NON "COMPLETE" CASE OF LOUVEAU-SIMPSON THEOREM

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In 1982 A. Louveau and S. G. Simpson proved the essential theorem on functions having the Baire property from the Ellentuck space:

Theorem 1 ([4]) *Let X be a metric space and let $f\colon [\omega]^\omega \to X$ be such a mapping that the inverse image of any open set is completely Ramsey. Then there is an infinite subset T of ω such that $f([\omega]^\omega)$ is separable.*

Unfortuntely, their result was restricted to complete spaces. Our results shows that this restriction can be erased, more precisely we showed that

Theorem 2 ([2]) *Let Y be a metric space and M be a non-completely Ramsey subset in Ellentuck space $[\omega]^\omega$. Let $f\colon M \to Y$ be such a mapping that $f^{-1}(U)$ is completely Ramsey for any open set $U \subset Y$. Then there is an infinite set $T \subset \omega$ such that $f([T]^\omega \cap M)$ is separable.*

This result is strictly connected with the following theorem

Theorem 3 ([2]) *Let M be a non-completely Ramsey subset in Ellentuck space. and let $\mathcal{F}$ be a partition of M into nowhere Ramsey sets. Then there is $\mathcal{F}' \subset \mathcal{F}$ such that $\bigcup \mathcal{F}'$ is not completely Ramsey.*

The research around partitions of structures onto "small" sets was started by K. Kuratowski in 1935 and continnued by L. Bukovský with using advanced set-theoretical methods, (see [1]).

The methods used for proving Theorem 2 and Theorem 3 allows to get a dip generalization of Kuratowski - Ryll-Nardzewski Theorem on selectors, (see e.g. [3, CH. XIV]) for Ellentuck structure and arbitrary metric space (not only Polish space as was originally).

This is the joint work with Ryszard Frankiewicz.

References

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