CLUB STATIONARY REFLECTION AND THE TREE PROPERTY

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It is known that the consistency of the tree property and the stationary reflection at $\aleph_2$ can be obtained from a weakly compact cardinal. More precisely, if λ is weakly compact, then forcing with the Mitchell poset $\mathbb{M}(\aleph_0, \lambda)$ produces a model in which $\lambda = \aleph_2$, there are no $\aleph_2$-Aronszajn trees and every stationary subset of $\aleph_2 \cap \mathrm{cof}(\aleph_0)$ reflects at a point of $\aleph_1$.

We improve this result by showing that it is consistent with the tree property that every stationary subset S of $\aleph_2 \cap \mathrm{cof}(\aleph_0)$ reflects in a stronger sense, i.e. there is a club $C \subseteq \aleph_2$ such that every point of C of cofinality $\aleph_1$ is a reflection point for S (this stronger form of reflection was introduced by [2]). We use the optimal consistency assumption of a weakly compact cardinal.

Our method is based on forcing with an iteration of club shooting posets after forcing with the Mitchell forcing, and hence it represents a progress with regard to the more general question of which kinds of forcing preserve the tree property. The result will appear soon [1].

References

[1] Maxwell Levine and Šárka Stejskalová. *Club stationary reflection and the tree property*, To be submitted.

[2] Menachem Magidor. *Reflecting stationary sets*, The Journal of Symbolic Logic, 47(4):755-751, 1982.

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