DISCRETE SETS, CELLULAR FAMILIES AND THE LINDELÖF PROPERTY

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Juhász, Tkachuk and Wilson [4] define a space to be *almost discretely Lindelöf* if every discrete set can be covered by a Lindelöf subspace. Inspired by Arhangel'skii's Theorem on the cardinality of Lindelöf Hausdorff first-countable spaces, they ask whether the cardinality of every Hausdorff first-countable almost discretely Lindelöf space is bounded by the continuum (see [4] and also [3]). We will give a consistent answer to their question. As a matter of fact, we will show that the cardinality of a sequential almost discretely Lindelöf space with points G_δ is bounded by the continuum under $2^{<\mathfrak{c}} = \mathfrak{c}$.

A space X is *cellular-Lindelöf* if for every family of pairwise disjoint non-empty open sets $\mathcal{U}$ there is a Lindelöf subspace $L \subset X$ which meets every member of $\mathcal{U}$. The class of cellular-Lindelöf spaces contains both almost discretely Lindelöf spaces and spaces with the countable chain condition. It's an open question whether the cardinality of a cellular-Lindelöf first-countable Hausdorff space is bounded by the continuum. A positive answer would lead to a common generalization of two seemingly unrelated fundamental results in the theory of cardinal functions: Arhangel'skii's Theorem and the Hajnal-Juhász bound on the cardinality of first-countable ccc spaces. We will show that the answer is positive for normal spaces under $2^{<\mathfrak{c}} = \mathfrak{c}$ and, time permitting, present various other results about the classes of almost discretely Lindelöf and cellular-Lindelöf spaces.

References

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