YET ANOTHER IDEAL VERSION OF THE BOUNDING NUMBER

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This is a joint work with Rafał Filipów.

Let $\mathcal{I}$ be an ideal on ω. Denote by $\mathcal{D}_{\mathcal{I}}$ the family of all functions $f \in \omega^\omega$ satisfying $f^{-1}[\{n\}] \in \mathcal{I}$ for all $n \in \omega$. Given $f, g \in \omega^\omega$, write $g \leq_{\mathcal{I}} f$ whenever $\{n \in \omega : g(n) > f(n)\} \in \mathcal{I}$.

We study two closely connected ideal versions of the bounding number:

$$\mathfrak{b}(\mathcal{D}_{\mathrm{Fin}}, \geq_{\mathcal{I}}) = \min\{|\mathcal{F}| : \mathcal{F} \subseteq \mathcal{D}_{\mathrm{Fin}} \wedge \neg(\exists g \in \mathcal{D}_{\mathrm{Fin}}\, \forall f \in \mathcal{F}\, (g \leq_{\mathcal{I}} f))\};$$

$$\mathfrak{b}(\mathcal{D}_{\mathcal{I}}, \geq_{\mathcal{I}}) = \min\{|\mathcal{F}| : \mathcal{F} \subseteq \mathcal{D}_{\mathcal{I}} \wedge \neg(\exists g \in \mathcal{D}_{\mathcal{I}}\, \forall f \in \mathcal{F}\, (g \leq_{\mathcal{I}} f))\}.$$

It is known that $\mathfrak{b}(\mathcal{D}_{\mathrm{Fin}}, \geq_{\mathrm{Fin}}) = \mathfrak{b}$. We study those two invariants in the case of nice ideals (ideals with the Baire property, coanalytic ideals, P-ideals, etc.) as well as show some consistency results distinguishing $\mathfrak{b}$, $\mathfrak{b}(\mathcal{D}_{\mathrm{Fin}}, \geq_{\mathcal{I}})$ and $\mathfrak{b}(\mathcal{D}_{\mathcal{I}}, \geq_{\mathcal{I}})$.

Although the topic is interesting itself, we are also motivated by the studies of ideal versions of QN-spaces, as $\mathfrak{b}(\mathcal{D}_{\mathrm{Fin}}, \geq_{\mathcal{I}})$ and $\mathfrak{b}(\mathcal{D}_{\mathcal{I}}, \geq_{\mathcal{I}})$ describe uniformity numbers of such spaces. This topic is intensively studied by Lev Bukovský and his group.

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