CARDINAL SEQUENCES AND UNIVERSAL SPACES

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If X is a locally compact, scattered Hausdorff (in short: LCS) space, we let $CS(X)$ denote the cardinal sequence of X, i.e. the sequence of the cardinalities of the Cantor Bendixson levels of X.

If α is an ordinal, let $\mathcal{C}(\alpha)$ denote the class of all cardinal sequences of LCS spaces of height α and put

$$\mathcal{C}_\lambda(\alpha) = \{s \in \mathcal{C}(\alpha) : s(0) = \lambda \wedge \forall \beta < \alpha \; s(\beta) \geq \lambda\}.$$

Given a family $\mathcal{C}$ of sequences of cardinals we say that an LCS space X is *universal for* $\mathcal{C}$ if $CS(X) \in \mathcal{C}$, and for each $s \in \mathcal{C}$ there is an open subspace $Y \subset X$ with $CS(Y) = s$.

Constructing universal spaces we will prove theorems claiming that certain $\mathcal{C}_\lambda(\alpha)$-classes are quite rich in elements. For example, we can prove the following generalization of a classical result of Baumgartner and Shelah:

Theorem. (Martinez, S, [3]) It is consistent, that 2^ω is as large as you wish and for each $\delta < \omega_3$ we have

$$\{f \in {}^{\delta}([\omega, 2^\omega] \cap Card) : f(\alpha) = \omega \text{ whenever } \alpha = 0 \text{ or } cf(\alpha) = \omega_2\} \subset \mathcal{C}_\omega(\delta).$$

References

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[3] J. C. Martinez , L. Soukup , *On cardinal sequences of length* $< \omega_3$, Top. Appl, to appear.

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