# COMPACTIFIABLE CLASSES AND THEIR COMPLEXITY

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I will present several results from the joint work with J. Bobok, J. van Mill, P. Pyrih, and B. Vejnar [1], [2].

We say that a class of continua $\mathcal{C}$ is *compactifiable* if there is a metrizable compactum whose set of components is equivalent to $\mathcal{C}$. (We call two classes of spaces equivalent if every member of one class has a homeomorphic copy in the other class and vice versa.) It turns out that a class of continua $\mathcal{C}$ is compactifiable if and only if there is a continuous map $q\colon A \to B$ between some metrizable compacta $A$ and $B$ such that the family of fibers $\{q^{-1}(b) : b \in B\}$ is equivalent to $\mathcal{C}$. This condition may be easily generalized, so we define compactifiable classes of compacta in the obvious way. We also define *Polishable classes by a weaker condition* – it is enough if the witnessing spaces $A$ and $B$ are Polish. Moreover, we define *strongly compactifiable* and *strongly Polishable* classes by the extra requirement that the map $q$ is closed and open. The motivation for these modified notions is their close connection to hyperspaces.

Since the Hilbert cube $[0,1]^\omega$ is universal for metrizable compacta, every class of compacta may be realized by an equivalent family $\mathcal{F} \subseteq \mathcal{K}([0,1]^\omega)$. Having such a realization, we may talk about its topological properties and about its complexity with respect to the Borel hierarchy. We have proved that a class of metrizable compacta $\mathcal{C}$ is strongly compactifiable if and only if it can be realized by a closed family or equivalently by an $F_\sigma$ family, and that $\mathcal{C}$ is strongly Polishable if and only if it can be realized by a $G_\delta$ family or equivalently by an analytic family.

## References

[1] A. Bartoš, J. Bobok, J. van Mill, P. Pyrih, B. Vejnar, *Compactifiable classes of compacta*, submitted to Topology Appl., arXiv:1801.01826.

[2] Adam Bartoš, *Borel complexity up to the equivalence*, submitted to Fund. Math., arXiv:1812.00484.

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