SPACES OF SMALL CELLULARITY HAVE NOWHERE CONSTANT CONTINUOUS IMAGES OF SMALL WEIGHT

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We call a continuous map $f : X \to Y$ *nowhere constant* if it is not constant on any non-empty open subset of its domain X. Clearly, this is equivalent with the assumption that every fiber $f^{-1}(y)$ of f is nowhere dense in X. We call the continuous map $f : X \to Y$ *pseudo-open* if for each nowhere dense $Z \subset Y$ its inverse image $f^{-1}(Z)$ is nowhere dense in X. Clearly, if Y is crowded, i.e. has no isolated points, then f is nowhere constant.

How "small" nowhere constant, resp. pseudo-open continuous images can "large" spaces have? We give two answers to these questions, both of them involve the cardinal function $\widehat{c}(X)$, the "hat version" of cellularity, defined as the smallest cardinal κ such that there is no κ-sized disjoint family of open sets in X. (Thus, for instance, $\widehat{c}(X) = \omega_1$ means that X is CCC.)

THEOREM A. Any crowded Tychonov space X has a crowded Tychonov nowhere constant continuous image Y of weight $w(Y) \le \widehat{c}(X)$. Moreover, in this statement $\le$ may be replaced with $<$ iff there are no $\widehat{c}(X)$-Suslin lines (or trees).

THEOREM B. Any crowded Tychonov space X has a crowded Tychonov pseudo-open continuous image Y of weight $w(Y) \le 2^{<\widehat{c}(X)}$.

The latter result is sharp, at least consistently, because if Martin's axiom holds then there is a CCC crowded Tychonov space X such that for any crowded Hausdorff pseudo-open continuous image Y of X we have $w(Y) \ge \mathfrak{c}\,(= 2^{<\omega_1})$.

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