

Riešenia úloh na precvičenie VIII.  
15. máj 2014

A.

c) jsou ortogonální v  $L_2(0, \infty)$  s vahou  $\rho = e^{-x}$ .

114. Dokažte rekurentní vztahy pro Hermitovy polynomy

$$\xi H_n(\xi) = n H_{n-1}(\xi) + \frac{1}{2} H_{n+1}(\xi)$$

$$H'_n = 2n H_{n-1}.$$

115. Určete hodnotu výrazu

$$\langle \xi \rangle = \int_{-\infty}^{\infty} \psi_n^2(\xi) \xi^2 d\xi, \quad \psi_n = (\gamma n! 2^n)^{1/2} \pi H_n(\xi) \cdot \exp\left(-\frac{\xi^2}{h}\right).$$

Poznámka: tento výraz představuje, až na numerickou konstantu, střední kvadratickou odchylku od střední hodnoty souřadnice kvantového lineárního harmonického oscilátoru s energií  $E_n = h(n + \frac{1}{2})$ .

Výsledky kapitoly 1

1.1.

**1** Postup jako v příkladě A.: a) užitím sudosti funkce  $\cos nx$  a sudého prodloužení funkce  $f \in L_2(0, 2\pi)$ ; b) podle věty 5. s volbou  $p = \rho = 1$ ,  $q = 0$ ,  $\alpha = \gamma = 0$ ,  $\beta = \delta = 1$ . **3**  $\{\sin(2n+1)t\}_{n=0}^{\infty}$ ; koeficienty funkce  $f$  odpovídají koeficientům (vzhledem k systému (6)) funkce  $f$ , kterou dostaneme nejprve „sudým“ prodloužením funkce  $f$  kolem bodu  $\frac{\pi}{2}$  (tj. platí  $\tilde{f}(\frac{\pi}{2}+t) = f(\frac{\pi}{2}-t)$ ), jinak řečeno  $\tilde{f}(\pi-t) = f(t)$ ) a pak lichým prodloužením takto vzniklé funkce kolem počátku. **4**  $\{\cos(2n+1)t\}_{n=0}^{\infty}$ . **5**  $\{\mu^n \cos \frac{\mu n t}{t} + H t \sin \frac{\mu n t}{n}\}_{n=1}^{\infty}$ ,  $t \in (0, l)$ , kde  $\mu_n$  jsou kladné kořeny rovnice  $2 \cot \mu = \frac{\mu}{H} - \frac{H}{\mu}$ . **6**  $\{\sin nx\}_{n=0}^{\infty}$  (rozdíl proti příkladu A. je pouze ve vlastních číslech  $\lambda_n$ ). **7** a)  $P_0 = 1, P_1 = t, P_2 = \frac{3}{2}t - \frac{1}{2}, P_3 = \frac{5}{2}t^3 - \frac{7}{2}t$ ; b) ne: z podmínky  $\alpha P_0(-1) + \beta P_0'(-1) = 0$  dostaneme  $\alpha = 0$ , podobně dostaneme  $\gamma = 0$ ; z podmínky  $\beta P_1'(-1) = \delta P_1''(1) = 0$  plyne  $\beta = \delta = 0$ . **8** Užití výsledků příkladů A. a B. a tvrzení věty 7.

(a)

1.2.

- 9**  $a_{2k} = b_{2k} = 0$ , **10**  $a_{4k+1} = b_{4k+1} = 0$ , **11**  $a_{2k} = b_k = 0$ .
- 12**  $a_k = b_k = 0$ , **13**  $a_{2k+1} = b_k = 0$ , **14**  $a_k = b_{k+1} = 0$ .
- 15**  $f(-z) = f(z), f(\pi-z) = -f(z)$ , **16**  $\frac{1}{2} - \frac{1}{2} \cos 2z + \frac{1}{3} \cos 4z$ .
- 17**  $\frac{1}{2} \sum_{n=1}^{\infty} \frac{\cos 2nz}{2n-1}$ , **18**  $\frac{1}{2} \sum_{n=1}^{\infty} \frac{\cos 2nz}{2n-1} \sin \frac{(2n-1)z}{2}$ .
- 19**  $\frac{1}{2} \sum_{n=1}^{\infty} \frac{\cos 2nz}{2n-1}$ , **20**  $\sum_{n=1}^{\infty} \frac{\cos 2nz}{2n-1} \cos z$ .
- 21**  $\frac{1}{2} \sum_{n=1}^{\infty} \frac{\cos 2nz}{2n-1} \cos z$ .
- 22**  $\frac{1}{2} \sum_{n=1}^{\infty} \frac{\cos 2nz}{2n-1} \cos z$ .
- 23**  $\frac{1}{2} \sum_{n=1}^{\infty} \frac{\cos 2nz}{2n-1} \cos z$ .
- 24**  $\frac{1}{2} \sum_{n=1}^{\infty} \frac{\cos 2nz}{2n-1} \cos z$ .
- 25**  $\frac{1}{2} \sum_{n=1}^{\infty} \frac{\cos 2nz}{2n-1} \cos z$ .
- 26**  $\frac{1}{2} \sum_{n=1}^{\infty} \frac{\cos 2nz}{2n-1} \cos z$ .
- 27**  $2 \sin 4z \left( \frac{1}{2} + \sum_{n=1}^{\infty} \frac{\cos 2nz}{2n-1} \right)$ .
- 28**  $1 - \frac{1}{2} \cos z + \frac{1}{3} \cos 3z - \frac{1}{4} \cos 5z + \dots$ .
- 29**  $1 - \frac{1}{2} \cos z + \frac{1}{3} \cos 3z - \frac{1}{4} \cos 5z + \dots$ .
- 30**  $\frac{1}{2} \sum_{n=1}^{\infty} \frac{\cos 2nz}{2n-1} \cos z$ .
- 31**  $\frac{1}{2} \sum_{n=1}^{\infty} \frac{\cos 2nz}{2n-1} \cos z$ .
- 32**  $\frac{1}{2} \sum_{n=1}^{\infty} \frac{\cos 2nz}{2n-1} \cos z$ .
- 33**  $\frac{1}{2} \sum_{n=1}^{\infty} \frac{\cos 2nz}{2n-1} \cos z$ .
- 34**  $\frac{1}{2} \sum_{n=1}^{\infty} \frac{\cos 2nz}{2n-1} \cos z$ .
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- 37**  $\frac{1}{2} \sum_{n=1}^{\infty} \frac{\cos 2nz}{2n-1} \cos z$ .
- 38**  $\frac{1}{2} \sum_{n=1}^{\infty} \frac{\cos 2nz}{2n-1} \cos z$ .
- 39**  $\frac{1}{2} \sum_{n=1}^{\infty} \frac{\cos 2nz}{2n-1} \cos z$ .
- 40** a)  $\sin z$ ; b) viz příklad 35. **41** a)  $2z \sum_{n=1}^{\infty} (-1)^{n+1} \sin z$ .
- 42** a)  $2z \sum_{n=1}^{\infty} (-1)^{n+1} \sin z$ ; b)  $\frac{1}{2} + 4 \sum_{n=1}^{\infty} (-1)^{n+1} \sin z$ .
- 43** a)  $\frac{1}{2} + 6z \sum_{n=1}^{\infty} (-1)^{n+1} \sin z$ ; b)  $\frac{1}{2} + 6z \sum_{n=1}^{\infty} (-1)^{n+1} \sin z$ .
- 44** a)  $\frac{1}{2} + 6z \sum_{n=1}^{\infty} (-1)^{n+1} \sin z$ ; b)  $\frac{1}{2} + 6z \sum_{n=1}^{\infty} (-1)^{n+1} \sin z$ .
- 45**  $\frac{1}{2} + 6z \sum_{n=1}^{\infty} (-1)^{n+1} \sin z$ .

(b)

- 46**  $\frac{1}{2} + \frac{1}{2} \sum_{n=1}^{\infty} \frac{\cos 2nz}{2n-1}$ .
- 47**  $\frac{1}{2} + \frac{1}{2} \sum_{n=1}^{\infty} \frac{\cos 2nz}{2n-1}$ .
- 48**  $\frac{1}{2} + \frac{1}{2} \sum_{n=1}^{\infty} \frac{\cos 2nz}{2n-1}$ .
- 49**  $\frac{1}{2} + \frac{1}{2} \sum_{n=1}^{\infty} \frac{\cos 2nz}{2n-1}$ .
- 50**  $\frac{1}{2} + \frac{1}{2} \sum_{n=1}^{\infty} \frac{\cos 2nz}{2n-1}$ .
- 51**  $\frac{1}{2} + \frac{1}{2} \sum_{n=1}^{\infty} \frac{\cos 2nz}{2n-1}$ .
- 52**  $\frac{1}{2} \ln(1+\sqrt{2}) + \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n-1} \cos(2n-1)z$ .
- 53**  $\ln 2 - \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n-1} \cos(2n-1)z$ .
- 54**  $\ln 2 - \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n-1} \cos(2n-1)z$ .
- 55**  $2 \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n-1} \cos(2n-1)z$ .
- 56**  $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n-1} \cos(2n-1)z$ .
- 57**  $\frac{1}{2} \sum_{n=1}^{\infty} \frac{\cos 2nz}{2n-1}$ .
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- 75**  $\frac{1}{2} \sum_{n=1}^{\infty} \frac{\cos 2nz}{2n-1}$ .
- 76**  $\frac{1}{2} \sum_{n=1}^{\infty} \frac{\cos 2nz}{2n-1}$ .
- 77**  $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n-1} \cos(2n-1)z$ .
- 78**  $\frac{1}{2} \sum_{n=1}^{\infty} \frac{\cos 2nz}{2n-1}$ .
- 79**  $\frac{1}{2} \sum_{n=1}^{\infty} \frac{\cos 2nz}{2n-1}$ .
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- 90**  $\frac{1}{2} \sum_{n=1}^{\infty} \frac{\cos 2nz}{2n-1}$ .
- 91**  $\frac{1}{2} \sum_{n=1}^{\infty} \frac{\cos 2nz}{2n-1}$ .
- 92**  $C_1 = \sqrt{\frac{2}{\pi}} \sin(\frac{\pi}{2})$ ,  $D_1 = 0$ , **93**  $C_1 = -\sqrt{\frac{2}{\pi}} \sin(\frac{\pi}{2})$ ,  $D_1 = 0$ .
- 94**  $C_1 = \sqrt{\frac{2}{\pi}} \sin(\frac{\pi}{2})$ ,  $D_1 = 0$ .

1.4.

**92**  $C_1 = \sqrt{\frac{2}{\pi}} \sin(\frac{\pi}{2})$ ,  $D_1 = 0$ , **93**  $C_1 = -\sqrt{\frac{2}{\pi}} \sin(\frac{\pi}{2})$ ,  $D_1 = 0$ .  
**94**  $C_1 = \sqrt{\frac{2}{\pi}} \sin(\frac{\pi}{2})$ ,  $D_1 = 0$ .  
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**98**  $C_1 = \sqrt{\frac{2}{\pi}} \sin(\frac{\pi}{2})$ ,  $D_1 = 0$ .  
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**114**  $C_1 = \sqrt{\frac{2}{\pi}} \sin(\frac{\pi}{2})$ ,  $D_1 = 0$ .  
**115**  $C_1 = \sqrt{\frac{2}{\pi}} \sin(\frac{\pi}{2})$ ,  $D_1 = 0$ .

**A1. Pomocou Charpitovej metódy vyriešte rovnicu.**

1.  $au(x, y) = \frac{(ax+y)^2}{2} + c$

5.  $x + \frac{y}{a+yu} = b$

2.  $u(x, y) = bx^a y^{\frac{1}{a}}$

6.  $u(x, y) = 2axe^{-y} - \frac{a^2}{2}e^{-2y} + b$

3.  $u(x, y)^2 = (ax + b)^2 + a^2y^2$

7.  $u(x, y) = a(a + \frac{y}{x}) + bx^2$

4.  $u(x, y)^2 = (ay + b)^2 + a^2x^2$

8.  $\ln(u(x, y) - ax) = y - a \ln(a + y) + b$

**B. Sčítajte nasledujúce rady na daných intervaloch.**

1.  $-\ln(2 \sin \frac{x}{2})$

5.  $\frac{xy}{4}$

2.  $\int_0^x \int_0^y \ln(2 \sin \frac{t}{2}) dt dy + \sum_{n=1}^{\infty} \frac{1}{n^3}$

6.  $\Sigma = \begin{cases} \frac{\pi}{4}, & 0 < x < 2y, \\ 0, & 2y < x < 2(\pi - y), \\ -\frac{\pi}{4}, & 2(\pi - y) < x < 2\pi, \\ \frac{\pi}{8}, & 0 < x = 2y < \pi, \\ -\frac{\pi}{8}, & \pi < x = 2(\pi - y) < 2\pi, \end{cases}$

3.  $\frac{x^2}{4} - \frac{\pi^2}{12}$

4.  $\frac{1}{2} \int_0^x \int_0^y \ln(2 \tan \frac{t}{2}) dt dy + \sum_{k \text{ je nep.}}^{\infty} \frac{1}{k^3}$

7.  $e^{\cos x} \cos(\sin x)$

**C.**

$a_0 = \frac{2}{\pi}, b_n = 0, a_n = \frac{2(-1+(-1)^{1+n})}{\pi(n^2-1)}, n \geq 1$

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{16n^2 - 1} = \frac{1}{2} - \frac{\sqrt{2}\pi}{8}$$

**D. Vypočítajte.**

1.  $\frac{\pi^2}{6}$

3.  $\frac{\pi^3}{32} - 1$

2.  $\frac{\pi^4}{90}$

4.  $\frac{\pi^3}{12}$

**E.**

Pre  $f$  sú  $a_0 = a_n = 0$  a  $b_n = -\frac{4(-1+(-1)^n)}{n^3\pi}$ .

Pre  $f'$  sú  $a_0 = 0, a_n = -\frac{4(-1+(-1)^n)}{n^2\pi} \geq 0$  a  $b_n = 0$ .

**G.**

Pre  $|x|$  sú  $a_0 = \frac{\pi}{2}, a_n = \frac{2(-1+(-1)^n)}{n^2\pi}$  a  $b_n = 0$ .

**H.**

$$f(x) = \sum_{n=0}^{\infty} A_n L_n(x),$$

kde

$$A_n = \left(-\frac{1}{2}\right)^{\frac{n-1}{2}} \frac{(2n+1)(n-2)!!}{2 \left(\frac{n+1}{2}\right)!}, n \text{ nepárne}, A_n = 0, n \text{ párne}$$

**K. Použitím Fourierových radov nájdite riešenie začiatočnej, resp. okrajovej úlohy.**

1.  $\sum_{n=1}^{\infty} \frac{6(-1)^n}{\pi n(\pi^2 n^2 - 2)} \sin \pi n x$

2.  $\frac{6 - \pi^2}{3\pi^4} + \frac{6}{\pi^4} \cos \pi x + \frac{\pi^2 - 4}{\pi^5} \sin \pi x + \sum_{n=2}^{\infty} \left( \frac{4(-1)^{n+1}}{\pi^4 n^2} \cos \pi n x + \frac{2(-1)^{n+1}}{\pi^3 n} \sin \pi n x \right)$

3.  $-e^x \sum_{n=1}^{\infty} \frac{1}{n!(n^2 + 1)} + \sum_{n=1}^{\infty} \left( \frac{1}{(n-1)!(n^2 + 1)} \sin n x + \frac{1}{n!(n^2 + 1)} \cos n x \right)$

4.  $\sum_{n=1}^{\infty} \frac{2(-1 + (-1)^n n^2 \pi^2 \cos 1 - (-1)^n n^2 \pi^2 + (-1)^n)}{\pi n(\pi^2 n^2 - 1)} \sin \pi n x$

**M.**

$$y(x) = \sum_{n=1}^{\infty} \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L} \left( C_n \cos \frac{n\pi ct}{L} + D_n \sin \frac{n\pi ct}{L} \right),$$

kde

$$C_n = \sqrt{\frac{2}{L}} \frac{4hL}{n^2 \pi^2} \sin \frac{n\pi}{2}, D_n = 0$$

**R. Nájdite Fourierove transformácie nasledujúcich funkcií.**

1. 
$$-\frac{\sqrt{\pi a} \text{ie}^{-\frac{w^2}{4a}}}{2}$$

2. 
$$\frac{2 \sin w}{w} - \frac{4 \sin^2(w/2)}{w^2}$$

3. 
$$\frac{2 \sin(aw)}{w}$$

4. 
$$\frac{4 \sin^2(aw/2)}{aw^2}$$

5. 
$$\frac{4 \sin(u/2) \sin(v/2)}{uv}$$

**T. Spočítajte**

1. 
$$E(\alpha x) * E(\beta x) = \frac{E(\alpha x) - E(\beta x)}{\beta - \alpha}, \alpha \neq \beta$$

2. 
$$E(\alpha x) * E(\alpha x) = xE(\alpha x)$$

3. 
$$(f * f)(x) = \begin{cases} \frac{x^3}{6} - x^2 + x, & x \in \langle 0, 1 \rangle, \\ -\frac{x^3}{6} + x^2 - 2x + \frac{4}{3}, & x \in \langle 1, 2 \rangle, \\ 0, & \text{inak.} \end{cases}$$

4. 
$$\Lambda(x)\Lambda(y). \text{ Využite fakt, že } \Pi(x, y) * \Pi(x, y) = \Pi(x)\Pi(y) * \Pi(x)\Pi(y)$$

**U. Nájdite riešenie integrálnej rovnice.**

1. 
$$y(x) = \frac{2(x - \sin x)}{\pi x^2}$$

2. 
$$\psi(x) = \frac{2x}{\pi(x^2 + 1)}$$

3. 
$$y(x) = \frac{\sqrt{2}e^{-2x^2}}{\pi^{\frac{1}{4}}}$$

**V. Pomocou Fourierovej transformácie vyriešte diferenciálnu rovnicu.**

1. 
$$y(x) = 3e^{-2x} \begin{cases} \frac{e^{3x}}{3}, & x \leq 0 \\ e^x - \frac{2}{3}, & \text{inak} \end{cases}$$

2. 
$$y(x) = \begin{cases} \frac{1}{10} \sin x - \frac{3}{10} \cos x - \frac{e^{-2x}}{5} - \frac{e^{-x}}{2}, & x \geq 0 \\ 0, & \text{inak} \end{cases}$$

**W. Pomocou Fourierovej transformácie riešte rovnicu.**

1.  $u(x, t) = g(x - t^2/3)$

2.  $u(x, t) = h(x - t)e^{-t}$

3.  $u(x, t) = \int_{\mathbb{R}} \frac{e^{-\frac{(x-y)^2}{4c^2t}}}{2c\sqrt{\pi t}} f(y) dy$

4.  $u(x, t) = 1/2 \left( \frac{1}{1+(x+t)^2} + \frac{1}{1+(x-t)^2} \right)$

1.  $u(x, t) = \frac{\sqrt{2}}{\sqrt{\pi t}} \left( \frac{\sin(t+cx)}{t+cx} + \frac{\sin(t-cx)}{t-cx} \right)$

2.  $u(x, t) = -\frac{x}{2} \operatorname{erf} \left( \frac{x}{2c\sqrt{t}} \right) + \frac{1}{4c\sqrt{\pi t}} \left( e^{-\frac{(x-2)^2}{4t}} + e^{-\frac{(x+2)^2}{4t}} - 2e^{-\frac{x^2}{4t}} \right) + \frac{x+2}{4} \operatorname{erf} \left( \frac{x+2}{2c\sqrt{t}} \right) + \frac{x-2}{4} \operatorname{erf} \left( \frac{x-2}{2c\sqrt{t}} \right)$

3.  $u(x, t) = 100$

**X.**

$u(x, t) = (\operatorname{Heaviside}(x) - \operatorname{Heaviside}(x - t))e^{-bx}$