THE UNIFORM BOUNDEDNESS PRINCIPLE FOR (ULTRA)FILTERS

BEN DE BONDT AND HANS VERNAEVE

Let X be an infinite dimensional Banach space. We formulate for each filter $\mathcal{F}$ on the set of natural numbers a corresponding uniform boundedness principle (UBP) in X, which can either hold true or fail, depending on the properties of the filter $\mathcal{F}$. For the Fréchet filter of cofinite sets, this UBP follows from the classic Banach-Steinhaus theorem. More surprisingly, it follows from a theorem by Benedikt [1] that this UBP holds as well for selective ultrafilters. We will discuss recent work together with Hans Vernaeve which gives a combinatorial characterization of those filters $\mathcal{F}$ for which this filter version of the uniform boundedness principle holds.

References

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(Ben De Bondt) Ghent University, Department of Mathematics: Analysis, Logic and Discrete Mathematics, Krijgslaan 281, S8, 9000 Ghent, Belgium
E-mail address, Ben De Bondt: `ben.debondt@ugent.be`

(Hans Vernaeve) Ghent University, Department of Mathematics: Analysis, Logic and Discrete Mathematics, Krijgslaan 281, S8, 9000 Ghent, Belgium
E-mail address, Hans Vernaeve: `hans.vernaeve@ugent.be`

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