CLOSED HEREDITARY COREFLECTIVE SUBCATEGORIES IN CERTAIN CATEGORIES OF TOPOLOGICAL SPACES

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Denote by Tych (ZD) the category of all Tychonoff (zero-dimensional) spaces. Let A be an epireflective subcategory of the category Top of all topological spaces such that ZD ⊆ A ⊆ Tych. Our goal is to describe closed hereditary coreflective subcategories of A.

Let α be a regular cardinal. By Top(α) we denote the subcategory of Top consisting of such spaces X that if U is a non-empty family of open subsets of X with |U| < α, then the intersection \( \bigcap_{U \in \mathcal{U}} U \) is open in X. The subcategories Top(α) ∩ A are closed hereditary and coreflective in A.

Let C(α) be the space on the set \( \alpha \cup \{\alpha\} \) such that a subset U is open in C(α) if and only if \( \alpha \notin U \) or \( |\alpha \setminus U| < \alpha \). In [1] we showed that if MA (Martin’s Axiom) holds and measurable cardinals do not exist, then the closed hereditary coreflective hull of the space C(\( \omega_0 \)) in A is the whole category A.

In our talk we show that if MA holds and measurable cardinals do not exist, then the closed hereditary coreflective hull of the space C(α) in A is Top(α) ∩ A for any regular cardinal α. We obtain that if B is a closed hereditary coreflective subcategory of A such that B ≠ Top(α) ∩ A and B ≠ Dis (the category of all discrete spaces), then B consists only of sums of connected spaces. Hence, the only closed hereditary coreflective subcategories of ZD are Dis and Top(α) ∩ ZD, where α is a regular cardinal.

References


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