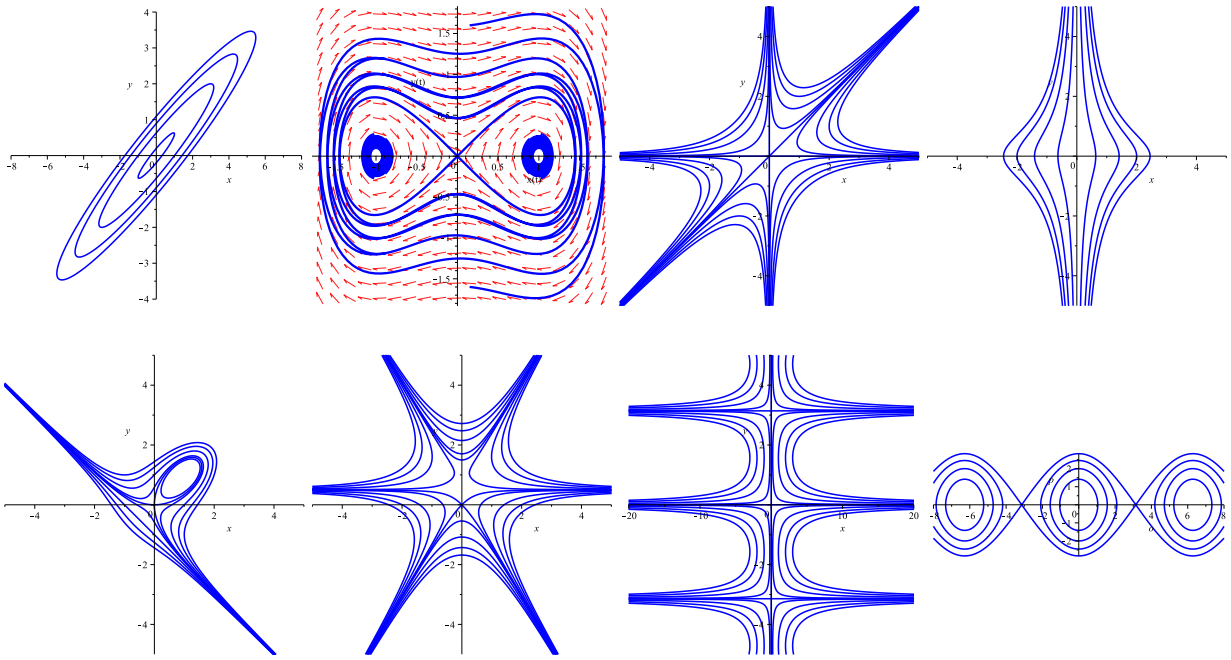
1. $I_1 = x + y + z, \;\; I_2 = x^2 + y^2 + z^2$
2. $I_1 = (x - z)\mathrm{e}^t, \;\; I_2 = (y - z)\mathrm{e}^t$
3. $I_1 = x + z, \;\; I_2 = x(x + y + z) - \dfrac{y^2}{2}$
4. $I_1 = z + \dfrac{z^2}{2} + x^2, \;\; I_2 = x^2 + y^2$
5. $I_1 = \dfrac{z^2}{2} - z + x^2, \;\; I_2 = z + \dfrac{z^2}{2} - y^2$
6. $I_1 = z^2 + z - y^2 - y, \;\; I_2 = \dfrac{y^3}{3} + \dfrac{y^2}{2} - x^2$
7. $I_1 = Ap^2 + Bq^2 + Cr^2, \;\; I_2 = A^2p^2 + B^2q^2 + C^2r^2$ (okrem prípadu $A = B = C$)
8. $I_1 = z^2 + y^2, \;\; I_2 = yz - x$
9. $I_1 = \dfrac{y}{x}, \;\; I_2 = xy - 2\sqrt{z^2 + 1}$
10. $I_1 = \dfrac{z}{y}, \;\; I_2 = \dfrac{x}{y} - \dfrac{z^2}{y} - y$
11. $I_1 = \dfrac{z}{y}, \;\; I_2 = y + \dfrac{y^3}{x^2}$
12. $I_1 = \arcsin(y) + \ln|x|, \;\; I_2 = zy + \dfrac{ax^2}{2}$

C1. Zistite, ktoré zo systémov sú gradientné/Hamiltonove a nájdite potenciál/Hamiltonián.

1. $U(x, y) = -\mathrm{e}^{-x^2 - y^2}$
2. "ani ani"
3. "ani ani"
4. "ani ani"
5. $U(x, y) = -x \sin y$
6. $H(x, y) = \dfrac{y^2 - x^2}{2} + xy^2 - \dfrac{x^3}{3}$
7. "ani ani"
8. $U(x, y) = -x^3 + (x + 1)\mathrm{e}^{-2y}$

C2. Ukážte, že systém je Hamiltonov, nájdite Hamiltonián a načrtnite fázový portrét.

1. $H(x,y) = \frac{5}{2}y^2 + x^2 - 3xy$
2. $H(x,y) = \frac{y^2 - x^2}{2} + \frac{x^4}{4}$
3. $H(x,y) = x^2y - xy^2$
4. $H(x,y) = \frac{x^2(y^2+1)}{2}$
5. $H(x,y) = y^3 - 3xy + x^3$
6. $H(x,y) = \frac{y^2 - x^2}{2} + x^2y - \frac{y^3}{3}$
7. $H(x,y) = x \sin y$
8. $H(\omega,\phi) = \frac{\phi^2}{2} - \frac{g}{l}\cos\omega, \quad g,l > 0$



D. Ukážte, že pre netlmenú Duffingovu rovnicu $x'' + x' + \beta x^3 = \gamma \cos \omega t$ existuje Hamiltonián.

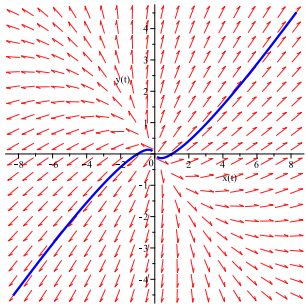
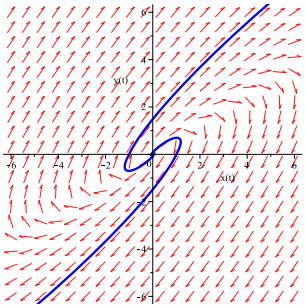
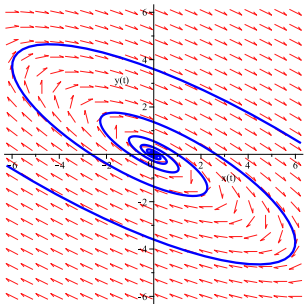
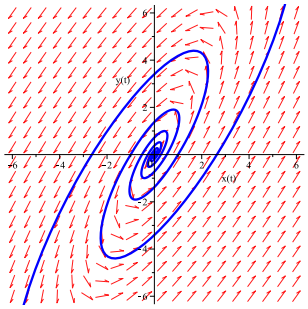
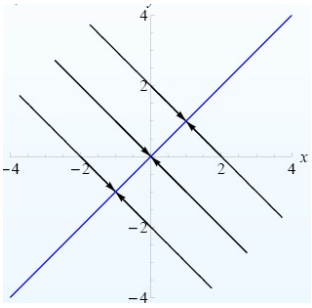
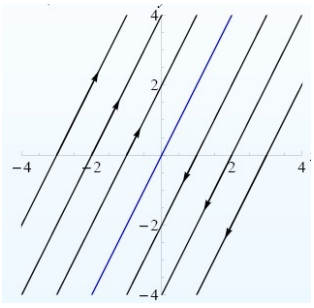
$H(x,x',t) = \frac{x^2 + (x')^2}{2} + \frac{\beta x^4}{4} - \gamma x \cos \omega t$

E. Ukážte, že systém je Hamiltonov, nájdite Hamiltonián.

1. $H(\mathbf{x}) = 2x_2x_3x_4 - x_1 - x_3 + \dfrac{x_4^3}{3} - \cos x_2$
2. $H(\mathbf{q}) = q_1^2 - q_1q_2 + q_2^2 + \dfrac{(q_1')^2 + (q_2')^2}{2}$
3. $H(\mathbf{q}) = \dfrac{(q_1')^2 + (q_2')^2}{2} - \dfrac{1}{\sqrt{q_1^2 + q_2^2}}$
4. $H(x, y, u, v) = \dfrac{Ax^2 + By^2 + u^2 + v^2}{2} - x^2y - \dfrac{\epsilon y^3}{3}$
5. $H(\mathbf{q}) = \frac{(q_1')^2+(q_2')^2}{2} + \frac{q_1^4+q_2^4}{4} + \frac{a}{2}\, q_1^2q_2^2$
6. $H(\mathbf{q}) = \frac{(q_1')^2+(q_2')^2}{2} + \frac{q_1^4+q_2^4}{4} - \frac{q_1^2+q_2^2}{2} + q_1^2q_2^2$
7. $H(\mathbf{q}) = \frac{(q_1')^2+(q_2')^2}{2} - \frac{a}{3}\,(q_1^3 + q_2^3) + \frac{\nu}{2}\,(q_1^2 + q_2^2)$

F. Vyšetrite stabilitu a načrtnite fázový portrét sústavy dvoch lineárnych diferenciálnych rovníc.

1. $\lambda_1 = 2, \lambda_2 = 4 \Rightarrow$ nest. uzol
2. $\lambda_{1,2} = 2 \pm 2\mathrm{i} \Rightarrow$ nest. ohnisko
3. $\lambda_{1,2} = \frac{1}{2} \pm \frac{\sqrt{15}}{2}\mathrm{i} \Rightarrow$ nest. ohnisko
4. $\lambda_{1,2} = \frac{1}{2} \pm \frac{\sqrt{15}}{2}\mathrm{i} \Rightarrow$ nest. ohnisko
5. Priamka $y = x$ sú stab. SB
6. Priamka $2y = x$ sú nestab. SB

G. Nájdite riešenie systému.

$$x(t) = c_1 \mathrm{e}^{t} + c_2 \mathrm{e}^{t(t-1)} + \frac{\sqrt{\pi}}{2} \mathrm{e}^{t(t-1)+1} \operatorname{erf}(t-1),$$

$$y(t) = c_1 \mathrm{e}^{t} - c_2 \mathrm{e}^{t(t-1)} - \frac{\sqrt{\pi}}{2} \mathrm{e}^{t(t-1)+1} \operatorname{erf}(t-1).$$

H. Využite Routheove-Stodolove-Hurwitzove kritérium na určenie regiónu asymptotickej stability systémov.