

# DIVISIBILITY IN $\beta N$ AND $*N$

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We investigate a quasiorder  $\tilde{|}$  on the set  $\beta N$  of ultrafilters on the set  $N$  of natural numbers that is a natural extension of the divisibility relation on  $N$ . The "lower" ultrafilters of the  $\tilde{|}$ -hierarchy are nicely organized in  $\omega$ -many levels, resembling divisibility on  $N$ . Above these levels the situation is more complex. The connection of  $\beta N$  with nonstandard extensions  $*N$  of  $N$  via monads of ultrafilters proves to be useful in finding out more about the relation  $\tilde{|}$ , as it turns out that two ultrafilters are divisible if and only if there are divisible hypernatural numbers in their respective monads. Some recent results on this connection by Di Nasso and Luperi Baglini provide another useful tool.

## REFERENCES

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