

SMALL $u(\kappa)$ FOR A SINGULAR κ WITH COMPACTNESS AT
 κ^{++}

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We study the question whether the compactness principles at λ^+ have a non-trivial effect on the generalized cardinal invariants in the neighbourhood of λ (for instance at λ or the cardinal predecessor of λ unless λ is a limit cardinal).

As a first result in this direction, we show that it is consistent that there is a singular strong limit cardinal κ (with countable or uncountable cofinality) such that $u(\kappa) = \kappa^+$, $2^\kappa > \kappa^+$, and the tree property, stationary reflection and the failure of approachability hold at κ^{++} . The proof is based on the methods of [1] and [2] for the small ultrafilter number $u(\kappa)$ and [3] for the tree property argument. The result will soon appear as a preprint, see [4].

REFERENCES

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