

ON A SEMITOPOLOGICAL EXTENDED BICYCLIC SEMIGROUP WITH ADJOINED ZERO

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The *bicyclic monoid* $\mathcal{C}(p, q)$ is the semigroup with the identity 1 generated by two elements p and q subject only to the condition $pq = 1$. The *extended bicyclic semigroup* $\mathcal{C}_{\mathbb{Z}}$ was introduced in [7] and it is a generalizations of the bicyclic monoid.

The following dichotomy for the bicyclic monoid with adjoined zero $\mathcal{C}^0 = \mathcal{C}(p, q) \sqcup \{\mathbf{0}\}$ was proved in [3]: every Hausdorff locally compact semitopological bicyclic monoid with adjoined zero \mathcal{C}^0 is either compact or discrete. The above dichotomy was extended in [1] to locally compact λ -polycyclic semitopological monoids, in [2] to locally compact semitopological graph inverse semigroups in [5] to locally compact semitopological interassociates of the bicyclic monoid with an adjoined zero, to other generalizations of the bicyclic monoid with adjoined zero in [6], and they are extended in [4] to locally compact semitopological 0-bisimple inverse ω -semigroups with compact maximal subgroups.

We show that every Hausdorff locally compact semigroup topology on the extended bicyclic semigroup with zero $\mathcal{C}_{\mathbb{Z}}^0$ is discrete, but on $\mathcal{C}_{\mathbb{Z}}^0$ there exist \mathfrak{c} many Hausdorff locally compact non-compact shift-continuous topologies. Also, we construct minimal shift-continuous, minimal semigroup and minimal inverse semigroup topologies on $\mathcal{C}_{\mathbb{Z}}^0$ and establish their property.

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