CONVERGENCE OF MEASURES ON MINIMALLY GENERATED BOOLEAN ALGEBRAS

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Minimally generated Boolean algebras were introduced and investigated by Koppelberg in 1989. Later on they were intensively used and studied in the context of the Efimov problem (Geschke, Dow and Pichardo-Mendoza, Dow and Shelah), measure theory (Borodulin-Nadzieja) or both (Džamonja and Plebanek). During my talk we will follow the latter way.

It is already a folklore fact that each minimally generated Boolean algebra $\mathcal{A}$ admits a sequence of signed finite measures $\big(\mu_n\big)$ for which the integrals $\int_{St(\mathcal{A})} f d\mu_n$ converge to 0 for every continuous real-valued function f on the Stone space $St(\mathcal{A})$ but not for every bounded Borel real-valued function (i.e. $\mathcal{A}$ does not have the Grothendieck property). We will show that this fact can always be witnessed even by a sequence of measures being a finite linear combination of point measures. Assuming CH, we will also provide an example of a minimally generated Boolean algebra $\mathcal{A}$ having the property that for every sequence of measures $\big(\mu_n\big)$ on $\mathcal{A}$ such that the integrals $\int_{St(\mathcal{A})} 1_A d\mu_n$ $(= \mu_n(A))$ converge to 0 for every $A \in \mathcal{A}$, the integrals $\int_{St(\mathcal{A})} f d\mu_n$ converge to 0 for every $f \in C(St(\mathcal{A}))$, too (i.e. $\mathcal{A}$ has the Nikodym property). The Stone space of such a minimally generated Boolean algebra must necessarily be an Efimov space (but we will show that under CH the converse does not hold). The existence of $\mathcal{A}$ has important measure-theoretic consequences.

This is a joint work with Lyubomyr Zdomskyy.

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