ON $\mathcal{S}$-APPROXIMATELY CONTINUOUS FUNCTIONS

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It is well known that a point $x_0 \in \mathbb{R}$ is a density point of a Lebesgue measurable set A if

$$\lim_{h \to 0^+} \frac{\lambda(A \cap [x_0 - h, x_0 + h])}{2h} = 1,$$

where $\lambda(A)$ denote the Lebesgue measure of a measurable set A.

In the paper [1] is presented generalization of notion of density point. By $\mathcal{S}$ we will denote a **sequence** of sets with positive Lebesgue measure $\{S_n\}_{n \in \mathbb{N}}$ **tending to zero**, that means $\operatorname{diam}\{S_n \cup \{0\}\} \underset{n\to\infty}{\longrightarrow} 0$.

We shall say that a point $x_0 \in \mathbb{R}$ is a $\mathcal{S}$–**density point** of a set $A \in \mathcal{L}$, if

$$\lim_{n \to \infty} \frac{\lambda(A \cap (S_n + x_0))}{\lambda(S_n)} = 1.$$

Let

$$\Phi_{\mathcal{S}}(A) = \{x \in \mathbb{R} : x \text{ is a } \mathcal{S}\text{-density point of } A\}$$

$$\mathcal{T}_S = \{A \in \mathcal{L} : A \subset \Phi_{\mathcal{S}}(A)\}.$$

Then family $\mathcal{T}_S$ contains topology natural topology.

For sequnce $\mathcal{S}$ of measurable sets tending to zero we consider four families of continuous functions defined as follows:

$$\begin{aligned}
\mathcal{C}_{nat,nat} =& \{f : (\mathbb{R}, \mathcal{T}_{nat}) \to (\mathbb{R}, \mathcal{T}_{nat})\},\\
\mathcal{C}_{nat,\mathcal{S}} =& \{f : (\mathbb{R}, \mathcal{T}_{nat}) \to (\mathbb{R}, \mathcal{T}_{S})\},\\
\mathcal{C}_{\mathcal{S},nat} =& \{f : (\mathbb{R}, \mathcal{T}_{S}) \to (\mathbb{R}, \mathcal{T}_{nat})\},\\
\mathcal{C}_{\mathcal{S},\mathcal{S}} =& \{f : (\mathbb{R}, \mathcal{T}_{S}) \to (\mathbb{R}, \mathcal{T}_{S})\}.
\end{aligned}$$

The aim of the presentation are the properties of continuous functions equipped with the $\mathcal{S}$–density topology or natural topology in the domain or the range.

References

[1] F. Strobin, R. Wiertelak, *On a generalization of density topologies on the real line*, Topology Appl. 199 (2016), 1–16.

[2] W. Wilczyński, *Density topologies*, Handbook of Measure Theory, Ed. E. Pap. Elsevier, chapter 15(2002), 675-702.

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